$u_t|_{t=t_0} = \varphi_3,\ u_t|_{t=t_1} = \varphi_4,\ \varphi_i \in U_1\ (i = \overline{1,4})\}$: $U = C^2([t_0, t_1]; U_1)$, $V = C([t_0, t_1]; V_1)$, U_1, V_1 are linear normed spaces, $U_1 \subseteq V_1$. The set $W \subseteq U_1$ is determined by the boundary conditions.

Assume that the nonlocal bilinear form $\Phi(\cdot,\cdot) \equiv \int\limits_{t_0}^{t_1} \langle\cdot,\cdot\rangle : V \times V \to \mathfrak{R}$ is symmetric and nondegenerate.

We use the notation and terminology of [1-3].

Theorem 1. *Suppose that S_1, S_2 are generators of symmetries of equation (1) and the operator N is potential on $D(N)$ (2) with respect to bilinear form (3). Then*

$$I_1[t,u] = D_t \langle P_{2u}S_2(u), S_1(u)\rangle - \langle 2P_{3u}(u_tS_2(u)) + 2P_{2u}D_tS_2(u) + P_{1u}S_2(u), S_1(u)\rangle$$

is a first integral of the given equation.

Theorem 2. *Suppose that the operator N is not potential on $D(N)$ (2) with respect to bilinear form (3), S_1 is a generator of symmetry of equation (1) and there exists an operator S_2 such that $N_u'^*S_2(u) \overset{(1)}{=} 0$. Then*

$$I_2[t,u] = D_t \langle P_{2u}S_1(u), S_2(u)\rangle - \langle 2P_{3u}(u_tS_1(u)) + 2P_{2u}D_tS_1(u) + P_{1u}S_1(u), S_2(u)\rangle$$

is a first integral of the given equation.

The theoretical results are illustrated by some examples.

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TRAJECTORY PLANNING FOR MECHATRONIC DRIVES ON THE BASE OF HOLONOMIC AUTOMATIC SYSTEMS APPROACH

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A large quantity of mechatronic drives is based on trajectory constructing on topological manifolds (particularly on curves and surfaces) with given accuracy [1]. For automated control of these systems it is preferable for trajectory planning to be given in the form of finite equation or system of equations. These systems with the behavior which is established with exactness up to variety intersection are related to the class of holonomic systems [2].

The problem of synthesis of holonomic systems presupposes the synthesis of differential analyzer in the form of differential equations for which the solution is the reproducible program of movement.

The building of control systems based on differential analyzers gives such advantages as:

- simplification of control algorithm and automatic control system structure;
- possibility to control the speed of affix movement without considerable complication of structure of automatic control system;
- possibility of change-over of control system parameters for reproduction of affix movement on different topological manifold without considerable complication of control system structure;
- possibility of optimal curves programming, i.e. such control design that provides maximum performance of drive while surface handling.

Any multi-variable function can be presented in the form of finite superposition of continuous functions of one-variable functions. Suppose an implicit function of n variables:

$$F(x_1, x_2, \ldots, x_n) = 0, \tag{1}$$

which is differentiable in established range of variables M and is not a hyper-transcendental function. We shall look for the first-order system of differential equations:

$$\frac{dx_i}{d\varphi} = f_i, \quad i = 1, 2, \ldots, n, \tag{2}$$

the solution of which satisfies the equation (1) in established range of variables M.

The variable φ in (2) is an argument of the designed differential analyzer. In is necessary to define the functions f_i. After differentiation of equation (1) by parameter φ, we shall get:

$$\sum_{i=1}^{n} \frac{\partial F}{\partial x_i} \frac{dx_i}{d\varphi} = 0. \tag{3}$$

If the system (2) solution turns into identity the equation (1), then system (2) turns into identity the equation (3). Thus, the functions f_i definition can be based on analytical condition (1). This problem has a solution set, at that functions f_i in all cases depend on partial derivatives $\frac{\partial F}{\partial x_i}$. For analytical algorithm simplification let us concern the functions f_i are linear functions of mentioned above partial derivatives. This method of differential analyzers synthesis has an essential advantage: the argument φ, which is concerned to be a system parameter, can be any analytical function, what specifically lets realize the argument control, which is necessary for differential analyzer structure simplification.

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THE NASH'S OPTIMAL CONTROL OF FOREST ECOSYSTEM

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In [1] was offered the next model 4-tier mosaic forest communities, characterized by productivity x and the soil fertility measure y:

$$\begin{cases} \dfrac{dx}{dt} = -\alpha x^5 - x^3 k - x^2 m - xa - w, \\ \dfrac{dy}{dt} = -\gamma y^3 + \gamma y p - \delta \cdot (w + (w_0 - w\_)), \end{cases} \tag{1}$$

$$0 < w_- < w_0 < w_+,$$

$$t \in [0, T],$$

where m is mosaic state, k is interspecific and intraspecific competition, a is the anthropogenic impact, w is soil moisture, p is the measure of soil type and $\alpha, \gamma, \delta > 0$ are constants. Here $k = 0, a = 0, w = 0$ are the boundaries of ecological stability of phytocenosis, and $x = 0$ is characteristic observed value of productivity in the absence of strong changes in external factors.

Position control $\{u_1^* = m^*(x,y), u_2^* = k^*(x,y), u_3^* = a^*(x,y), u_4^* = w^*(x,y), u_5^* = p^*(x,y)\}$ are said to constitute a *Nash's optimal control* if

$$J_i(u_1^*, u_2^*, u_i^*, ..., u_N^*) \leq J_i(u_1^*, u_2^*, ..., u_{i-1}^*, u_i, u_{i+1}^*, ..., u_N^*), \quad \forall u_i, \quad i = 1, ..., N, \ N = 5,$$

where

$$J_i(u_1^*, u_2^*, u_i^*, ..., u_N^*) = \int_0^{+\infty} [Q_i(x) + u_i^2] dt.$$