The Painlevé test for nonlinear system of differential equations with complex chaotic behavior

2017 J. Phys.: Conf. Ser. 788 012035

(http://iopscience.iop.org/1742-6596/788/1/012035)

The Painlevé test for nonlinear system of differential equations with complex chaotic behavior

V Tsegel'nik

Belarusian State University of Informatics and Radioelectronics, P. Brovki Str., 6, Minsk, 220013, Belarus

E-mail: tsegvv@bsuir.by

Abstract. The Painlevé-analysis was performed for solutions of nonlinear third-order autonomous system of differential equations with quadratic nonlinearities on their right-hand sides. At certain values of two constant parameters incorporated into the system, the latter exhibits complex chaotic behavior. When the parameters attain the values corresponding to complex chaotic behavior, the system was found not to possess the Painlevé property.

The system of differential equations

$$\dot{x} = z, \quad \dot{y} = -ay - xz, \quad \dot{z} = z - bz^2 + xy \tag{1}$$

with unknown functions x, y, z of the independent variable t and constant parameters a, b belongs [1, 2] to the class of chaotic systems

$$\dot{x} = y, \;\; \dot{y} = -x + yz, \;\; \dot{z} = -x - axy - bxz, \qquad (LE_1)$$

$$\dot{x} = y, \;\; \dot{y} = -x + yz, \;\; \dot{z} = -y - axy - bxz, \qquad (LE_2)$$

$$\dot{x} = y, \;\; \dot{y} = -x + yz, \;\; \dot{z} = x^2 - axy - bxz, \qquad (LE_3)$$

$$\dot{x} = y, \;\; \dot{y} = -x + yz, \;\; \dot{z} = -axy - bxz - yz, \qquad (LE_4)$$

$$\dot{x} = y, \;\; \dot{y} = -ax + yz, \;\; \dot{z} = -x^2 + y^2 - bxy, \qquad (LE_5)$$

$$\dot{x} = y, \;\; \dot{y} = -x + yz, \;\; \dot{z} = ay^2 - xy - bxz, \qquad (LE6)$$

$$\dot{x} = z, \;\; \dot{y} = x + yz, \;\; \dot{z} = ax^2 - xy - byz, \qquad (LE_7)$$

$$\dot{x} = z, \;\; \dot{y} = x - yz, \;\; \dot{z} = -ax^2 + xy + bxz, \qquad (LE_8)$$

with hidden attractors and line equilibrium. Such systems are important and potentially problematic in engineering applications, because they allow unexpected and potentially disastrous responses to perturbations in a structure like a bridge or an airplane wing. In case $a = 1.62$, $b = 0.2$ the system (1) possesses a hidden attractor. Considering, that t is a complex variable, find out, at what values a, b the general solution of the system (1) has no movable critical singular points, i.e., whether for (1) the so-called Painlevé property is carried out.

To solve this problem let us apply to the system (1) formal Painlevé test [3].

The following statements are true

Theorem 1 The system (1) passes the Painlevé test under parameters values $a = -1$, $b = 0$.

Theorem 2 The system (1) when $a = -1$, $b = 0$ is a system of Painlevé-type and does not possess chaotic behavior.

The validity of **Theorem 2** follows from the facts, that

1. The system (1) is equivalent to the equation

$$x\dddot{x} + (a-1)x\ddot{x} + 2bx\dot{x}\ddot{x} - \dot{x}\ddot{x} - b\dot{x}^3 + abx\dot{x}^2 + \dot{x}^2 + x^3\dot{x} - ax\dot{x} = 0 \qquad (2)$$

2. The equation (2) when $a = -1$, $b = 0$ has the first integral

$$(\ddot{x} - \dot{x})^2 = C^2x^2e^{2t} - x^2\dot{x}^2, \qquad (3)$$

where C is the arbitrary constant. The equation (3) possesses the Painlevé property, since its solutions are expressed through solutions of the third Painlevé equation under the private values of the parameters.

3. The unknown functions y, z, incorporated into the system (1), in a rational way are expressed through x, $\dot{x}$.

The statement of **Theorem 2** is consistent with the well-known hypothesis [4], according to which the execution for the system Painlevé property with a high degree of confidence is considered to be incompatible with the randomness of its behavior.

Theorem 3 The equation (2) under parameters values $a = 1.62$, $b = 0.2$ possesses chaotic behavior and does not pass the Painlevé test.

Theorem 4 The system (1) in case $a = 0$ does not possess chaotic behavior.

Indeed, when $a = 0$ the system (1) has autonomous first integral $\frac{x^2}{2} + y = C$, where C is the constant of integration. The presence of this integral allows us to reduce (1) to two-dimensional autonomous system. According to [5] solutions of two-dimensional autonomous systems cannot be chaotic.

Theorem 5 The equation

$$x\dddot{x} - 2x\ddot{x} - Ax^3\dot{x} - Bx\dot{x} + H(\dot{x}\ddot{x} - \dot{x}^2) = 0 \qquad (4)$$

under parameters values

a) either $H = 0$;

b) or $H = -1$, $A = B = -1$

is the equation of Painlevé-type.

Indeed, when $H = 0$ the equation (4) has first integral

$$\ddot{x} = Ax^3 + Kx^2 - Bx, \qquad (5)$$

where K is the constant of integration. Integration of the equation (5) is carried out in elliptic functions.

In case b) the equation (4) coincides with (2) when $a = -1$, $b = 0$.

References

[1] Jafari S and Sprott J C 2013 Simple chaotic flows with a line equilibrium *Chaos, solitons and fractals* **57** 79–84

[2] Jafari S and Sprott J C 2015 Erratum to: Simple chaotic flows with a line equilibrium *Chaos, solitons and fractals* **77** 341–42

[3] Ablowitz M J, Ramani A and Segur H 1980 A connection between nonlinear evolution equations and ordinary differential equations of P-type *J. Math. Phys.* **21** 715–21

[4] Goriely A 2001 *Integrability and nonintegrability of dynamical systems* (*Advanced series in nonlinear dynamics* vol 19) (Singapore: World Scientific Publishing Company)

[5] Moon F C 1987 *Chaotic vibrations* (New York: John Wiley and Sons)