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MODELS DESCRIBING THE DEGRADATION OF FUNCTIONAL PARAMETERS OF ELECTRONIC DEVICES BASED ON THE WEIBULL–GNEDENKO DISTRIBUTION

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Abstract: The authors offer the possibility for obtaining the mathematical model of degradation of a functional parameter in the form of conditional density of its distribution over a given operating time period on the basis of the 3-parametric Weibull–Gnedenko distribution. This model provides reliability prediction errors for samples of electronic devices smaller, than the errors after using the degradation model based on normal distribution of the functional parameter.

Key words: degradation model of parameter, reliability prediction of electronic devices, semiconductor devices, experiment for reliability prediction.

1. Introduction

During operation of the electronic device its electrical functional parameter (denoted by y) changes, or is degraded, and can be considered as a function of time t. The degradation of the functional parameter displays reflects the appearance of the gradual failures of electronic devices. The criteria for the failure of the electronic device are the changes of functional parameter below or above some levels which are set by technical documentation and requirements of the consumer, is considered as failed. Gradual failures define such a concept as parametric reliability which characterizes the ability of electronic devices to maintain the level of functional parameter $y(t)$ within the norms (from $y_{\min}$ to $y_{\max}$) during specified operating time t_o under selected operating conditions and modes. A quantitative value of the parametric reliability is the probability of finding the parameter $y(t)$ within these norms during t_o, i.e.

$$P\left(t_0\right) = \text{Probability}\left\{y_{\min} \le y\left(t\right) \le y_{\max}, t \le t_0\right\} = \\ = P\left\{y_{\min} \le y\left(t\right) \le y_{\max}, t \le t_0\right\}, \quad (1)$$

where “Probability” will be replaced by symbol “P”.

The prediction of parametric reliability for electronic devices can be performed according to the model of degradation of the functional parameter (the model of gradual failures). The model is commonly used for highly reliable products and allows effective valuation of the level of parametric reliability during the production process for samples of electronic devices supplied to consumers [1, 2].

2. The relevance of research

For predicting the parametric reliability for samples of electronic devices, a mathematical model of degradation of the functional parameter $y(t)$ in the form of conditional density of its distribution $f(y|t)$ for the specified operating time t was proposed in [3, 4]. This model is obtained once for electronic devices of the type of interest with a preliminary experimental investigation (called “learning experiment”) of certain sample chosen randomly from the batch of electronic devices [3, 4]. The obtained model of degradation of the functional parameter $y(t)$ can be used at the initial time ($t = 0$) for predicting the parametric reliability of new samples of the same type of electronic devices for the specified operating time in the future. New samples are the samples of the same batch of electronic devices which were not used during the learning experiment.

Reliability prediction about new samples of electronic devices is based on the hypothesis that the quantitative characteristic of the parametric reliability $P(t_o)$ defined by the expression (1) can be obtained on the basis of the known law of distribution of the functional parameter $y(t)$ at the initial time, for example the conditional distribution density $f(y|t = 0)$ and its changes during the operating time of electronic devices (Fig. 1).

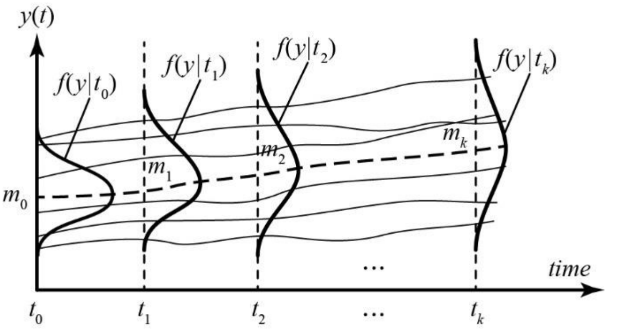

Fig. 1. Changing of the density distribution of functional parameter $y(t)$ for the device at timepoints $t_0, t_1, ..., t_k$; $m_0, m_1, ..., m_k$ are average values of the parameter y at the timepoints.

Value of interest $P(t_o)$ is the result of changes in the statistical distribution $f(y \mid t)$ during operating time of electronic devices.

