Lorentz symmetry violation due to interactions of photons with the graviton background

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Abstract

The average time delay of photons due to multiple interactions with gravitons of the background is computed in a frame of the model of low-energy quantum gravity by the author. The two variants of evaluation of the lifetime of a virtual photon are considered: 1) on a basis of the uncertainties relation (it is a common place in physics of particles) and 2) using a conjecture about constancy of the proper lifetime of a virtual photon. It is shown that in the first case Lorentz violation is negligible: the ratio of the average time delay of photons to their propagation time is equal approximately to 10^{-28}; in the second one (with a new free parameter of the model), the time-lag is proportional to the difference $\sqrt{E_{01}} - \sqrt{E_{02}}$, where E_{01}, E_{02} are initial energies of photons, and more energetic photons should arrive later, also as in the first case. The effect of graviton pairing is taken into account, too.

1 Introduction

Lorentz invariance is the cornerstone of physics of elementary particles, and a degree of its possible violation is of a great interest (see the review [1]). Possible Lorentz violation is often connected in our minds with quantum gravity

e ects; and it is alm ost com m only accepted that these e ects should reveal them selves at the P lank scales of energies and distances. It is another story that dealing w ith the P lank scale of distances suggests that our know ledge of gravity (general relativity) is true up to this scale [2]; but it is not a proofed fact. I w ould like to cite the recent paper [3] as a typical one in this direction; the authors speak about days or m onths of tim e–lags for photons of G R B 's in som e theoretical cases.

But in m y m odel of low -energy quantum gravity [4], gravity reveals asym ptotic freedom at very short distances beginning from 10^{11} 10^{13} m eter for di erent particles [5], i.e. very-very far from the P lank scale. In this paper, I have com puted the average tim e delay of photons due to m ul-tiple interactions w ith gravitons of the background in a fram e of the m odel [4]. T he two variants of evaluation of the lifetim e of a virtual photon are considered: 1) on a basis of the uncertainties relation (it is a com m on place in physics of particles) and 2) using a conjecture about constancy of the proper lifetim e of a virtual photon. It is show n that in the rst case Lorentz violation is negligible; in the second one (w ith a new free param eter of the m odel), the tim e–lag is proportional to the di erence $\sqrt{E_{01}}$ $\sqrt{E_{02}}$, w here E_{01}; E_{02} are initial energies of photons, and m ore energetic photons should arrive later, also as in the rst case. T he e ect of graviton pairing is taken into account, too.

2 T im e delay of photons due to interactions w ith gravitons

To com pute the average tim e delay of photons in the m odel [4], it is necessary to nd a num ber of collisions w ith gravitons of the graviton background on a sm all w ay dr and to evaluate a delay due to one act of interaction. Let us consider at rst the background of single gravitons. G iven the expression for H in the m odel, we can w rite for the num ber of collisions w ith gravitons having an energy $= h!$:

$$dN(\ )=\frac{|dE(\ )|}{\ }=E(r)\ \frac{dr}{c}\frac{1}{2}D\,f(!\,;T)d!\,;\qquad(1)$$

w here $f(!\,;T)$ is described by the P lank form ula. In the forehead collision, a photon loses the m om entum $=c$ and obtains the energy ; it m eans that for

a virtual photon we will have:

$$\frac{v}{c} = \frac{E}{E + \varepsilon}\ ;\quad 1 - \frac{v}{c} = \frac{2\varepsilon}{E + \varepsilon}\ ;\quad 1 - \frac{v^2}{c^2} = \frac{4\varepsilon E}{(E + \varepsilon)^2} : \tag{2}$$

2.1 Evaluation of the lifetime of a virtual photon on a basis of the uncertainties relation

The uncertainty of energy for a virtual photon is equal to $\Delta E = 2\varepsilon$. If we evaluate the lifetime using the uncertainties relation: $\Delta E \cdot \tau \sim h = 2\pi\hbar$, we get $\tau \sim h = 4\pi\hbar$. So as during the same time τ real photons overpass the way $c\tau$, and virtual ones overpass only the way $v\tau$, we have:

$$c\Delta t = c\tau - v\tau\ ;$$

where Δt is the time delay, and the last one will be equal to:

$$\Delta t(\varepsilon) = (1 - \frac{v}{c})\tau\quad h = 2\pi\hbar \frac{1}{E + \varepsilon} : \tag{3}$$

