Sensor Location Problem's Software Optimization

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***Abstract*—In this work we consider the application of the graph theory for construction the optimal and suboptimal solutions to the sensor location problem. That problem is named Sensor Location Problem for a graph (SLP). For constructing the solution of the SLP for a graph we presented the pseudocodes of the algorithm's for finding the flow arcs for the non observered part on the network. In the pseudocode of the algorithm 1 defines sensor configurations of the suboptimal solution and flows on the arcs on the unobserved part of the network.**

***Keywords*—software optimization, sensor location problem, sparse linear system, suboptimal and optimal solution**

I. INTRODUCTION

The problem of locating sensors on the network to monitoring flows has been object of growing interest in the past years, due to its relevance in the field of traffic management and control [1]–[4]. The basis for modeling the processes of estimating flows in network is a sparse underdetermined systems of linear algebraic equations of a special types [5], [6]. Sensors are located in the nodes of the network for the given traffic levels on arcs within range covered by the sensors, that would permit traffic on any unobserved flows on arcs to be exactly.

This work is devoted to the research of intelligent transport systems and their applications. The obtained theoretical and practical results are an important contribution to the solution of problems in the field of environmental monitoring. Technologies and algorithms for one practical solution problem of ecological monitoring and analysis of flows on the unobserved part of the transport network are developed.

The suboptimal solutions for the network programming problem are considered in [7]. The common solutions for the sparse underdetermined systems of linear algebraic equations are obtained in [8]. In this work we research the numerical results for constructing the suboptimal solutions of SLP problem for various values of the intensity threshold.

II. SENSOR LOCATION PROBLEM

Let's introduce the finite connected directed graph $G = (I, U)$. The set U is defined on $I \times I$ ($|I| < \infty, |U| < \infty$). We assume, that the graph G is symmetric: that is: if $(i,j) \in U$, then $(j,i) \in U$. We note that the graph G is not undirected: the flow on arc (i,j), in general, will not be the same as the flow on arc (j,i). To designate this distinction, we refer to the graph $G = (I, U)$ as a two way directed graph.

We represent the traffic flow by a network flow function $x : U \to \mathbb{R}$ that satisfies the following system:

$$\sum_{j \in I_i^+(U)} x_{i,j} - \sum_{j \in I_i^-(U)} x_{j,i} = \begin{cases} x_i, & i \in I^*, \\ 0, & i \in I \setminus I^* \end{cases} \tag{1}$$

where I^* is the set of nodes with variable intensities, x_i is the variable intensity of node $i \in I^*$, $I_i^+(U) = \{j \in I : (i,j) \in U\}$ and $I_i^-(U) = \{j \in I : (j,i) \in U\}$. If the variable intensity x_i of node i is positive, the node i is a source; if it is negative, this node i is a sink. For system (1) is true the following condition: $\sum_{i \in I} x_i = 0$. According [9] if $I^* \neq \emptyset$, then the rank of the matrix of system (1) for a connected graph $G = (I, U)$ is equal to $|I|$.

In order to obtain information about the network flow function x and variables x_i of nodes $i \in I^*$, sensors are placed at the nodes of the graph $G = (I, U)$. The nodes in the graph $G = (I, U)$ with sensors we call monitored ones and denote the set of monitored nodes by M, $M \subseteq I$. We assume that if a node i is monitored, we know the values of flows on all outgoing and all incoming arcs for the node $i \in M$:

$$x_{ij} = f_{ij},\, j \in I_i^+(U), \quad x_{ji} = f_{ji},\, j \in I_i^-(U), \quad i \in M.$$

If the set M includes the nodes from the set I^*, then we also know the values $x_i = f_i,\ i \in M \bigcap I^*$. So, we have

$$\begin{gathered} x_{ij} = f_{ij},\, j \in I_i^+(U), \quad x_{ji} = f_{ji},\, j \in I_i^-(U), \\ i \in M; \\ x_i = f_i,\ i \in M \bigcap I^*. \end{gathered} \tag{2}$$

Consider any node i of the network. For every outgoing arc $(i,j) \in U$ for this node i determine a real number $p_{ij} \in (0,1]$ which denotes the part of the total outgoing flow $\sum\limits_{j\in I_i^+(U)} x_{ij}$ from node i corresponding to the arc (i,j). That is,

$$x_{ij} = p_{ij} \sum_{j\in I_i^+(U)} x_{ij}.$$

If $|I_i^+(U)| \geq 2$ for the node $i \in I$ then we can write the arc flow along all outgoing arcs from node i (except any selected arc) in terms of arc flow a single outgoing arc, for example, $(i,v_i), v_i \in I_i^+(U)$:

$$x_{i,j} = \frac{p_{i,j}}{p_{i,v_i}} x_{i,v_i}, \quad j \in I_i^+(U) \setminus v_i. \tag{3}$$

We continue this process for each node $i \in I$, if $|I_i^+(U)| \geq 2$.