The approximate analytical expression for the conditional distribution density $f(y \mid t)$ for any specified operating time $t = t_o$ can be obtained by mathematical transformations of the initial distribution $f(y \mid t = 0)$:

$$f(y|t=t_0) = y\left[f(y|t=0), t_0\right], \qquad (1)$$

where ψ is a symbol of functional dependence.

Physicochemical characteristics of degradation of the functional parameter $y(t)$ obtained by averaging the test sample of electronic devices will be included as coefficients in the right-hand side of expression (2).

The predictive value of the quantitative characteristics of the parametric reliability $P(t_o)$ is determined by the accepted rules of finding probabilities (1) using the distribution law of random variables of probability theory [5].

In [2–4, 6] the normal distribution law of the functional parameter $y(t)$, traditionally used in electronics, is taken as a basis for obtaining the degradation model. In this case, the conditional distribution density $f(y|t)$ for the considered timepoint t depends on two parameters: mean value $m(y|t)$ and standard deviation $\sigma(y|t)$ of the functional parameter $y(t)$ in a timepoint t. According to the expression (2) values $m(y|t)$ and $\sigma(y|t)$ are determined as the functions of operating time t and the values $m(y|t = 0)$ and $\sigma(y|t = 0)$, which are the parameters of the normal law at initial time ($t = 0$):

$$m(y\,|\,t) = j_1\left[t, m(y\,|\,t=0), s\,(y\,|\,t=0)\right]; \qquad (3)$$

$$s\,(y\,|\,t) = j_2\left[t, m(y\,|\,t=0), s\,(y\,|\,t=0)\right]. \qquad (4)$$

The degradation model based on the normal distribution law is a classic model which has its disadvantages due to the fact that in some cases the distribution law of functional parameters of electronic devices can significantly differ from a normal one. For example, for high power electronic devices, performing output control operations (namely, the selection by the values of the parameters), training and then the further operation can significantly distort the normal distribution law of the parameter $y(t)$ in timepoints. This is confirmed by distribution histograms of such functional parameters of the high power bipolar transistor (BT) KT872A (whose prototype is a transistor BU508A) as β (static base current gained in circuit with a common emitter) and $V_{CE(sat)}$ (saturation collectoremitter voltage). Fig. 2-5 show the distribution histograms plotted for parameters β and $V_{CE(sat)}$ according to experimental data for timepoints of $t = 0$ and $t = 17280$ hours. Vertical axes indicate the relative frequencies p (in percentage terms) which show functional parameters belonging to specified value ranges.

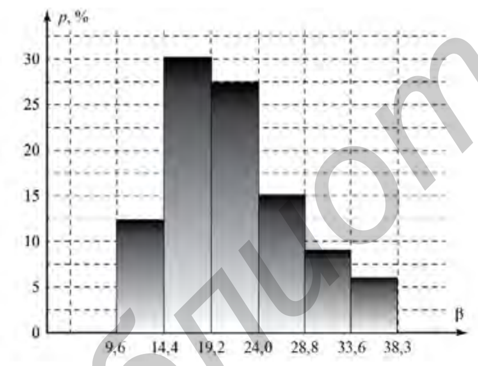

Fig. 2. Histogram of β distribution for timepoint t = 0.

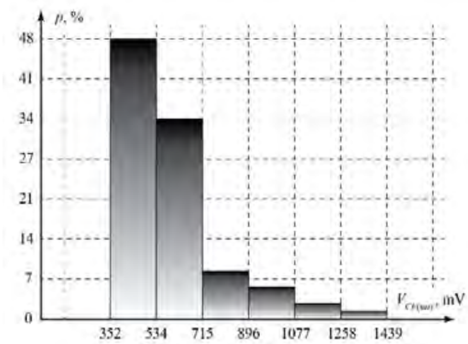

Fig. 3. Histogram of $V_{CE(sat)}$ *distribution for timepoint t = 0.*

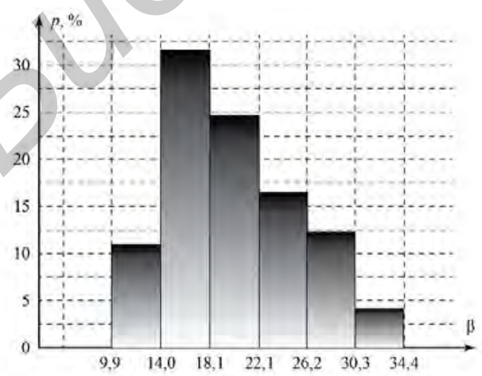

Fig. 4. Histogram of β distribution for timepoint t = 17280 h.