The full time delay due to gravitons with an energy ε is: $d\Delta t(\varepsilon) = \Delta t(\varepsilon)dN(\varepsilon)$. Taking into account all frequencies, we find the full time delay on the way dr:

$$d\Delta t \int_0^1 \frac{h}{2\pi} \frac{E}{E + \varepsilon} \cdot \frac{dr}{c} \frac{1}{2} D f(\omega; T) d\omega : \tag{4}$$

The one will be maximal for $\frac{\varepsilon}{E} \ll 1$; and it is easy to evaluate it:

$$d\Delta t_1 \sim \frac{h}{4\pi} \frac{dr}{c} D \sigma T^4 : \tag{5}$$

On the way r the time delay is:

$$\Delta t_1(r) \sim \frac{h}{4\pi} \frac{r}{c} D \sigma T^4 : \tag{6}$$

In this model: $r(z) = c/H \cdot \ln(1 + z)$; let us introduce a constant $h = 4\pi D \sigma T^4 = H = 37.2 \cdot 10^{12} s$, then

$$\Delta t_1(z) \sim \ln(1 + z): \tag{7}$$

We see that for $z' \sim 2$ the maximal time delay is equal to ~ 40 ps, i.e. the one is negligible.

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In the rest frame of a virtual photon, a single parameter, which may be juxtaposed with an energy uncertainty, is mc^2. Accepting $E = mc^2$ in this frame, we'll get:

$$t(z) = 2\tau \ln(1 + z) \qquad (8)$$

with the same τ; now this estimate doesn't depend on E.

2.2 The case of constancy of the proper lifetime of a virtual photon

Taking into account that for a virtual photon after a collision $(E^0 = c)^2 - p^{02} > 0$, we may consider another possibility of lifetime estimation, for example, $\tau_0 = const$, where τ_0 is the proper lifetime of a virtual photon (it should be considered as a new parameter of the model). Now it is necessary to transit to the reference frame of observer:

$$\tau = \tau_0 (1 - \frac{v^2}{c^2})^{-1=2} = \tau_0 \frac{E + \epsilon}{2\sqrt{\epsilon E}}; \qquad (9)$$

accordingly:

$$t(\epsilon) = \tau(1 - \frac{v}{c}) = \tau_0 \sqrt{\frac{\epsilon}{E}}; \qquad (10)$$

Then the full time delay due to gravitons with an energy ϵ is:

$$dt(\epsilon) = t(\epsilon)dN(\epsilon) = \tau_0 \sqrt{\frac{\epsilon}{E}} \frac{dr}{c} \frac{1}{2} D f(\omega, T)d\omega; \qquad (11)$$

and integrating it, we get:

$$dt = \tau_0 \sqrt{\frac{\epsilon}{E(r)}} \frac{dr}{c} \frac{1}{2} D \int_0^\infty \sqrt{\frac{\epsilon}{\epsilon}} f(\omega, T)d\omega: \qquad (12)$$

The integral in this expression is equal to:

$$\int_0^\infty \sqrt{\frac{\epsilon}{\epsilon}} f(\omega, T)d\omega \simeq \frac{1}{4\pi^2 c^2} \frac{(kT)^{9=2}}{h^3} I_6; \qquad (13)$$

where a new constant I_6 is the following integral:

$$I_6 \equiv \int_0^\infty \frac{x^{7=2}dx}{\exp x - 1} = 12.2681: \qquad (14)$$

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In this model, the energy of a photon decreases as [4]: $E(r) = E_0 \exp(-Hr/c)$. The full delay on the way r now is:

$$t(r) = \epsilon_0 \frac{D}{8\pi^3 c^2} \frac{(kT)^{9/2}}{h^3} \int_0^r \sqrt{E(r')} \frac{dr'}{c} = \tag{15}$$

$$= \epsilon_0 \frac{D}{8\pi^3 c^2} \frac{(kT)^{9/2}}{h^3} \frac{2}{H} (\sqrt{E_0} - \sqrt{E(r)}).$$

Let us introduce a new constant Φ_0 for which:

$$\frac{1}{\sqrt{\Phi_0}} = \epsilon_0 \frac{D}{8\pi^3 c^2} \frac{(kT)^{9/2}}{h^3} \frac{2}{H};$$

so $\Phi_0 = 2.391 \cdot 10^4$ eV; then

$$t(r) = \frac{\epsilon_0}{\sqrt{\Phi_0}} (\sqrt{E_0} - \sqrt{E(r)}) = \epsilon_0 \sqrt{\frac{E_0}{\Phi_0}} (1 - \exp(-Hr/2c)); \tag{16}$$

where E_0 is an initial photon energy. This delay as a function of redshift is:

$$t(z) = \epsilon_0 \sqrt{\frac{E_0}{\Phi_0}} \cdot \frac{\sqrt{1+z} - 1}{\sqrt{1+z}}. \tag{17}$$

