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A new time-discretization for delay multiple-input nonlinear systems using the Taylor method and first order hold

Yuanliang Zhang [a], Olga Kostyukova [b], Kil To Chong [c,*]

[a] School of Mechanical Engineering, Huaihai Institute of Technology, Lianyungang, Jiangsu, China
[b] Institute of Mathematics National Academy of Science of Belarus, Minsk 220072, Belarus
[c] Institute of Information and Communication Chonbuk National University Chonju, 561-765, Republic of Korea

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ABSTRACT

A new discretization method is proposed for multi-input-driven nonlinear continuous systems with time-delays, based on a combination of the Taylor series expansion and the first-order hold (FOH) assumption. The mathematical structure of the new discretization scheme is explored. On the basis of this structure, the sampled-data representation of the time-delayed multi-input nonlinear system is derived. First the new approach is applied to nonlinear systems with two inputs, and then the delayed multi-input general equation is derived. The resulting time discretization method provides a finite-dimensional representation for multi-input nonlinear systems with time-delays, thereby enabling the application of existing controller design techniques to such systems. The performance of the proposed method is evaluated using a nonlinear system with time-delays (maneuvering an automobile). Various sampling rates, time-delay values and control inputs are considered to evaluate the proposed method. The results demonstrate that the proposed discretization scheme can meet the system requirements even when using a large sampling period with precision limitations. The discretization results of the FOH method are also compared with those of the zero order hold (ZOH) method. The precision of the FOH method in the discretization procedure combined with the Taylor series expansion is much higher than that of the ZOH method except in the case of constant inputs.

1. Introduction

Developing and evolving technologies that use the Internet are creating more interest in control systems that have time-delays. The convergence of communication and computation in control systems and the complex behavior of control systems with non-negligible time-delays are the two main motivations for the special attention that is being given to the study of the effects of time-delays. The presence of delays makes system analysis and control much more complicated [25]. Control systems with time-delays exhibit complex behavior because of their infinite dimensionality. Even in the case of linear time-invariant systems with constant time-delays in the input or in the states but has infinite dimensionality if expressed in the continuous time domain. The time-delay factors have, by and large, counteracting effects on the system behavior and most of the time this leads to poor performance. Therefore, the subject of Time-Delay Systems (TDS) has been investigated in the

* Corresponding author. Tel.: +82 63 270 2478; fax: +82 63 270 2451.
E-mail address: kitchong@chonbuk.ac.kr (K.T. Chong).

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form of functional differential equations over the past three decades. This has occupied a separate discipline in mathematical sciences falling between differential and difference equations [4].

The engineering literature dealing with time-delayed systems is very extensive. Most of the approaches proposed so far deal with linear time-delay control systems and, in particular, with the stability analysis and behavior of such systems with constant and/or uncertain time delays [8,5,9,18,23]. Quite recently and on the nonlinear front, nonlinear controllers were systematically synthesized for multivariable nonlinear systems in the presence of sensor and actuator dead time [22]. Time-delays are often encountered in various engineering systems, such as chemical processes, hydraulic actuators, and rolling mill systems, and the time-delays are frequently a source of instability. Many of these models are also significantly nonlinear which motivates research into the control of nonlinear systems with time-delays. A natural direction is to try to extend the ideas and results of nonlinear non-delay control to systems with delay. Such results include input–output linearization and decoupling, partial feedback linearization with delay term domination, and extension of control Lyapunov functions (CLF) to delay systems in the form of control Lyapunov–Razumikhin functions (CLRF) [10].

In practice, nonlinear control strategies are usually implemented using a microcontroller or a digital signal processor. As a direct consequence of this, the associated control algorithms must operate using discrete time intervals. The time discretization is based either on a continuous-time control law design assuming a continuous-time system, or on a discrete-time control law designed for a continuous-time system that results in a discrete-time system. It is apparent that the second approach is attractive for dealing directly with the issue of sampling. The effect of sampling on the system–theoretic properties of a continuous-time system is very important because it strongly influences the ability to meet the design objectives. It should be emphasized that, in both design approaches, the time discretization of either the controller or the system model is necessary. Furthermore, in the controller design for time-delay systems, the first approach is troublesome because of the infinite-dimensional nature of the underlying system dynamics. As a result, the second approach becomes more desirable [16] and will be pursued in the present study. In particular, we extend the well-known procedure of time discretization of linear time-delay systems to nonlinear input-driven systems with constant time-delay [21].

The proposed discretization scheme is based on the Taylor series and it uses a similar mathematical framework to the previously developed one for delay-free nonlinear systems [12]–[20]. However, it should be mentioned that conventional numerical techniques, such as the Euler and Runge–Kutta methods, have been employed in order to obtain a sampled-data representation of the original continuous-time delay-free system [16]. All of these approaches require a "small" time step in order to be deemed accurate; however, this may not be the case in control applications where large sampling periods are inevitably introduced due to physical and technical limitations. A thorough but non-exhaustive sample of other approaches of notable significance, yet with certain associated practical limitations, is reported in [15]–[6], and solid theoretical results based on the direct use of discrete time approximations in the control of sampled-data nonlinear systems can be found in [17]–[24]. The effect of this approach on the system–theoretic properties of nonlinear systems, such as their equilibrium properties, relative order, stability, zero dynamics, and minimum-phase characteristics has also been studied and it reveals the natural and transparent manner in which Taylor methods permeate the relevant theoretical aspects of such systems [21].

Moreover, the sampling period can be selected after the analogue control system is designed; hence the continuous-time closed-loop bandwidth is known. The performance of this method is significantly affected by the discretization method and the sampling interval that are chosen. Standard methods such as bilinear transformation often require a high sampling rate to achieve adequate performance and retain closed-loop stability. In certain cases, however, the sampling rate is constrained by either the computational speed of the microprocessor for digital control or by the measurement scheme, and it must be set to a low value [3].

In these large sample period systems, the Taylor series method was used to improve the performance of the controller [11]. However, in previous papers, the Zero-Order Hold (ZOH) assumption was used in the discretization method. The performance of the ZOH is highly dependent on the input signal, and the sample time should be short enough for the desired control precision.

A high-order method is one that provides increased accuracy with only a modest increase in the computational cost [2,7,1]. Except for the square wave and unit step input signals, the ZOH assumption no longer assures good control performance when a large sampling interval is adopted. Therefore, the First-Order Hold (FOH) assumption is introduced in this paper to enhance the performance in situations where a large sampling interval is unavoidable.

In particular, the present study aims to develop a new method for the time discretization of multi-input-driven nonlinear dynamic systems with time delay, based on the Taylor series and FOH assumption. This discretization method can provide a finite dimensional representation, which allows the direct application of existing nonlinear control system design techniques. Secondly, the performance evaluation of the proposed algorithm is presented using a numerical simulation with different sampling periods, time-delays and inputs, and the results are compared to those obtained by using the ZOH method.

This paper is organized as follows. Section 2 contains a brief comparative analysis of ZOH and FOH assumptions and Section 3 presents the time discretization method for delay-free nonlinear systems. Section 4 includes the main results of this paper, in which a new time-discretization method for multi-input nonlinear systems with time-delays is introduced. Finally, numerical simulations with varying sampling periods, time-delays, and kinds of inputs are presented in Section 5 to demonstrate the effectiveness of the proposed discretization scheme while Section 6 provides a few concluding remarks drawn from this study.