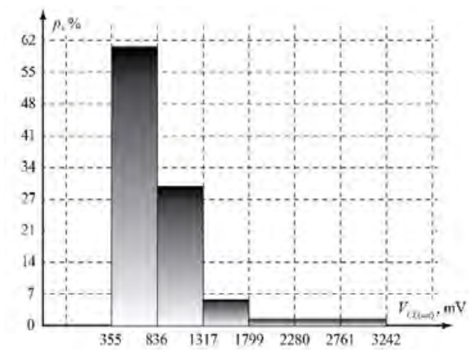

Fig. 5. Histogram of $V_{CE(sat)}$ *distribution for timepoint t = 17280 h*

After the analysis of Fig. 2–5, it is possible to draw two important conclusions:

1) for some parameters of electronic devices in the time point t = 0 the normal distribution law does not describe the dispersion of the parameter properly (example is $V_{CE(sat)}$). This could be caused by manufacturing operations providing the selection of electronic devices according to parameter values corresponding to the requirements of technical documentation for this product group;

2) in the course of operating time the form of the distribution density of the parameter is deformed, values of the characteristics of the distribution law change in comparison with values in the initial time point t = 0. And here there are two possibilities:

- normal distribution describing the change of parameter in the time point t = 0 quite well, is distorted (deformed); and in time points t >> 0 the density distribution of the parameter is noticeably different from normal law; an example of this case is the parameter β;
- the form of the graph of distribution density of the parameter for operating time t >> 0 is maintained, but the characteristics of distribution law of the parameter significantly change; an example of this case is the parameter *VCE*(*sat*).

The use of the degradation model of the parameter based on the hypothesis of normal distribution law [6] when the operating time significantly differs from zero leads to significant errors in prediction of parametric reliability for new samples of electronic devices. And the longer operating time is, the higher prediction errors are. Therefore, the actual problem is the choice of such a model of degradation that would provide good prediction results in the case of normal distribution of the functional parameter of electronic devices and, at the same time, would be able to respond to possible deviations from the normal distribution law. In this case it is desirable to obtain the degradation model of functional parameters for electronic devices based on one universal distribution law.

3. The new degradation model

The new degradation model of parameters for electronic devices has been proposed by the authors in [7].

This model is based on the use of the translated (or 3-parameter) Weibull–Gnedenko distribution [7], according to which the conditional distribution density of the functional parameter y of sample of electronic devices at any timepoint t (mathematical model of degradation) can be written as

$$f\left(y \mid t\right) = \frac{b}{a}\left(\frac{y \mid t - c}{a}\right)^{b-1} \exp\left[-\left(\frac{y \mid -c}{a}\right)^{b}\right], \quad (5)$$

where $y|t$ is the value of the considered functional parameter of electronic devices which corresponds to operating time t ($y|t$ > c); a, b, c are distribution parameters having been found for the timepoint t; a is a scale parameter (a > 0), b is a shape parameter (b > 0), c is a translation parameter (displacement) indicating displacement of y relative to its zero value.

The argument in favour of choosing 3-parameter Weibull–Gnedenko distribution as the basis of degradation model for the electronic devices is the form of distribution histograms of functional parameters obtained from experimental data (see Fig. 2–5).

Fig. 4 shows that the envelope line for histogram of distribution of parameter β for t = 17280 hours reminds a bell-shaped curve similar to a Gaussian curve, however, the hypothesis of normal distribution is not confirmed by a chi-squared test (Pearson's χ^2) and the Kolmogorov-Smirnov test. The hypothesis of 3-parameter Weibull–Gnedenko distribution does not conflict with statistical criteria. The assessment of parameters a, b and c of this distribution was obtained from the experimental data, see Table 1.

