COLLISIONS AND STABILITY OF QUANTUM WAVE PACKETS

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Abstract – Time evolution of the quantum wave packets is discussed in context of the non-linear cubic Shrödinger equation and equivalent hydrodynamical description. In hydrodynamical description, the quantum Hamilton-Jacobi equation for action is rewritten in variables: probability density and probability flow density. These variables are smooth at the node points. The studied dynamical systems have finite dimensions and impenetrable walls, they have been analyzed at the different initial conditions including the Gaussian-like form. Our interest in investigation of the properties of dynamical non-linear equations is caused by existence of stable solutions which correspond to the quantum non-spreading wave packets. Behavior of the localized wave packet in one-dimensional system is characterized by classic-like trajectory and collisions against walls. The packet keeps localized form during some time interval and can oscillate around some "stable" profile. To describe the time evolution of two wave packets on plane we have to integrate the non-stationary two dimensional Shrödinger equation for the two-particle wave function taking into account the symmetry properties. But, as first step, in present investigation the problem was essentially simplified. The particles were considered as spinless, and wave function was presented in the product form of two functions. Now, the collisions between quantum wave packets of two particles will also occur. During some time interval, fragmentations of packets are generated. Then they return to its original shape and move as classical particles. In both cases, non-linearity plays self-organizing role in comparison to the regimes when non-linear cubic term is absent.

I. SINGLE WAVE PACKET DYNAMICS

In simple case, we investigated solutions to the one-dimensional Shrödinger equation

$$i\frac{\partial\Psi}{\partial t} = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Psi + k\left|\Psi\right|^2\Psi + U\,\Psi \tag{1}$$

for wave function $\Psi(x,t)$ with different initial conditions in the form $\Psi_0 = C\cdot sch(x)\cdot\exp(iV_0 x)$ and $\Psi_0 = 1/\sqrt[4]{2\pi\sigma^2}\cdot\exp\left\{-(x-x_0)^2/4\sigma^2\right\}$. The equivalent description is given by hydrodynamical equations

$$\begin{aligned}
&\frac{\partial N}{\partial t} = -\frac{\partial J}{\partial x}, \\
&\frac{\partial J}{\partial t} = \frac{\partial}{\partial x}\left(-\frac{J^2}{N} + \frac{1}{4}\left(\frac{\partial^2 N}{\partial x^2} - \frac{1}{N}\left(\frac{\partial N}{\partial x}\right)^2\right) + \kappa N^2\right) - \frac{\partial U}{\partial x}N,\ \ \kappa = -k.
\end{aligned} \tag{2}$$

Here, N, J are probability density and probability flow density, respectively. The variables (N, J) are smooth at the node points. Stationary solutions (1) and (2) were also studied. Our interest to the properties Eq. 1 is caused by stable solutions which correspond to the quantum non-spreading wave packets [1]. In context of listed equations, we investigated soliton-like wave packets, fragmentations under collisions against the walls of potential well and self-reconstruction to its original form caused by nonlinearity. The comparison with solutions at $\kappa = 0$ is carried out.

II. INTERPACKET COLLISIONS (COLLISIONS BETWEEN PARTICLES)

To describe two wave packets at the same time we offer the simplified dynamical model where collisions between packets can be described with potential terms as follows

$$\begin{aligned}
i\frac{\partial\Psi_1}{\partial t} &= -\frac{1}{2}\Delta\Psi_1 - \kappa\left|\Psi_1\right|^2\Psi_1 + U_{1\leftarrow 2}(\Psi_2), \\
i\frac{\partial\Psi_2}{\partial t} &= -\frac{1}{2}\Delta\Psi_2 - \kappa\left|\Psi_2\right|^2\Psi_2 + U_{2\leftarrow 1}(\Psi_1).
\end{aligned} \tag{3}$$

Here, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$; $U_{1\leftarrow 2}(\Psi_2)$, $U_{2\leftarrow 1}(\Psi_1)$ are modeled terms describing collisions. It is essentially to propose that mean coordinates of two colliding particles behave by nearly classical way. Our calculations confirm this statement. It is important to note both packets show light fragmentation when they are placed nearby. For initial specified packets and $\kappa = 5$, the time evolution is presented in Figs. 1-4.