2. ZOH and FOH assumptions

Nonlinear continuous-time control systems with time-delayed single-input are considered with a state–space representation of the form:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t)) + g(x(t))u(t-D) \tag{1}$$

where $x \in X \subset R^n$ is the vector of the states representing an open and connected set, $u \in R$ is the input variable and D is the system's constant time-delay (dead-time) that directly affects the input. It is assumed that $f(x)$ and $g(x)$ are real analytic vector fields on X.

An equidistant grid on the time axis with mesh $T = t_{k+1} - t_k > 0$ is considered where sampling interval is $[t_k, t_{k+1}) = [kT, (k+1)T)$ and T is the sampling period.

Furthermore, we suppose the time-delay D and mesh T are related as follows

$$D = qT + \gamma \tag{2}$$

where $q \in \{0, 1, 2, \ldots\}$ and $0 \leq \gamma < T$. Equivalently, the time-delay D is customarily represented as an integer multiple of the sampling period plus a fractional part of T [16]–[21].

In this paper it is also assumed that an original piecewise continuous input function is approximated by a piecewise linear one or, in other words, we assume that system (1) is driven by an input that is piecewise linear over the sampling interval, i.e. the FOH assumption holds true.

Remember that, for the ZOH assumption, it is assumed that the original piecewise continuous input function is approximated by a piecewise constant one, i.e. it is assumed that system (1) is driven by an input that is piecewise constant over the sampling interval.

Let us consider ZOH and FOH assumptions in more detail.

Under ZOH assumption, in the delay free case, while $D = 0$, we have

$$u_Z(t) = v(k) = \text{constant}, \quad \text{for } kT \leq t < kT + T \tag{3}$$

where $v(k) := u(kT + 0)$. Based on the above notation one can deduce that for $D > 0$ the "delayed" input variable attains the following two distinct values within the sampling interval [11]:

$$u_Z(t-D) = \begin{cases} v(k-q-1) & \text{if } kT \leq t < kT + \gamma, \\ v(k-q) & \text{if } kT + \gamma \leq t < kT + T. \end{cases} \tag{4}$$

Here subscript Z denotes that input approximation is performed under ZOH assumption.

Under the FOH assumption, while $D = 0$, we have

$$u_F(t) = v(k) + \Delta v(k)(t - kT) \quad \text{for } kT \leq t \leq kT + T. \tag{5}$$

Here and in what follows:

$$\begin{aligned} & v(k) := u(kT+0), \\ & \Delta v(k) := \frac{u((k+1)T - 0) - u(kT+0)}{T} \quad \text{or} \quad \Delta v(k) := \frac{\mathrm{d}u(kT+0)}{\mathrm{d}t}, \quad k = 0, \pm 1, \pm 2, \ldots. \end{aligned} \tag{6}$$

For $D > 0$, it is rather straightforward to verify that the "delayed" input variable attains the following values within the sampling interval

$$u_F(t-D) \equiv \begin{cases} v(k-q-1) + \Delta v(k-q-1)[t - kT + (T-\gamma)], & t \in [kT, kT+\gamma), \\ v(k-q) + \Delta v(k-q)[t - kT - \gamma], & t \in [kT+\gamma, kT+T). \end{cases} \tag{7}$$

Here subscript F denotes that input approximation is performed under FOH assumption.

It follows from the relations presented above that input function $u_F(t), t \geq 0$, obtained under the FOH assumption approximates the original input (control) function $u(t), t \geq 0$, better than the corresponding function $u_Z(t), t \geq 0$, obtained under the ZOH assumption.

In fact, under the FOH assumption, for every the sampling interval, we have more parameters for approximation than under the ZOH one: there are two parameters $v(k)$ and $\Delta v(k)$ in (5) while there is only one parameter $v(k)$ in (2). Notice that (2) and (4) are particular cases of (5) and (7) respectively: putting $\Delta v(k) = 0, k = 0, 1, \ldots$ in (5) and (7), we immediately get (2) and (4).

Suppose that $D = 0$. and original control function $u(t)$, $t \geq 0$, is continuous over the sampling interval $t \in (kT, kT + T)$. Then we have

$$\begin{aligned}|u(t) - u_Z(t)| &= |u(kT + \Delta t) - u(kT + 0)| \\ &= \left| u(kT + 0) + \frac{\mathrm{d}u(kT + 0)}{\mathrm{d}t}\Delta t + o(\Delta t) - u(kT + 0) \right| = O(\Delta t) \leq O(T), \\ |u(t) - u_F(t)| &= |u(kT + \Delta t) - u(kT + 0) - \Delta v(k)\Delta t| \\ &= \left| u(kT + 0) + \frac{\mathrm{d}u(kT + 0)}{\mathrm{d}t}\Delta t + o(\Delta t) - u(kT + 0) - \Delta v(k)\Delta t \right| \\ &= \left| \frac{\mathrm{d}u(kT + 0)}{\mathrm{d}t}\Delta t - \Delta v(k)\Delta t \right| + o(\Delta t) = o(\Delta t) \leq o(T),\end{aligned}$$

where $\Delta t = t - kT$ and $\Delta v(k)$ is defined in (6). Similar estimations can be obtained and for $D > 0$.

Based on these facts we can conclude that the precision of the approximation $u_F(t)$, $t \geq 0$, is higher than that of the approximation $u_Z(t)$, $t \geq 0$. Hence we may expect that a discretization based on the FOH assumption will be better than a discretization that based on the ZOH one.

3. Discretization of nonlinear systems with delay-free single input

At this point, it would be methodologically appropriate to succinctly present and delineate the time-discretization method available for delay-free ($D = 0$) nonlinear control systems, that is based on the Taylor series with the ZOH assumption and reported in [13] and the discretization method that is based on the Taylor series with the FOH assumption. The ensuing brief description of the time discretization methods for delay-free nonlinear systems will serve as a natural point of departure for the development of a discretization scheme for multi-input time-delay nonlinear systems.