For the timepoints t = 0 and t = 17280 hours, as you can conclude from Fig. 3 and 5, the distribution of the functional parameter $V_{CE(sat)}$ significantly differs from the normal distribution law. Its experimental data does not conflict with the hypothesis of 3-parameter Weibull–Gnedenko distribution with assessments of distribution parameters a (mV), b and c (mV), shown in Table 1. For both functional parameters (β and $V_{CE(sat)}$) in Table 1 you can see assessments of characteristics m and σ under the assumption of the normal distribution law.

The proposed degradation model (5) is universal to a certain extent because it describes the change of functional parameter y quite well for any kind of its degradation for the sample of electronic devices, i.e. in any form of the curve of its density distribution which can really take place for electronic devices. For example, for b = 1 distribution (5) coincides with the 2-parameter exponential distribution, for c = 0 with classical (2-parameter) Weibull-Gnedenko distribution, for b = 2 with Rayleigh distribution with $a/2^{0,5}$ scale parameter. In the case b > 3–5 distribution (5) insignificantly differs from the normal distribution law.

Table 1

The experimental values of distribution parameters

Functional parameter of BT	Timepoint *t*, hours	Parameters of normal distribution		Parameters of Weibull-Gnedenko distribution		
		m	σ	*a*	*b*	*c*
β	0	22,12	5,83	17,04	3,02	6,94
	280	19,82	5,16	14,53	2,73	6,91
$V_{CE(sat)}$	0	604,1 mV	187,1 mV	181,8 mV	0,98	mV
	280	878,4 mV	479,0 mV	350,4 mV	0,82	mV

According to the expression (2), values of a, b and c of the model (5) for a given operating time t can be obtained as the functions of operating time t and values a_0, b_0 and c_0, which are the parameters of the 3-parameter Weibull–Gnedenko distribution in timepoint $t = 0$ (initial time). However, for practical applications it is more interesting to obtain the values a, b and c as functions of operating time t and basic numerical characteristics of functional parameter $y(t)$ at the initial time, that is, the average value $m(y|t = 0)$ and standard deviation $\sigma(y|t = 0)$:

$$a = j_1\left[t, m(y\,|\,t = 0), s\ (y\,|\,t = 0)\right]; \qquad (6)$$

$$b = j_2\left[t, m(y\,|\,t = 0), s\ (y\,|\,t = 0)\right], \qquad (7)$$

$$c = j_3\left[t, m(y\,|\,t = 0), s\ (y\,|\,t = 0)\right]. \qquad (8)$$

For obtaining operators φ_1, φ_2, φ_3 of functions (6)–(8) the method [7] is proposed, according to which after the completion of the accelerated tests a training sample (TS) is divided into several groups, and a table with the results of passive factor experiment (Table 2) is formed. In Table 2 the following symbols are accepted: n_1, n_2, …, n_q is the number of the instance of training sample corresponding to the last item of a relevant group; q is the number of groups; k is the number of time points; t_i is a time point, for which the values of functional parameters and their degradation during the accelerated tests conducted on the electronic devices from the training sample were monitored, $i = 0, 1, \ldots, k$.

The first two columns of Table 2 set the number of a test of the passive experiment. Values m_0, σ_0 and t_i (where $i = 0, 1, \ldots, k$) are considered as factors (arguments), and the parameters of interest a, b and c of the model (5) as response functions. The subscript zero index of values m and σ shows that they belong to the initial time $t = 0$. Superscript index of all values indicates the group number of the value. In the last two columns values of $m(y|t_i)$, $\sigma(y \mid t_i)$ are placed, they are also considered as the response functions and are used to construct degradation model based on the hypothesis of normal distribution of the functional parameter $y(t)$ in timepoints, when $m^{(r)}(y \mid t_0) = m_0^{(r)}$ and $\sigma^{(r)}(y \mid t_0) = \sigma_0^{(r)}$, (where $r = 1, 2, \ldots, q$).