Let $|I_i^+(U)| \geq 2$ for any node $i \in I$ and x_{i,v_i} is known for the arc (i,v_i) and equal to f_{i,v_i}. Then we can write the unknown arc flow along all outgoing arcs from node i (except any selected arc (i,v_i)) in terms of arc flow for a single outgoing arc (i,v_i), where x_{i,v_i} is known and equal to f_{i,v_i}.

Let's substitute the calculated arc flows according to (2) and (3) in the equations of system (1). Let's delete from graph $G=(I,U)$ the set of the arcs on which the arc flow are known. Let's delete from graph G the set of the nodes $i \in M$. Then we have a new graph $\overline{G} = (\overline{I}, \overline{U})$. A new set of nodes with variable intensity for a new graph $\overline{G}$ is $\overline{I}^*$, where $\overline{I}^* = I^* \setminus (M \bigcap I^*)$. The new graph $\overline{G}$ can be non-connected. The graph $\overline{G}$ consists of connected components. Some connected components may contain no nodes of the set $\overline{I}^*$. The system (1) for graph $\overline{G} = (\overline{I}, \overline{U})$ will be the following one:

$$\sum_{j\in I_i^+(\overline{U})} x_{i,j} - \sum_{j\in I_i^-(\overline{U})} x_{j,i} = \begin{cases} x_i + b_i, & i \in \overline{I}^*, \\ a_i, & i \in I \setminus \overline{I}^* \end{cases} \tag{4}$$

$$\sum_{(i,j)\in\overline{U}} \lambda_{ij}^p x_{ij} = 0, \; p = \overline{1,\ q}, \tag{5}$$

where $a_i,\ b_i,\ \lambda_{ij}^p$ – are constants.

So formulate the optimal solution to the Sensor Location Problem: *what is the minimum number of monitored nodes $|M|$ such that system (4)–(5) has an unique solution?*

In [10] was proof that SLP problem is NP-complete.

III. Optimal solution to the sensor location problem

In Figure 1 we show a finite connected directed symmetric graph G with the set of nodes I and the set of arcs U where

$$I = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\},$$

$$U = \{(1,2),(1,3),(2,1),(2,4),(2,6),(3,1),(3,5),(4,2),(4,6),(4,5),(5,3),(5,4),(5,6),(6,2),(6,4),(6,5)\},$$

$$I^* = \{2,\ 4,\ 5,\ 6\}.$$

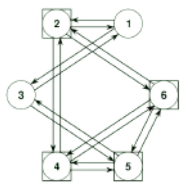

Figure 1. Finite connected directed symmetric graph G

For the graph $G=(I,U)$ (see Figure 1) we write the system of linear algebraic equations in the form:

$$\begin{aligned} x_{1,2}+x_{1,3}-x_{2,1}-x_{3,1} &= 0 \\ x_{2,4}+x_{2,6}+x_{2,1}-x_{4,2}-x_{1,2}-x_{6,2} &= x_2 \\ x_{3,1}+x_{3,5}-x_{1,3}-x_{5,3} &= 0 \\ x_{4,2}+x_{4,5}+x_{4,6}-x_{2,4}-x_{5,4}-x_{6,4} &= x_4 \\ x_{5,4}+x_{5,3}+x_{5,6}-x_{3,5}-x_{4,5}-x_{6,5} &= x_5 \\ x_{6,2}+x_{6,4}+x_{6,5}-x_{2,6}-x_{4,6}-x_{5,6} &= x_6 \end{aligned} \tag{6}$$

Suppose that the set of monitoring nodes is $M=\{2\}$ for the graph shown in Figure 1. Construct the cut $CC(M)$ with respect to the set M. We form the sets

$$M^+ = I(CC(M)) \setminus M = \{1,4,6\};$$

$$M^* = M \bigcup M^+ = \{1,2,4,6\};$$

$$I \setminus M^* = \{3,5\}.$$

In the Sensor Location Problem (SLP) the values of flows on all incoming and outgoing arcs for the each node i of the set M (monitored nodes) are known and we also know the values $x_i = f_i,\ i \in M \bigcap I^*$:

$$x_{1,2} = f_{1,2},\ x_{2,1} = f_{2,1},\ x_{2,4} = f_{2,4},$$

$$x_{4,2} = f_{4,2},\ x_{2,6} = f_{2,6},\ x_{6,2} = f_{6,2},\ x_2 = f_2. \tag{7}$$