Initially, delay-free ($D = 0$) nonlinear control systems are considered with a state–space representation of the form,

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t)) + g(x(t))u(t). \tag{8}$$

Within the sampling interval and under the ZOH assumption, the solution of (8) is expanded in a uniformly convergent Taylor series [14]:

$$x((k+1)T) = x(kT) + \sum_{l=1}^{\infty} \frac{T^l}{l!} \left. \frac{\mathrm{d}^l x(t)}{\mathrm{d}t^l} \right|_{t=kT} = x(kT) + \sum_{l=1}^{\infty} B^{[l]}(x(kT), v(k)) \frac{T^l}{l!}$$

where $x(kT)$ is the value of the state vector x at time $t = kT$ and $B^{[l]}(x, u)$ are determined recursively by:

$$B^{[1]}(x, u) = f(x) + ug(x), \qquad B^{[l+1]}(x, u) = \frac{\partial B^{[l]}(x, u)}{\partial x}(f(x) + ug(x)) \quad \text{with } l = 1, 2, 3, \ldots.$$

Based on the Taylor series with the FOH assumption and within the sampling interval, the system (8) can be discretized in the following form

$$x((k+1)T) = x(kT) + \sum_{\ell=1}^{\infty} A^{[\ell]}(x(kT), v(k), \Delta v(k)) \frac{T^\ell}{\ell!} \tag{9}$$

where $A^{[\ell]}(x, u, \Delta u)$ are determined recursively by:

$$\begin{aligned} &A^{[1]}(x, u, \Delta u) = f(x) + ug(x), \\ &A^{[\ell+1]}(x, u, \Delta u) = \frac{\partial A^{[\ell]}(x, u, \Delta u)}{\partial x} A^{[1]}(x, u, \Delta u) + \frac{\partial A^{[\ell]}(x, u, \Delta u)}{\partial u} \Delta u \end{aligned} \tag{10}$$

where $\ell = 1, 2, 3, \ldots$. Notice that and the resulting coefficients $A^{[\ell]}(x(kT), v(k), \Delta v(k))$ can be easily computed by taking successive partial derivatives of the right hand-side of (8).

Therefore, an exact sampled-data representation (ESDR) of (8) can be derived by retaining the full infinite series of (9),

$$x((k+1)T) = \Phi_T(x(kT), v(k), \Delta v(k)) := x(kT) + \sum_{\ell=1}^{\infty} A^{[\ell]}(x(kT), v(k), \Delta v(k)) \frac{T^\ell}{\ell!}. \tag{11}$$

Simultaneously, an approximate sampled-data representation (ASDR) of Eq. (8) is obtained from a truncation of the Taylor series order N,

$$x^N((k+1)T) = \Phi_T^N(x(kT), v(k), \Delta v(k)) := x(kT) + \sum_{\ell=1}^{N} A^{[\ell]}(x(kT), v(k), \Delta v(k)) \frac{T^\ell}{\ell!} \tag{12}$$

where the subscript T of the mapping Φ_T^N denotes the dependence on the sampling period, and the superscript N denotes the finite series truncation order associated with the ASDR of Eq. (12).

It is important to observe that the ESDR of (8) (see Eq. (11)) represents the nonlinear analogue of the exact discretization scheme available for linear systems under the FOH assumption. Indeed, consider the linear delay-free control system with a state–space representation of the form:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = Ax(t) + bu(t), \tag{13}$$

where A and b are constant matrices of appropriate dimensions. Applying the Cauchy formula to Eq. (13) within the sampling interval and under the FOH assumption results in,

$$\begin{aligned} x((k+1)T) &= \mathrm{e}^{AT}x(kT) + \int_{kT}^{kT+T} \mathrm{e}^{A(kT+T-\tau)}bu(\tau)\mathrm{d}\tau = \mathrm{e}^{AT}x(kT) + \int_0^T \mathrm{e}^{A(T-t)}bu(kT+t)\mathrm{d}t \\ &= \mathrm{e}^{AT}x(kT) + \int_0^T \mathrm{e}^{A(T-t)}b(v(k) + \Delta v(k)t)\mathrm{d}t \end{aligned} \tag{14}$$

where the exponential matrix is defined through the uniformly convergent power series:

$$\mathrm{e}^{At} \equiv \sum_{\ell=0}^{\infty} \frac{A^{\ell}t^{\ell}}{\ell!}. \tag{15}$$

Substituting (15) in (14), we get

$$\begin{aligned} x((k+1)T) &= \sum_{\ell=0}^{\infty} \frac{A^{\ell}T^{\ell}}{\ell!}x(kT) + \sum_{\ell=0}^{\infty} \frac{A^{\ell}b}{\ell!} \int_0^T (T-t)^{\ell}(v(k) + \Delta v(k)t)\mathrm{d}t \\ &= x(kT) + (Ax(kT) + bu(k))T + \sum_{\ell=0}^{\infty} A^{\ell}(A^2x(kT) + Abv(k) + b\Delta v(k))\frac{T^{\ell+2}}{(\ell+2)!}. \end{aligned} \tag{16}$$

It is easy to check that presentation (16) coincides with the ESDR (11) of the original linear continuous-time system (13).

4. Discretization of multi-input time-delay nonlinear systems

4.1. Discretization of single-input nonlinear time-delay nonlinear systems

Motivated by the delay-free approach described in Section 3, a similar line of thinking is adopted for the case $D > 0$ as well. Indeed, by applying the Taylor series discretization method for the nonlinear systems (1) over the subinterval $[Tk, Tk+\gamma)$ and taking into account (7), one can obtain the state vector evaluated at $Tk+\gamma$ as a function of $x(kT)$, $v(k-q-1)$, and $\Delta v(k-q-1)$

$$x(kT+\gamma) = x(kT) + \sum_{\ell=1}^{\infty} A^{[\ell]}(x(kT), v(k-q-1), \Delta v(k-q-1))\frac{\gamma^{\ell}}{\ell!} \tag{17}$$

where the subsequent calculation of the corresponding Taylor coefficients can be realized by means of the recursive Eq. (10).

Similarly, formula (7) and the application of the Taylor discretization method to the $[Tk+\gamma, Tk+T)$ subinterval yields the state vector evaluated at $(k+1)T$ as a function of $x(kT+\gamma)$, $v(k-q)$, and $\Delta v(k-q)$

$$x(kT+T) = x(kT+\gamma) + \sum_{\ell=1}^{\infty} A^{[\ell]}(x(kT+\gamma), v(k-q), \Delta v(k-q))\frac{(T-\gamma)^{\ell}}{\ell!}. \tag{18}$$

The ASDR of Eqs. (17) and (18) are obtained from the truncation of the Taylor series order N, as shown below,

$$x^N(kT+\gamma) = x(kT) + \sum_{\ell=1}^{N} A^{[\ell]}(x(kT), v(k-q-1), \Delta v(k-q-1))\frac{\gamma^{\ell}}{\ell!}, \tag{19}$$

$$x^N(kT+T) = x^N(kT+\gamma) + \sum_{\ell=1}^{N} A^{[\ell]}(x^N(kT+\gamma), v(k-q), \Delta v(k-q))\frac{(T-\gamma)^{\ell}}{\ell!}. \tag{20}$$

4.2. Discretization of nonlinear time-delay systems with two inputs

The time discretization method for the single input case can be expanded to the multi-input case. The discretization method of a general nonlinear system with multi-input delay is developed using the Taylor series expansion with the FOH assumption. A system with only two time-delayed inputs will be considered for simplicity in this subsection, and then the general case will be presented in next subsection.

A time-delayed two-input nonlinear continuous system can be expressed with the following state–space form

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t)) + u_1(t - D_1)g_1(x(t)) + u_2(t - D_2)g_2(x(t)). \tag{21}$$

Similar to (2), we assume that the delays of the inputs in Eq. (21) can be presented as follows: $D_1 = q_1T + \gamma_1$, $D_2 = q_2T + \gamma_2$, where D_1 and D_2 have the same meaning as D, γ_1 and γ_2 have the same meaning as γ, q_1 and q_2 have the same meaning as q.