The type of operators φ_1, φ_2, φ_3 in expressions (6)–(8) is determined by processing the results of passive factor experiment. Conditional density function (5) obtained by taking into account expressions (6)–(8), is a mathematical model of the degradation of the considered functional parameter $y(t)$ and can be used to predict the parametric reliability of new samples of electronic devices of the same type.

4. Group prediction of the parametric reliability by the proposed model of degradation

During group prediction the forecast is obtained as the probability that a functional parameter $y(t)$ of the sample of electronic devices for the interest operating time t_i does not go beyond the norms (from $y_{\min}$ to $y_{\max}$) specified in the technical documentation or established by the customer of electronic devices [2]. To calculate this probability $P(t_i)_{pr}$ in accordance with expression (2) and degradation model (5) the following formula was obtained:

$$P(t_i)_{pr} = \exp\left\{-\left[\frac{y_{\min} - c(t_i)}{a(t_i)}\right]^{b(t_i)}\right\} - \exp\left\{-\left[\frac{y_{\max} - c(t_i)}{a(t_i)}\right]^{b(t_i)}\right\}, \qquad (9)$$

where $y_{\min}$, $y_{\max}$ are the lower and upper limits of norms established for the parameter y; $a(t_i)$, $b(t_i)$ and $c(t_i)$ are values of a, b and c of a model (5) calculated by the expressions (6)–(8) for operating time t_i.

Table 2

The results of passive factorial experiment

Number of instance for TS	Group number	Factors			Response function				
		$m_0 = m(y\|t = 0)$	$\sigma_0 = \sigma(y\|t = 0)$	Timepoint t_i	a	b	c	$m(y\|t_i)$	$\sigma(y\|t_i)$
$1…n_1$	1	$m_{0(1)}$	$\sigma_{0(1)}$	t_0	$a_{(1)}(t_0)$	$b_{(1)}(t_0)$	$c_{(1)}(t_0)$	$m_{(1)}(y\|t_0)$	$m_{(1)}(y\|t_0)$
$(n_1 + 1)…n_2$	2	$m_{0(2)}$	$\sigma_{0(2)}$	t_0	$a_{(2)}(t_0)$	$b_{(2)}(t_0)$	$c_{(2)}(t_0)$	$m_{(2)}(y\|t_0)$	$m_{(2)}(y\|t_0)$
…	…	…	…	…	…	…	…	…	…
$(n_{q-1} + 1)…n_q$	q	$m_{0(q)}$	$\sigma_{0(q)}$	t_0	$a_{(q)}(t_0)$	$b_{(q)}(t_0)$	$c_{(q)}(t_0)$	$m_{(q)}(y\|t_0)$	$m_{(q)}(y\|t_0)$
$1…n_1$	1	$m_{0(1)}$	$\sigma_{0(1)}$	t_1	$a_{(1)}(t_1)$	$b_{(1)}(t_1)$	$c_{(1)}(t_1)$	$m_{(1)}(y\|t_1)$	$m_{(1)}(y\|t_1)$
$(n_1 + 1)…n_2$	2	$m_{0(2)}$	$\sigma_{0(2)}$	t_1	$a_{(2)}(t_1)$	$b_{(2)}(t_1)$	$c_{(2)}(t_1)$	$m_{(2)}(y\|t_1)$	$m_{(2)}(y\|t_1)$
…	…	…	…	…	…	…	…	…	…
$(n_{q-1} + 1)…n_q$	q	$m_{0(q)}$	$\sigma_{0(q)}$	t_1	$a_{(q)}(t_1)$	$b_{(q)}(t_1)$	$c_{(q)}(t_1)$	$m_{(q)}(y\|t_1)$	$m_{(q)}(y\|t_1)$
…	…	…	…	…	…	…	…	…	…
$1…n_1$	1	$m_{0(1)}$	$\sigma_{0(1)}$	t_k	$a_{(1)}(t_k)$	$b_{(1)}(t_k)$	$c_{(1)}(t_k)$	$m_{(1)}(y\|t_k)$	$m_{(1)}(y\|t_k)$
$(n_1 + 1)…n_2$	2	$m_{0(2)}$	$\sigma_{0(2)}$	t_k	$a_{(2)}(t_k)$	$b_{(2)}(t_k)$	$c_{(2)}(t_k)$	$m_{(2)}(y\|t_k)$	$m_{(2)}(y\|t_k)$
…	…	…	…	…	…	…	…	…	…
$(n_{q-1} + 1)…n_q$	q	$m_{0(q)}$	$\sigma_{0(q)}$	t_k	$a_{(q)}(t_k)$	$b_{(q)}(t_k)$	$c_{(q)}(t_k)$	$m_{(q)}(y\|t_k)$	$m_{(q)}(y\|t_k)$