We substitute the known values of the variables (7) to the system of equations (6) and delete the corresponding arcs from the graph G. Also, we delete the nodes $i \in M$ from the graph G. The graph G' obtained after deleting the arcs corresponding to the variables (7) and nodes $i \in M$ from graph G is shown in Figure 2. The rest of the flows for the outgoing arcs from the nodes of the set $M^+ = I(CC(M)) \setminus M = \{1,4,6\}$, can be expressed from the flows of the outgoing arcs for $M^+ = \{1,4,6\}$ by the following equations:

$$x_{1,3} = \frac{p_{1,3}}{p_{1,2}} f_{1,2}, \quad x_{4,5} = \frac{p_{4,5}}{p_{4,2}} f_{4,2},$$

$$x_{4,6} = \frac{p_{4,6}}{p_{4,2}} f_{4,2}, \tag{8}$$

$$x_{6,4} = \frac{p_{6,4}}{p_{6,2}} f_{6,2}, \quad x_{6,5} = \frac{p_{6,5}}{p_{6,2}} f_{6,2}.$$

Let us substitute (8) to the system of linear equations (6). We delete from the graph G arcs which correspond to the known values of the arc flows (7) and (8). The graph $\overline{G} = (\overline{I}, \overline{U})$ obtained by deleting the arcs corresponding to variables (8) from the graph G' is shown in Figure 3. The system (6) for the graph $\overline{G} = (\overline{I}, \overline{U})$ (see Figure 3) transforms to the form (9).

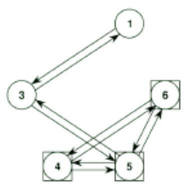

Figure 2. Graph G'

$$f_{1,2} + \frac{p_{1,3}}{p_{1,2}} f_{1,2} - f_{2,1} - x_{3,1} = 0,$$

$$f_{2,1} + f_{2,4} + f_{2,6} - f_{1,2} - f_{4,2} - f_{6,2} = f_2,$$

$$x_{3,1} + x_{3,5} - \frac{p_{1,3}}{p_{1,2}} f_{1,2} - x_{5,3} = 0,$$

$$f_{4,2} + \frac{p_{4,5}}{p_{4,2}} f_{4,2} + \frac{p_{4,6}}{p_{4,2}} f_{4,2} - \\ -f_{2,4} - x_{5,4} - \frac{p_{6,4}}{p_{6,2}} f_{6,2} = x_4, \tag{9}$$

$$x_{5,4} + x_{5,3} + x_{5,6} - x_{3,5} - \frac{p_{4,5}}{p_{4,2}} f_{4,2} - \\ -\frac{p_{6,5}}{p_{6,2}} f_{6,2} = x_5,$$

$$f_{6,2} + \frac{p_{6,4}}{p_{6,2}} f_{6,2} + \frac{p_{6,5}}{p_{6,2}} f_{6,2} - \\ -f_{2,6} - \frac{p_{4,6}}{p_{4,2}} f_{4,2} - x_{5,6} = x_6.$$

Arc flows $x_{i,j}, (i,j) \in \overline{U}$, corresponding to the arcs outgoing from node set $I \setminus M^* = \{3,5\}$ are unknown. For these unknown flows $x_{i,j}, (i,j) \in \overline{U}$ we form the additional equations.

- Choose arbitrary outgoing arc that starts from a node set i of set $I \setminus M^* = \{3,5\}$, for example, for the node $i = 3$ we choose the arc $(3,5)$. Let us express the arc flows to all other arcs outgoing from the node $i = 3$ through the arc flow of $x_{3,5}$ for the chosen outgoing arc $(3,5)$.
- Choose any outgoing arc from the node $i = 5$, for example $(5,6)$. Let us express the arc flows to all other arcs outgoing from the node $i = 5$ through the arc flow of $x_{5,6}$.