Under the FOH assumption, the inputs over interval $[kT, kT + T)$ are as follows

$$u_i(t - D_i) = \begin{cases} v_i(k - q_i - 1) + \Delta v_i(k - q_i - 1)\,(t - kT + (T - \gamma_i)), & t \in [kT, kT + \gamma_i), \\ v_i(k - q_i) + \Delta v_i(k - q_i)\,(t - kT - \gamma_i), & t \in [kT + \gamma_i, kT + T), \end{cases} \quad i = 1, 2, \tag{22}$$

where

$$v_i(k) = u_i(Tk + 0),\ \Delta v_i(k) = \frac{u_i(Tk + T - 0) - u_i(Tk + 0)}{T}, \quad i = 1, 2.$$

It is convenient to assume that $\gamma_1 < \gamma_2$, case $\gamma_1 \geq \gamma_2$ is similar to this case.

Taking into account (22), we have

(i) For $t \in [kT, kT + \gamma_1)$
ESDR:

$$x(kT + \gamma_1) = x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), V_1(k), \Delta V_1(k))\frac{\gamma_1^l}{l!}.$$

ASDR:

$$x^N(kT + \gamma_1) = x(kT) + \sum_{l=1}^{N} A^l(x(kT), V_1(k), \Delta V_1(k))\frac{\gamma_1^l}{l!},$$

where

$$V_1(k) = (v_1(k - q_1 - 1), v_2(k - q_2 - 1)), \qquad \Delta V_1(k) = (\Delta v_1(k - q_1 - 1), \Delta v_2(k - q_2 - 1)).$$

(ii) For $t \in [kT + \gamma_1, kT + \gamma_2)$.
ESDR:

$$x(kT + \gamma_2) = x(kT + \gamma_1) + \sum_{l=1}^{\infty} A^l(x(kT + \gamma_1), V_2(k), \Delta V_2(k))\frac{(\gamma_2 - \gamma_1)^l}{l!}.$$

ASDR:

$$x^N(kT + \gamma_2) = x^N(kT + \gamma_1) + \sum_{l=1}^{N} A^l(x^N(kT + \gamma_1), V_2(k), \Delta V_2(k))\frac{(\gamma_2 - \gamma_1)^l}{l!}$$

where

$$V_2(k) = (v_1(k - q_1), v_2(k - q_2 - 1)), \qquad \Delta V_2(k) = (\Delta v_1(k - q_1), \Delta v_2(k - q_2 - 1)),$$

(iii) for $t \in [kT + \gamma_2, kT + T)$.
ESDR:

$$x(kT + T) = x(kT + \gamma_2) + \sum_{l=1}^{\infty} A^l(x(kT + \gamma_2), V_3(k), \Delta V_3(k))\frac{(T - \gamma_2)^l}{l!}.$$

ASDR:

$$x^N(kT + T) = x^N(kT + \gamma_2) + \sum_{l=1}^{N} A^l(x^N(kT + \gamma_2), V_3(k), \Delta V_3(k))\frac{(T - \gamma_2)^l}{l!}$$

where

$$V_3(k) = (v_1(k - q_1), v_2(k - q_2)), \qquad \Delta V_2(k) = (\Delta v_1(k - q_1), \Delta v_2(k - q_2)).$$

Here $k = 0, 1, 2, 3, \ldots$ and the parameters $A^{[l]}(x, V, \Delta V)$ with $V = (v_1, \ldots, v_m)$, $\Delta V = (\Delta v_1, \ldots, \Delta v_m)$ can be obtained similar to Eq. (10):

$$\begin{aligned} A^{[1]}(x, V, \Delta V) &= f(x) + \sum_{i=1}^{m} g_i(x)v_i, \\ A^{[\ell+1]}(x, V, \Delta V) &= \frac{\partial A^{[\ell]}(x, V, \Delta V)}{\partial x}A^{[1]}(x, V, \Delta V) + \sum_{i=1}^{m}\frac{\partial A^{[\ell]}(x, V, \Delta V)}{\partial v_i}\Delta v_i, \quad \ell = 1, 2, \ldots. \end{aligned} \tag{23}$$

4.3. Discretization of multi-input time-delay nonlinear systems

The general multi-input nonlinear system in state space form with time delays can be represented as follows.

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f(x(t)) + \sum_{i=1}^{m} g_i(x(t))u_i(t - D_i). \tag{24}$$

From the last subsection, the general time-discretization equation of nonlinear systems with multi-input time-delays can be derived as follows.

Similar to (2) the delays of the inputs as in Eq. (24) can be presented as follows

$$D_i = q_i T + \gamma_i, \quad i = 1, \ldots, m,$$

where $D_i,\ i = 1, \ldots, m$, have the same meaning as D and $\gamma_i,\ i = 1, \ldots, m$, have the same meaning as γ, $q_i,\ i = 1, \ldots, m$, have the same meaning as q.

Due to the FOH assumption, the inputs are as follows;

$$u_i(t - D_i) = \begin{cases} v_i(k - q_i - 1) + \Delta v_i(k - q_i - 1)\,[t - kT + (T - \gamma_i)], & t \in [kT, kT + \gamma_i), \\ v_i(k - q_i) + \Delta v_i(k - q_i)\,[t - kT - \gamma_i], & t \in [kT + \gamma_i, kT + T), \end{cases} \quad i = 1, \ldots, m, \tag{25}$$

where

$$v_i(k) = u_i(Tk + 0), \qquad \Delta v_i(k) = \frac{u_i(Tk + T - 0) - u_i(Tk + 0)}{T}, \quad i = 1, \ldots, m. \tag{26}$$

Without lost of generality one may consider that $\gamma_1 \le \gamma_2 \le \cdots \le \gamma_m$.

Assuming $\gamma_1 < \gamma_2 < \cdots < \gamma_m$ and taking into account (25), (26) we get

- for $t \in [kT, kT + \gamma_1)$

  ESDR:

$$x(kT + \gamma_1) = x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), V_1(k), \Delta V_1(k))\frac{\gamma_1^l}{l!}. \tag{27}$$

  ASDR:

$$x^N(kT + \gamma_1) = x(kT) + \sum_{l=1}^{N} A^l(x(kT), V_1(k), \Delta V_1(k))\frac{\gamma_1^l}{l!} \tag{28}$$

  where

$$V_1(k) = (v_j(k - q_j - 1),\ j = 1, \ldots, m), \qquad \Delta V_1(k) = (\Delta v_j(k - q_j - 1),\ j = 1, \ldots, m). \tag{29}$$

- for $t \in [kT + \gamma_{i-1}, kT + \gamma_i)$ where $2 \le i \le m + 1$ and $\gamma_{m+1} = T$

  ESDR:

$$x(kT + \gamma_i) = x(kT + \gamma_{i-1}) + \sum_{l=1}^{\infty} A^l(x(kT + \gamma_{i-1}), V_i(k), \Delta V_i(k))\frac{(\gamma_i - \gamma_{i-1})^l}{l!}. \tag{30}$$

  ASDR:

$$x^N(kT + \gamma_i) = x^N(kT + \gamma_{i-1}) + \sum_{l=1}^{N} A^l(x^N(kT + \gamma_{i-1}), V_i(k), \Delta V_i(k))\frac{(\gamma_i - \gamma_{i-1})^l}{l!} \tag{31}$$

  where

$$\begin{aligned} &V_i(k) = (v_j(k - q_j),\ j = 1, \ldots, i - 1,\ v_j(k - q_j - 1),\ j = i, \ldots, m), \\ &\Delta V_i(k) = (\Delta v_j(k - q_j),\ j = 1, \ldots, i - 1,\ \Delta v_j(k - q_j - 1), j = i, \ldots, m), \quad i = 2, \ldots, m; \\ &V_{m+1}(k) = (v_j(k - q_j),\ j = 1, \ldots, m), \qquad \Delta V_{m+1}(k) = (\Delta v_j(k - q_j),\ j = 1, \ldots, m). \end{aligned} \tag{32}$$

Here $k = 0, 1, 2, 3, \ldots$ and the parameters $A^{[l]}(x, V, \Delta V)$ are obtained using Eq. (23).