In most cases of practical use of formula (9) for electronic devices it necessary to take into account the unilateral standards established on the functional parameter $y(t)$, because the theoretical range of the parameter which meets customer needs can take values: $y_{\min} = c(t_i)$, $y_{\max} = \infty$ depending on the physical nature of the parameter. In these cases, either the left part of the residual in expression (9) takes the value equal to one, or right-hand side takes the value $\rightarrow 0$.

5. Experimental investigations and methods of their conducting

The proposed degradation model (5) was tested on the high power BT of types KT872A, KT8225A (whose prototype is BU941ZP STMicroelectronics) and КП723Г (the prototype is IRFZ44). As a practical example, the BT of type KT872A was chosen. For that type β and $V_{CE(sat)}$ were considered as functional parameters and its electrical measurement modes met the requirements of BT technical documentation. The degradation model (5) was based on the learning sample of volume $n = 100$ devices. For obtaining data of degradation of functional parameters, physical modeling of operating time of BT was used, which included their conducting of accelerated forced tests according to standard methods [9–10]. These tests were equivalent to operating time of 17280 hours for normal working conditions: ambient temperature $T = +55$ ºC, the coefficient of electrical load on the power dissipation $K = 0{,}5$. Normal operating conditions are taken into account with the standard modes of (thermal and electrical) operation of high power transistors as a part of electronic equipment.

Checking the efficiency of the constructed degradation model was performed on the control sample containing 100 instances. For this sample the problem of group parametric reliability prediction was solved by proposed mathematical degradation model with the use of expression (9) for four timepoints t_i (3840, 8320, 12800 and 17280 hours). Then accelerated forced tests of the transistors were performed and the values of their functional parameters β and $V_{CE(sat)}$ [collectively $y(t)$] were controlled at the indicated timepoints. Using the obtained data and taking into account the norms accepted for the parameters β and $V_{CE(sat)}$, experimental parametric reliability level assessments of the control sample were identified for specified time sections.

By comparing the experimental (actual) and predicted levels of parametric reliability an average prediction error $\Delta_{av}(y_{user})$ was calculated depending on the norm y_{user}, which was established by the user for the parameter $y(t)$. To determine $\Delta_{av}(y_{user})$ the expression [2] was used

$$\Delta_{av}\left(y_{user}\right)=\sqrt{\frac{1}{k}\sum_{i=1}^{k}\left(\frac{P_{pr\,i}-P_{ex\,i}}{P_{ex\,i}}\right)^{2}}\cdot 100\,\%, \qquad (10)$$

where k is a number of timepoints, for which predictive and experimental values of the parametric reliability level were found; $P_{pr\,i} = P(t_i)_{pr}$ is a predictive value of the parametric reliability level for electronic devices from the control sample obtained from the expression (9) for i-th timepoint ($i = 1, 2, \ldots, k$); $P_{ex\,i} = P(t_i)_{ex}$ is an experimental value of the parametric reliability level for electronic devices of the control sample calculated for i-th timepoint ($i = 1, 2, \ldots, k$).

In this case, the method of determining probability of the event by the frequency of its occurrence [5] was used to calculate $P(t_i)_{ex}$:

$$P\left(t_i\right)_{ex}=\frac{r\left(a\le y\le b\right)}{r},\; i=1,2,\ldots k., \qquad (11)$$

where $r(y_{\min} \le y \le y_{\max})$ is the number of instances in the control sample, for which the functional parameter $y(t)$ at the time t_i lies within the specified norms from $y_{\min}$ to $y_{\max}$; r is the total number of instances in the control sample (control sample volume).