In this case, the time-lag between photons emitted in one moment from the same source with different initial energies E_{01} and E_{02} will be proportional to the difference $\sqrt{E_{01}} - \sqrt{E_{02}}$, and more energetic photons should arrive later, also as in the first case. To find ϵ_0, we must compare the computed value of time-lag with future observations. An analysis of time-resolved emissions from the gamma-ray burst GRB 081126 [6] showed that the optical peak occurred (8.4 ± 3.9) s later than the second gamma peak; perhaps, it means that this delay is connected with the mechanism of burst.

2.3 An influence of graviton pairing

Graviton pairing of existing gravitons of the background is a necessary stage to ensure the Newtonian attraction in this model [7]. As it has been shown in the cited paper, the spectrum of pairs is the Planckian one, too, but with the smaller temperature $T_2 \approx 2^{-3/4}T$; this spectrum may be written as: $f(\omega_2, T_2)d\omega_2$, where $\omega_2 \approx 2\omega$. Then residual single gravitons will have

the new spectrum: $f(\omega;T)d\omega \to f(\omega_2;T_2)d\omega_2$, and we should also take into account an additional contribution of pairs into the time delay.

We shall have now:

$$dN(\omega) = E(r) \frac{dr}{c} \frac{1}{2\omega} D (f(\omega;T)d\omega \to f(\omega_2;T_2)d\omega_2); \qquad (18)$$

and for pairs with energies 2ω:

$$dN(2\omega) = \frac{|dE(2\omega)|}{2\omega} = E(r) \frac{dr}{c} \frac{1}{2\omega} D f(\omega_2;T_2)d\omega_2: \qquad (19)$$

After a collision of a photon with a pair, a virtual photon will have a velocity v_2: $v_2/c = (E-2\omega)/(E+2\omega)$, and a mass m_2: $m_2 c^2 = 2\sqrt{2\omega E}$.

For the case of subsection 2.1, after collisions with pairs: $E = 4\omega$, $\omega h = 8\omega$, and we get:

$$t(2\omega) \sim h = 2\omega \frac{1}{E+2\omega}: \qquad (20)$$

Then due to single gravitons and pairs:

$$dt_2(\omega) = dt^0(\omega) + dt(2\omega) \sim dt(\omega) \sim h = 2\omega \frac{E}{(E+\omega)(E+2\omega)} \frac{dr}{c} \frac{1}{2\omega} D f(\omega_2;T_2)d\omega_2; \qquad (21)$$

where $dt^0(\omega)$ is a reduced contribution of single gravitons, $dt(\omega)$ is its full contribution corresponding to formula (4). We see that if one takes into account graviton pairing, the estimate of delay became smaller. So as

$$\Delta E = (E+\omega)(E+2\omega) \to 0$$

by $\omega = E \to 0$; we have for the maximal delay in this case: $t_{21}(r) \to t_1(r)$, i.e. the maximal delay is the same as in subsection 2.1.

Repeating the above procedure for the case of subsection 2.2, we shall get:

$$t_2(r) = [1 + (1 - 1/\sqrt{2}) \cdot (T_2/T)^{9=2}] \cdot t(r) \simeq 1.028 \cdot t(r); \qquad (22)$$

where $t_2(r)$ takes into account graviton pairing, and $t(r)$ is described by formula (16). In this case, the full delay is bigger on about 2.8% than for single gravitons.

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3 Conclusion

Because in this model the propagation time for photons as a function of redshift is: $t(z) = r(z)/c = 1/H \cdot \ln(1 + z)$, the ratio of the average time delay of photons to their propagation time is equal approximately to 10^{-28} and doesn't depend on z in the first considered case. This very small quantity characterizes the degree of Lorentz violation in the model for the usually accepted manner of the lifetime evaluation. Even for remote astrophysical sources time-lags will be of the order of tens picoseconds, i.e. unmeasurable, and one may consider Lorentz symmetry as an exact one for any laboratory experiment. If the second considered case is realized in the nature, one should initially evaluate the free parameter of the model τ_0 from observations.

Some preliminary results of this work were used in my paper [8].

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