It follows from the above calculations (27)–(32) that the ESDR:

$$x((k + 1)T) = \Psi_{m+1}(x(kT), V(k), \Delta V(k)) \tag{33}$$

and the ASDR:

$$x^N((k + 1)T) = \Psi_{m+1}^N(x(kT), V(k), \Delta V(k)) \tag{34}$$

takes place with

$$V(k) = (v_j(k-q_j),\, v_j(k-q_j-1),\, j = 1, \ldots, m),$$
$$\Delta V(k) = (\Delta v_j(k-q_j),\, \Delta v_j(k-q_j-1),\, 1 = i, \ldots, m).$$

Here

$$\begin{aligned}
&\Psi_1(x, W, \Delta W) = x + \sum_{l=1}^{\infty} A^l(x, W_1, \Delta W_1)\frac{\gamma_1^l}{l!}, \qquad \Psi_1^N(x, W, \Delta W) = x + \sum_{l=1}^{N} A^l(x, W_1, \Delta W_1)\frac{\gamma_1^l}{l!}\\
&\Psi_i(x, W, \Delta W) = \Psi_{i-1}(x, W, \Delta W) + \sum_{l=1}^{\infty} A^l(\Psi_{i-1}(x, W, \Delta W), W_i, \Delta W_i)\frac{(\gamma_i-\gamma_{i-1})^l}{l!},\\
&\Psi_i^N(x, W, \Delta W) \;=\; \Psi_{i-1}^N(x, W, \Delta W) + \sum_{l=1}^{N} A^l(\Psi_{i-1}^N(x, W, \Delta W), W_i, \Delta W_i)\frac{(\gamma_i-\gamma_{i-1})^l}{l!},\\
&\qquad\qquad i = 2, \ldots, m+1,\;\; \gamma_{m+1} := T
\end{aligned} \tag{35}$$

with

$$\begin{aligned}
&W = (w_j, \bar{w}_j, j = 1, \ldots, m), \qquad \Delta W = (\Delta w_j, \Delta\bar{w}_j, j = 1, \ldots, m),\\
&W_1 = (\bar{w}_j,\, j = 1, \ldots, m), \qquad \Delta W_1 = (\Delta\bar{w}_j,\, j = 1, \ldots, m)\\
&W_i \;=\; (w_j,\, j = 1, \ldots, i-1, \bar{w}_j, j = i, \ldots, m), \qquad \Delta W_i = (\Delta w_j,\, j = 1, \ldots, i-1, \Delta\bar{w}_j,\, j = i, \ldots, m),\\
&\qquad i = 2, \ldots, m+1.
\end{aligned} \tag{36}$$

Remark 1. It is evident that the vectors $V(k), \Delta V(k), V_i(k), \Delta V_i(k),\ i = 1, \ldots, m+1$, depends on $q_i,\ i = 1, \ldots, m$. As it is also evident that the functions $\Psi_1(x, W, \Delta W), \Psi_1^N(x, W, \Delta W)$ depends on γ_1, functions $\Psi_i(x, W, \Delta W), \Psi_i^N(x, W, \Delta W)$ depend on $\gamma_1, \ldots, \gamma_{i+1},\ i = 2, \ldots, m$, functions $\Psi_{m+1}(x, W, \Delta W), \Psi_{m+1}^N(x, W, \Delta W)$ depend on $\gamma_1, \ldots, \gamma_m, T$. These dependences are not shown explicitly to simplify the notations.

Remark 2. We assumed above that $\gamma_1 < \gamma_2 < \cdots < \gamma_n$. Notice that in a case with $\gamma_{i-1} = \gamma_i$ for some $i, 2 \le i \le m$, calculations presented above are simplified. In fact it follows from (35) that if $\gamma_{i-1} = \gamma_i$ then $\Psi_i(x, W, \Delta W) = \Psi_{i-1}(x, W, \Delta W)$. In the more simple case when $\gamma_i = 0,\ i = 1, \ldots, m$, we have

$$\Psi_i(x, W, \Delta W) = x, \quad i = 1, \ldots, m, \qquad \Psi_{m+1}(x, W, \Delta W) = x + \sum_{l=1}^{\infty} A^l(x, W_{m+1}, \Delta W_{m+1})\frac{T^l}{l!}.$$

For nonlinear continuous systems with time-delayed mule-inputs, we have described above a new discretization method based on the combination of the Taylor series expansion and the FOH assumption. The advantage of the use of FOH was motivated in Section 2. Properties, numerical aspects (simplicity, convergence, stability) and advantages of the discretization methods based on the Taylor series expansion are described and justified in [14]. The same analysis can be carried out for the method proposed in this paper to justify the corresponding properties. For example, the following theorem shows us that equilibrium properties of the original continuous-time system (24) are invariant under the proposed time-discretization method.

Theorem 1. *Let x^0 be an equilibrium point of the original nonlinear continuous time system (24) that belongs to the equilibrium manifold*

$$E^c = \left\{ x \in R^n | \exists U = (u_i,\ i = 1, \ldots, m) : f(x) + \sum_{i=1}^{m} g_i(x)u_i = 0 \right\}$$

and $U^0 = (u_i^0,\ i = 1, \ldots, m)$ be the corresponding equilibrium input value:

$$f(x^0) + \sum_{i=1}^{m} g_i(x)u_i^0 = 0. \tag{37}$$

Then x^0 belongs to equilibrium manifolds

$$E^d = \{x \in R^n | \exists W = (w_j, \bar{w}_j, j = 1, \ldots, m), \Delta W = (\Delta w_j, \Delta\bar{w}_j,\ j = 1, \ldots, m) : \Psi_{m+1}(x, W, \Delta W) = 0\},$$
$$E^{dN} = \{x \in R^n | \exists W = (w_j, \bar{w}_j j = 1, \ldots, m), \Delta W = (\Delta w_j, \Delta\bar{w}_j,\ j = 1, \ldots, m) : \Psi_{m+1}^N(x, W, \Delta W) = 0\}$$

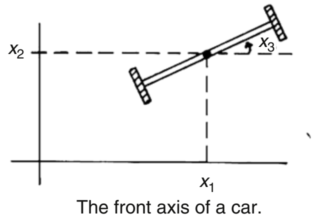

Fig. 1. The front axis of an automobile.