According to the values of found errors $\Delta_{av}(y_{user})$ the conclusions about the effectiveness of the proposed degradation model were made.

6. The effectiveness of mathematical degradation model

For obtaining expressions (6)–(8) used for calculating the parameters a, b and c for the proposed degradation model (5), the learning sample was divided into three groups with 33 instances in each group. For each group and for all considered timepoints t_i ($i = 0, 1, \ldots, 4$) the values a, b, c of the model (5) and the values $m(y \mid t)$, $\sigma(y \mid t)$ of degradation model based on the normal degradation law were obtained. Taking these values into account the results of passive factor experiment were formed (see Table 2).

According to the results of processing the passive factor experiment expressions (6)–(8) for parameter β were obtained in the form of

$$a=-7{,}1994m_0+30{,}8412s_0-0{,}442t^{0{,}2}; \qquad (12)$$

$$b=0{,}1391m_0-0{,}002246t^{0{,}5}, \qquad (13)$$

$$c=8{,}0045m_0-29{,}7228s_0, \qquad (14)$$

where $m_0 = m(y \mid t = 0)$, $\sigma_0 = \sigma(y \mid t = 0)$ are a mean value and standard deviation of the functional parameter $y(t)$, corresponding to the initial time for a new sample of electronic devices, for which parametric reliability is going to be predicted; t is specified operating time which is interesting according to parametric reliability of electronic devices by the parameter $y(t)$.

The regression equations similar to (12)–(14) were obtained for the parameter $V_{CE(sat)}$.

To compare the effectiveness of proposed degradation model (5) with a model based on the normal distribution of functional parameter $y(t)$ the regression equations were obtained for the values $m(y|t)$ and $\sigma(y|t)$ taking into account expressions (3) and (4).

During group prediction of the parametric reliability for the control sample, condition (1) for the function parameters β and $V_{CE(sat)}$ was accepted as $\beta \ge \beta_{user}$ and $V_{CE(sat)} \le V_{user}$, where β_{user}, V_{user} were norms of the corresponding functional parameter specified by the user. This prediction was performed for all time sections t_i, except of $t = 0$ (initial time). In the case of degradation model (5), characteristic of reliability $P(t_i)$ was determined by the expression (9) taking into account the parameters a, b and c calculated by regression equations (12)–(14) for the corresponding timepoint t_i ($i = 1, 2, \ldots, 4$). In the case of degradation model based on the hypothesis of a normal law generally accepted formula for determining the probability of random variable with entering in the range of values [5] was used for the prediction taking into account the values $m(y|t_i)$ and $\sigma(y|t_i)$ calculated by the received regression equations for timepoint t_i.

Values of average prediction error Δ_{av} calculated for different norms established for the functional parameters β and $V_{CE(sat)}$ are shown in Tables 3 and 4. The error is defined for two distribution hypotheses of β and $V_{CE(sat)}$ for timepoint t_i: normal distribution law and 3-parameter Weibull-Gnedenko distribution law.

Described regularities of deformation of the density distribution for functional parameters in the course of operating time take place for the other types of bipolar and field-effect transistors (КТ8225А, КП723Г). It is noticed that these regularities are the most evident for high-power semiconductor devices, i.e. for those devices whose discrete model is widely used in electronic equipment for various applications (automotive and industrial power electronics, control units of a variety of devices, etc.).

Table 3

Values of average prediction error $\Delta_{av}(y_{user})$ of parametric reliability, parameter β

Degradation model law	Average prediction error $\Delta_{av}(\beta_{user})$ in percentage terms at norm for parameter								
	15	16	17	18	19	20	21	22	23
Normal distribution	5,4	6,5	9,4	9,8	7,8	9,4	9,9	12,9	20,1
Weibull–Gnedenko distribution	3,2	3,8	3,4	4,0	5,2	5,4	5,4	6,9	5,8

Table 4

Values of average prediction error $\Delta_{av}(y_{user})$ parametric reliability, parameter $V_{CE(sat)}$