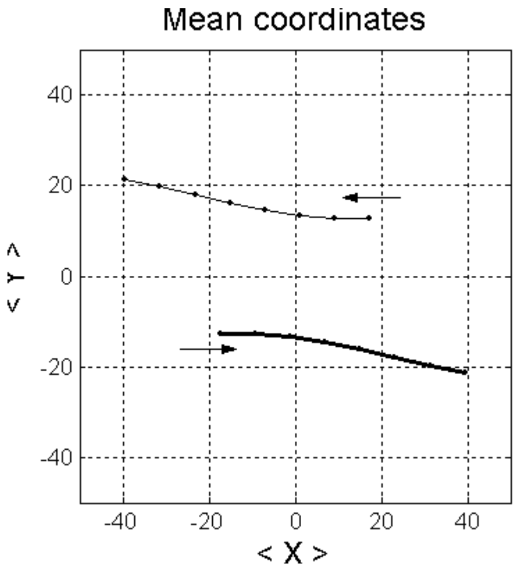

Figure 1 – Mean coordinates of particles

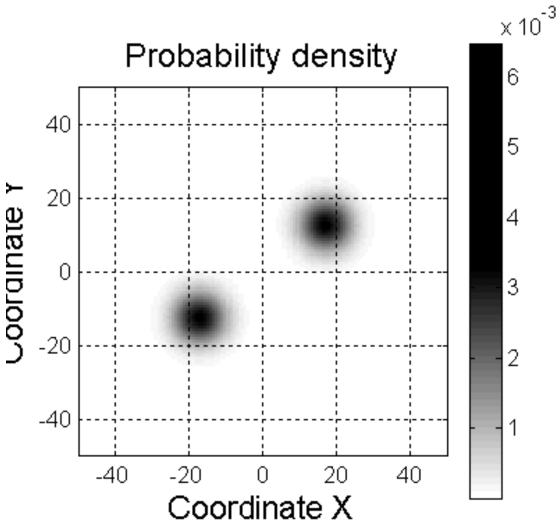

Figure 2 – Wave packets at t = 6.28

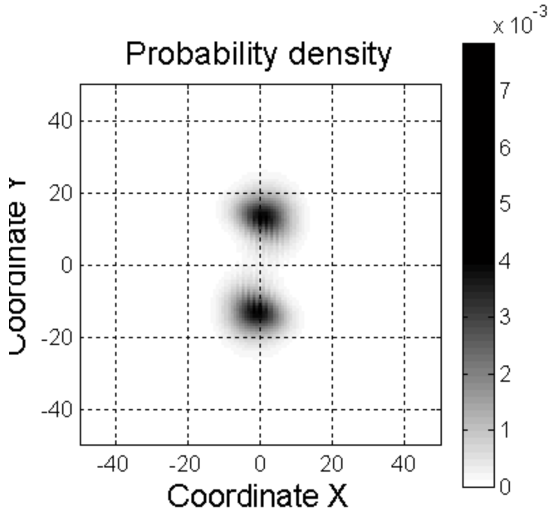

Figure 3 – Wave packets at t = 18.85

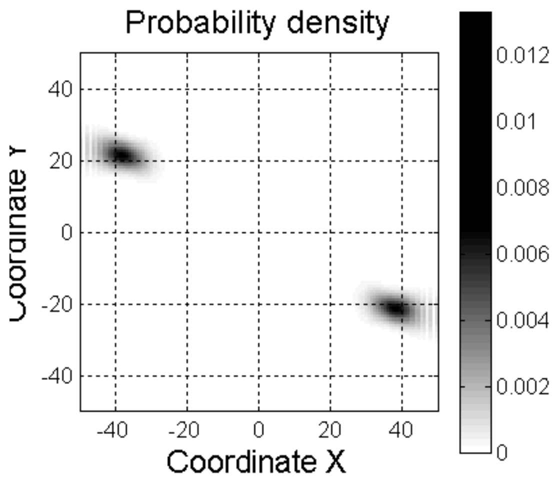

Figure 4 – Wave packets at t = 50.27

In both cases, non-linearity plays self-organizing role in comparison with regimes when non-linear cubic term is absent.

References

[1] A. L. Sanin. A. T. Bagmanov, Waves in quantum systems with nonlinearity and walls // Material Physics and Mechanics, V. 13, No. 2, 2012, P. 117-123.

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