of ESDR (34) and ASDR (35) obtained under the proposed discretization method and FOH assumption, with

$$W^0 = (w_j^0 = u_j^0,\ \bar{w}_j^0 = u_j^0,\ j = 1, \ldots, m), \qquad \Delta W^0 = (\Delta w_j^0 = 0,\ \Delta \bar{w}_j^0 = 0,\ j = 1, \ldots, m) \tag{38}$$

being the corresponding equilibrium input values:

$$\Psi_{m+1}(x^0, W^0, \Delta W^0) = x^0 \quad \text{and} \quad \Psi_{m+1}^N(x^0, W^0, \Delta W^0) = x^0.$$

Proof. It follows from (36) and (38) that

$$\begin{aligned} &W_1^0 = (w_j^0 = u_j^0,\ j = 1, \ldots, m), \qquad \Delta W_1^0 = (\Delta w_j^0 = 0,\ j = 1, \ldots, m), \\ &W_i^0 = (w_j^0 = u_j^0,\ j = 1, \ldots, i-1,\ \bar{w}_j^0 = u_j^0, j = i, \ldots, m) = W_1^0, \\ &\Delta W_i^0 = (\Delta w_j^0 = 0,\ j = 1, \ldots, i-1,\ \Delta \bar{w}_j^0 = 0, j = i, \ldots, m) = \Delta W_1^0, \quad i = 2, \ldots, m+1. \quad \square \end{aligned} \tag{39}$$

Taking into account (23), (37) and (39) we get

$$A^{[\ell]}(x^0, W_i^0, \Delta W_i^0) = 0, \quad i = 1, \ldots, m,\ \ell = 1, 2, 3, \ldots$$

and relating it with (35) takes the form

$$\Psi_1(x^0, W^0, \Delta W^0) = x^0, \qquad \Psi_i(x^0, W^0, \Delta W^0) = \Psi_{i-1}(x^0, W^0, \Delta W^0), \quad i = 2, \ldots, m+1.$$

The last equalities imply that $\Psi_{m+1}(x^0, W^0, \Delta W^0) = x^0$.

The equality $\Psi_{m+1}^N(x^0, W^0, \Delta W^0) = x^0$ can be proved in a similar way. The theorem is proved.

5. Case studies

One example is considered in a computer simulation and the example is a simplified model of maneuvering an automobile [19]. Different sampling periods, time delays and control inputs are introduced in the simulation. The partial derivative terms involved in the Taylor series expansion are determined recursively by MAPLE. The truncation order of the Taylor series is chosen as "3" since it can provide accurate enough discretization results and enlarging the truncation order to more than 3 does not improve accuracy. Exact solutions for the systems are required in order to validate the proposed discretization method of nonlinear systems with the delayed multi-input. The continuous Matlab ODE solver is used as an exact solution in this paper. The discrete values obtained using the Taylor series expansion with the FOH assumption method are compared to the values obtained through the continuous Matlab ODE solver at the moments $t = 1, 2, \ldots, 5$.

The front axle of a simplified automobile maneuvering system is shown in Fig. 1. The middle of the axles linking the front wheels has position $(x_1, x_2) \in R^2$, while the rotation of this axis is given by the angle x_3. The states x_1, x_2 related to rolling are directly controlled by input u_1 and the state x_3 related to rotation is directly controlled by u_2, thus the governing nonlinear differential equation can be obtained as follows;

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \sin x_3 \\ \cos x_3 \\ 0 \end{bmatrix} u_1(t - D_1) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2(t - D_2). \tag{40}$$

We first choose constant inputs to drive this nonlinear system. In this case the initial conditions are $x_1(0) = 0$, $x_2(0) = 0$, and $x_3(0) = \pi/6$, the sampling period is $T = 0.001$ s, the time delay values are $D_1 = 0.0122$ s and $D_2 = 0.0166$ s, and the control inputs are $u_1 = 2.5$ and $u_2 = 2.0$. The simulation results are shown in Table 1. The differences in the responses of the Taylor method with the FOH and ZOH assumptions and the Matlab solver are shown in Fig. 2.

Then we choose slope inputs to drive this nonlinear system. In Case 1, the sampling period is $T = 0.001$ s, the time delay values are $D_1 = 0.0143$ s and $D_2 = 0.0187$ s, and the control inputs are $u_1 = 0.5t$ and $u_2 = 0.5t$. The simulation results are shown in Table 2. The differences in the responses of the Taylor method with the FOH and ZOH assumptions and the Matlab solver are shown in Fig. 3.

Table 1
Simulation results using the constant inputs.

	MATLAB (x_1)	ZOH (x_1)	FOH (x_1)
1	2.0823	2.0822	2.0822
2	1.3628	1.3632	1.3632
3	−0.1353	−0.1352	−0.1352
4	1.8311	1.8309	1.8309
5	1.6926	1.6930	1.6930
	MATLAB (x_2)	ZOH (x_2)	FOH (x_2)
1	0.1417	0.1422	0.1422
2	−1.8351	−1.8348	−1.8348
3	−0.3582	−0.3583	−0.3583
4	0.3894	0.3898	0.3898
5	−1.7097	−1.7093	−1.7093

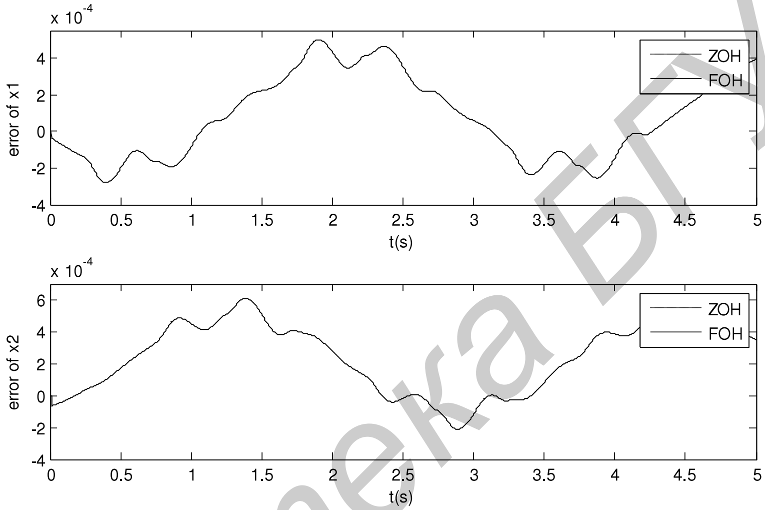

Fig. 2. State errors using the constant inputs.

Table 2
Simulation results using the slope inputs (Case 1).