Degradation model law	Average prediction error $\Delta_{av}(V_{user})$ in percentage terms at norm for parameter, V								
	0,7	0,8	0,9	1,0	1,2	1,4	1,6	1,8	2,0
Normal distribution	24,9	16,1	15,2	11,7	6,1	1,7	2,2	1,9	2,6
Weibull–Gnedenko distribution	5,6	6,2	3,2	2,4	2,3	0,8	0,9	0,8	0,9

7. Conclusion

On the example of bipolar transistors it is shown that mathematical model of the degradation of the functional parameter in the form of a conditional density of its distribution for a given operating time obtained on the basis of the 3-parameter Weibull–Gnedenko distribution provides smaller prediction errors of the parametric reliability of new BT samples, than the degradation model on the basis of normal distribution law for cases where the functional parameters in time sections are distributed according to the law both close to normal distribution law (functional parameter β, see Table 3), and significantly different from it (functional parameter $V_{CE(sat)}$, see Table 4).

The 3-parameter Weibull–Gnedenko distribution law to a certain extent should be viewed as universal because for samples of electronic devices it describes the degradation of the functional parameter quite well for any form of the experimental distribution law which may take place to the functional parameters of electronic devices in any time point (for any specified operating time).

References

[1] D. Coit, J. Evans, N. Vogt, and J. Thomson, “A Method for Correlating Field Life Degradation with Reliability Prediction for Electronic Modules”, *Quality and Reliability Engineering International*, vol. 21, issue 7, pp. 715–726, 29 September 2005.

[2] S. Borovikov, *Statistical Forecasting for the Rejection of Potentially Unreliable Electronic Devices*. Moscow, Russia: Novoye Znaniye, 2013. (Russian)

[3] S. Borovikov, A. Shalak, A. Beresnevich, and A. Emelyanov, “Physical-statistical models of degradation of functional parameters of electronic devices”, *Reports of The National Academy of Sciences of Belarus*, vol. 51, no. 6, pp. 105–109, 2007. (Russian)

[4] S. Borovikov, A. Shalak, A. Beresnevich, A. Emelyanov, and E. Shneiderov, “Reliability Forecasting of electronic devices on the basis of a mathematical model of degradation of functional parameter”, *Reports of Belarusian State University of Informatics and Radioelectronics: electronics, materials, technology, informatics*, vol. 36, no. 6, pp. 3239, 2008. (Russian)

[5] E. S. Ventcel, *Probability Theory*. Moscow, Russia: Nauka, 1969. (Russian)

[6] S. Borovikov, E. Shneiderov, and I. Burak, “Models based on the Weibull–Gnedenko distribution for describe the degradation of the functional parameters of electronic devices”, *Reports of The National Academy of Sciences of Belarus*, vol. 59, no. 3, pp. 109–115, 2015. (Russian)

[7] R. Vadzinskiy, *Handbook of Probability Distributions*. St. Petersburg, Russia: Nauka, 2001. (Russian)

[8] U. Miller, *Physical Bases of Integrated Circuits Reliability*. Moscow, Russia: Sovietskoye Radio, 1976. (Russian)

[9] *QuickLogic Reliability Report*. Sunnyvale, CA, USA: pASIC, Vialink and QuickLogic Corp., 1998.

[10] *Reliability Audit Report 1999. Life Test Data*. ON Semiconductor L.L.C., Formerly a Division of Motorola, 1999.

МОДЕЛІ ДЛЯ ОПИСУ ДЕГРАДАЦІЇ ФУНКЦІОНАЛЬНИХ ПАРАМЕТРІВ ЕЛЕКТРИЧНИХ ПРИЛАДІВ НА ОСНОВІ РОЗПОДІЛУ ВЕЙБУЛА–ГНЕДЕНКА

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Автори пропонують можливість отримання математичної моделі деградації функціонального параметра у вигляді умовної густини його

розподілу для заданого часу роботи на основі 3-параметричного розподілу Вейбула–Гнеденка. Ця модель забезпечує похибку прогнозування надійності для зразків електронних приладів, яка є меншою, ніж похибки після використання моделі деградації на основі нормального розподілу функціонального параметра.

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