	MATLAB (x_1)	ZOH (x_1)	FOH (x_1)
1	0.1501	0.1499	0.1501
2	0.8173	0.8168	0.8173
3	1.7986	1.7983	1.7986
4	1.0577	1.0588	1.0578
5	−0.0181	−0.0186	−0.0180
	MATLAB (x_2)	ZOH (x_2)	FOH (x_2)
1	0.1982	0.1980	0.1982
2	0.4991	0.4990	0.4991
3	−0.1376	−0.1368	−0.1375
4	−1.4815	−1.4813	−1.4815
5	−0.0329	−0.0341	−0.0330

In Case 2, the sampling period is $T = 0.005$ s, the time delay values are $D_1 = 0.0617$ s and $D_2 = 0.0435$ s, and the control inputs are $u_1 = 0.5t$ and $u_2 = 0.4t$. The simulation results are shown in Table 3. The differences in the responses of the Taylor method with the FOH and ZOH assumptions and the Matlab solver are shown in Fig. 4.

In Case 3 the sampling period is $T = 0.01$ s, the time delay values are $D_1 = 0.073$ s and $D_2 = 0.057$ s, and the control inputs are $u_1 = 0.5t$ and $u_2 = 0.4t$. The simulation results are shown in Table 4. The differences in the responses of the Taylor method with the FOH and ZOH assumptions and the Matlab solver are shown in Fig. 5.

Finally we choose sinusoidal inputs to drive this nonlinear system. In Case 1, the sampling period is $T = 0.001$ s, the time delay values are $D_1 = 0.0203$ s and $D_2 = 0.0157$ s, and the control inputs are $u_1 = 1.8\sin(0.8\pi t)$ and $u_2 = 1.5\sin(0.8\pi t)$. The simulation results are shown in Table 5. The differences in the responses of the Taylor method with the FOH and ZOH assumptions and the Matlab solver are shown in Fig. 6.

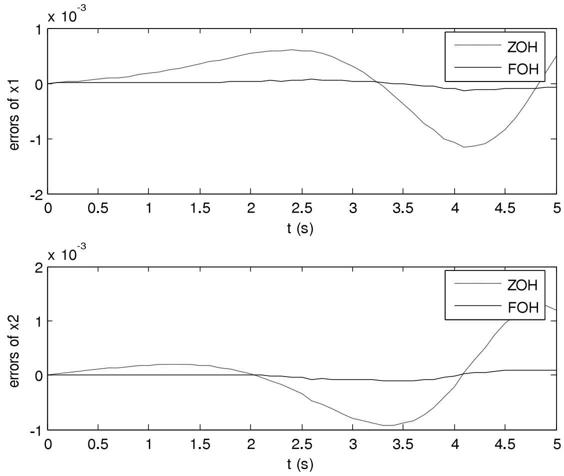

Fig. 3. State errors using the slope inputs for Case 1.

Table 3
Simulation results using the slope inputs (Case 2).

	MATLAB (x_1)	ZOH (x_1)	FOH (x_1)
1	0.1270	0.1263	0.1270
2	0.7230	0.7207	0.7230
3	1.8681	1.8652	1.8680
4	2.1492	2.1517	2.1493
5	0.2391	0.2439	0.2393
	MATLAB (x_2)	ZOH (x_2)	FOH (x_2)
1	0.1794	0.1785	0.1794
2	0.5635	0.5628	0.5635
3	0.3194	0.3219	0.3195
4	−1.2409	−1.2365	−1.2408
5	−1.5619	−1.5660	−1.5619

Table 4
Simulation results using the slope inputs (Case 3).

	MATLAB (x_1)	ZOH (x_1)	FOH (x_1)
1	0.1235	0.1220	0.1235
2	0.7120	0.7074	0.7119
3	1.8553	1.8496	1.8553
4	2.1647	2.1693	2.1648
5	0.2662	0.2758	0.2663
	MATLAB (x_2)	ZOH (x_2)	FOH (x_2)
1	0.1754	0.1737	0.1754
2	0.5613	0.5599	0.5612
3	0.3334	0.3381	0.3335
4	−1.2174	−1.2086	−1.2172
5	−1.5835	−1.5914	−1.5836

In Case 2 the sampling period is $T = 0.005$ s, the time delay values are $D_1 = 0.0817$ s and $D_2 = 0.0635$ s, and the control inputs are $u_1 = 1.8\sin(0.7\pi t)$ and $u_2 = 1.5\sin(0.7\pi t)$. The simulation results are shown in Table 6. The differences in the responses of the Taylor method with the FOH and ZOH assumptions and the Matlab solver are shown in Fig. 7.

In Case 3 the sampling period is $T = 0.01$ s, the time delay values are $D_1 = 0.043$ s and $D_2 = 0.087$ s, and the control inputs are $u_1 = 1.8\sin(0.7\pi t)$ and $u_2 = 1.5\sin(0.7\pi t)$. The simulation results are shown in Table 7. The differences in the responses of the Taylor method with the FOH and ZOH assumptions and the Matlab solver are shown in Fig. 8.

The difference between the proposed time discretization method and the MATLAB ODE solvers is small enough to demonstrate that the proposed time discretization method using Taylor series with the FOH assumption can be used to discretize multi-input time-delay nonlinear systems and provide satisfactory results. The errors become larger as the sampling period is increased and in this case, the Taylor series order can be enlarged to improve the performance. The Taylor series order should be chosen appropriately because the calculation burden improves quickly as the Taylor series

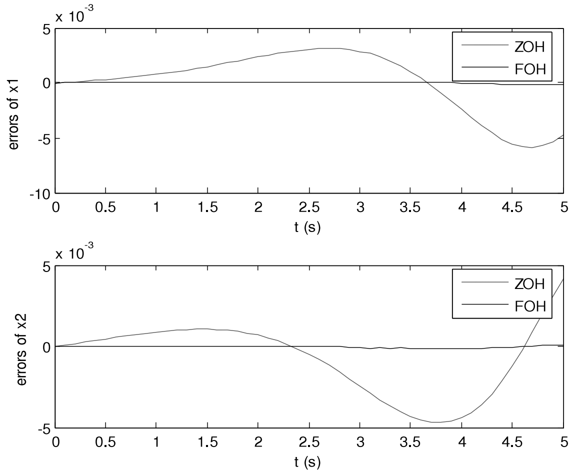

Fig. 4. State errors using the slope inputs for Case 2.

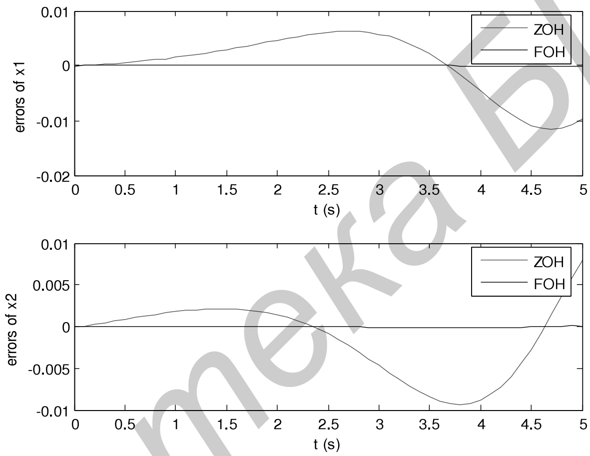

Fig. 5. State errors using the slope inputs for Case 3.

Table 5
Simulation results using the sinusoidal inputs (Case 1).

	MATLAB (x_1)	ZOH (x_1)	FOH (x_1)
1	1.0592	1.0587	1.0592
2	0.3607	0.3615	0.3607
3	0.3082	0.3075	0.3082
4	1.1084	1.1090	1.1086
5	0.0134	0.0134	0.0134
	MATLAB (x_2)	ZOH (x_2)	FOH (x_2)
1	0.5936	0.5937	0.5937
2	0.3747	0.3755	0.3750
3	0.3303	0.3301	0.3301
4	0.5779	0.5781	0.5782
5	0.0266	−0.0259	−0.0259

order is increased. From the simulation results, we can also see that the proposed method with the FOH assumption can provide much better performance than with the ZOH assumption except in the case where the inputs are constant.

The computational costs are considered at the end of this section where Table 8 shows the computing time used to get the discretization results of the proceeding simulations using Taylor series with the FOH assumption and the ZOH assumption respectively. The computing time is calculated on a computational process with 5000 steps. It can be seen from Table 8 that the computing time using the Taylor series with the FOH assumption is moderately longer compared to when we used ZOH assumption but the accuracy is much better for the process with FOH assumption.

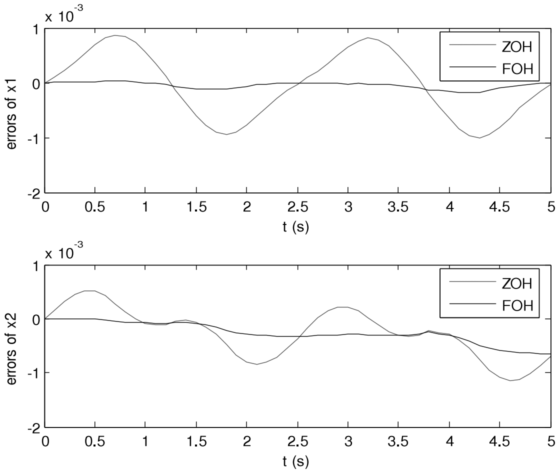

Fig. 6. State errors under the sinusoidal inputs for Case 1.

Table 6
Simulation results using the sinusoidal inputs (Case 2).

	MATLAB (x_1)	ZOH (x_1)	FOH (x_1)
1	0.9686	0.9645	0.9686
2	0.9999	1.0039	1.0000
3	0.0274	0.0271	0.0273
4	1.2023	1.1991	1.2024
5	0.7856	0.7900	0.7858
	MATLAB (x_2)	ZOH (x_2)	FOH (x_2)
1	0.5740	0.5739	0.5741
2	0.5604	0.5608	0.5605
3	−0.0557	−0.0559	−0.0555
4	0.5034	0.5041	0.5037
5	0.4578	0.4594	0.4582

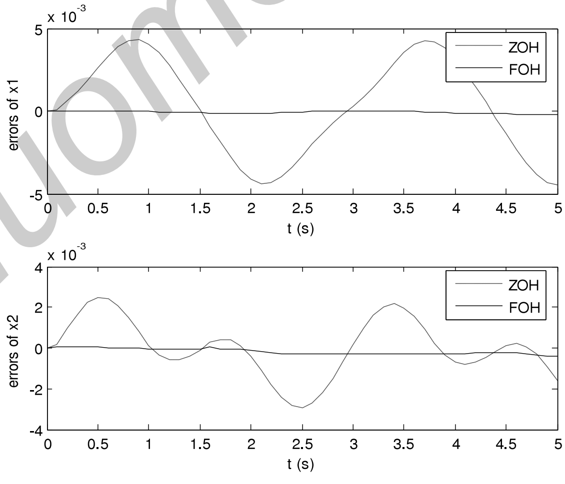

Fig. 7. State errors using the sinusoidal inputs for Case 2.

6. Conclusion

A method based on the Taylor series combined with the FOH assumption is proposed for the derivation of a discrete-time representation of a nonlinear control system with time delayed multi-input. The mathematical structure of the new discretization scheme is explored and characterized as useful for establishing the concrete connections between the numerical and system–theoretic properties. The derived time-discretization method provides a finite-dimensional representation for nonlinear control systems with time-delays, thereby enabling the application of existing nonlinear controller design techniques for such systems.

Table 7
Simulation results using the sinusoidal inputs (Case 3).

	MATLAB (x_1)	ZOH (x_1)	FOH (x_1)
1	0.9862	0.9785	0.9863
2	0.8987	0.9071	0.8989
3	−0.0472	−0.0482	−0.0473
4	1.1262	1.1203	1.1264
5	0.5962	0.6052	0.5965
	MATLAB (x_2)	ZOH (x_2)	FOH (x_2)
1	0.6571	0.6564	0.6571
2	0.6890	0.6894	0.6892
3	0.1659	0.1646	0.1661
4	0.8039	0.8047	0.8042
5	0.8047	0.8072	0.8053

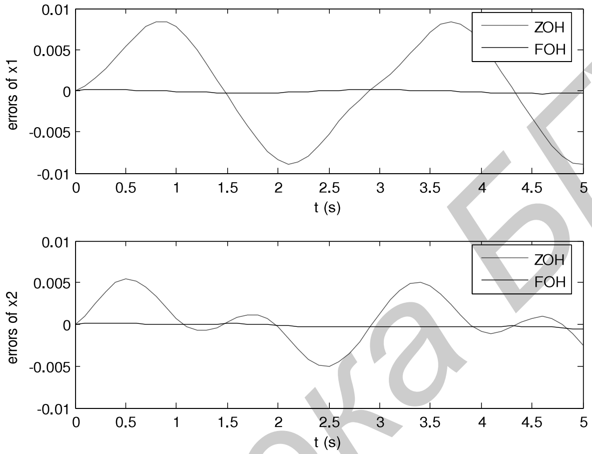

Fig. 8. State errors using the sinusoidal inputs for Case 3.

Table 8
The computing time using to do the proceeding simulations.

Computing time (5000 steps) (s)					
FOH	ZOH	FOH	ZOH	FOH	ZOH
Constant inputs (Case 1)		Constant inputs Case 2		Constant inputs Case 3	
7.61	5.43	7.58	5.56	7.50	5.39
Slope inputs (Case 1)		Slope inputs (Case 2)		Slope inputs (Case 3)	
8.79	6.03	9.09	6.09	9.18	6.08
Sinusoidal inputs (Case 1)		Sinusoidal inputs (Case 2)		Sinusoidal inputs (Case 3)	
14.89	9.61	14.98	9.88	14.96	9.45

The performance of the proposed time-discretization procedure is evaluated using the multi-input system of a simplified model of maneuvering an automobile where various sampling rates, time-delay values and control inputs are considered in the example studies. The simulation results are compared with those produced by MATLAB in order to verify the accuracy of the proposed method. The examples demonstrate how to use the proposed method to solve a real system. In cases even when the sampling time is large with input time-delay, the Taylor series combined with the FOH assumption can satisfy the accuracy requirement of the systems.

At the same time, some comparisons are made between the ZOH and FOH methods when combined with the Taylor series for the discretization procedure. Results show that although the computational cost is moderately bigger, the FOH method is much better than the ZOH method in achieving high precision for the input signals such as sinusoidal and unit slope.

More detailed comparisons of the FOH and ZOH methods will be the subject of future publications.

Acknowledgement

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