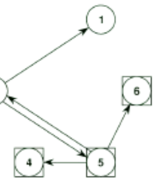

Figure 3. Graph $\overline{G} = (\overline{I}, \overline{U})$

$$x_{3,1} = \frac{p_{3,1}}{p_{3,5}} x_{3,5}, \quad x_{5,3} = \frac{p_{5,3}}{p_{5,6}} x_{5,6}, \quad x_{5,4} = \frac{p_{5,4}}{p_{5,6}} x_{5,6}.$$

Additional equations have the form:

$$x_{3,1} - \frac{p_{3,1}}{p_{3,5}} x_{3,5} = 0, \quad x_{5,3} - \frac{p_{5,3}}{p_{5,6}} x_{5,6} = 0, \\ x_{5,4} - \frac{p_{5,4}}{p_{5,6}} x_{5,6} = 0. \tag{10}$$

Part of the unknowns of the system (9), (10) makes up outgoing arc flows for arcs from node sets $I \setminus M^* = \{3,5\}$ for the graph $\overline{G}$:

$$x_{3,1}, \quad x_{3,5}, \quad x_{5,3}, \quad x_{5,4}, \quad x_{5,6}.$$

The remaining part of the unknowns of the system (9), (10) defines the variables x_i, $i \in \overline{I}^* = \{4,5,6\}$:

$$x_4, \quad x_5, \quad x_6.$$

Thus, the system (9), (10) is a system of full rank. The number of unknowns of the system (9), (10) equal to the rank of the matrix and is equal to 8. The system (9), (10) has the unique solution for given set of monitored nodes $M = \{2\}$.

IV. Pseudocode algorithms suboptimal solution to the sensor location problem

In the work [11] get the interval $[1, |I^*|]$ values changes of the number of $|M|$ nodes being viewed. Suboptimal ($t-$optimal) solution are constructed to the SLP problem of the establishment of full observability of the network for given intensity threshold t: $|x_i| \geq t, i \in I^*$. We presented the pseudocode algorithm's for finding the suboptimal solution to the sensor locations problem.

Algorithm 1 Pseudocode algorithm's for the suboptimal solution to the sensor locations problem

List of reference symbols:
arcs - array arc with flows value
FAKE-VERTEX - fake vertex
flow - value flow arc
iterations - count
lowerBound - value lower bound
normArcs - array arc notmalize with flow
numberThreads - count children threads
numberTrials - count value
observedArcs - array observible arcs
threshold - threshold of intensity

Input: arcs, threshold, iterations, numberTrials, lowerBound, numberThreads
Output: sensors configuration

$normArcs \leftarrow$ balance_nodes(arcs, treshold) ▷ Algorithm 2
$initialVertices \leftarrow$ get_start_nodes(normArcs) ▷ Algorithm 3
if $FAKE - VERTEX \in initialVertices$ **then**
    delete FAKE-VERTEX **from** initialVertices
end if
$randomSearch \leftarrow$ rsls(normArcs, initialVertices, iterations, numberTrials, lowerBound, numberThreads) ▷ Algorithm 4
$observedVertices \leftarrow$ **null**
for all $results \in randomSearch$ **do**
    if $residual \in result < 10^{-10}$ and cond $\in result < 10^6$ **then**
        $observedVertices \leftarrow vertices \in result$
        break
    end if
end for
$observedArcs \leftarrow$ get_monitored_nodes_with_network (normArcs, observedVertices) ▷ Algorithm 5
get_sensors_configuration(normArcs, observedVertices, observedArcs) ▷ Algorithm 6

Algorithm 2 – pseudocode of the algorithm to obtaining an array of normalized arcs.

Algorithm 3 – pseudocode of the algorithm to obtaining an array of initial nodes with variable intensity.

Algorithm 4 – pseudocode of the algorithm of the random search location sensors.

Algorithm 5 – pseudocode of the algorithm for obtaining the array of observed arcs.

Algorithm 6 – pseudocode of the algorithm to obtaining of the sensor configurations for the suboptimal solutions.

V. Suboptimal solution to the sensor location problem

The example of a suboptimal solution show in the figure 4 for the graph $G = (I, U)$, $|I| = 9$, $|U| = 18$ $I^* = \{2, 4, 5, 6, 7, 9\}$.

After location in the network G the $|M| = 6$ sencors in the nodes $M = \{2, 4, 5, 6, 7, 9\}$, (fig. 4), we have the suboptimal solution.

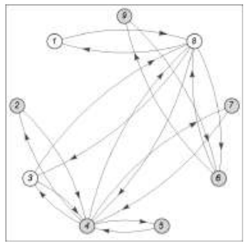

Figure 4. Suboptimal solution: $M = \{2, 4, 5, 6, 7, 9\}$

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ОПТИМИЗАЦИЯ ПРОГРАММНОГО ОБЕСПЕЧЕНИЯ ПРОБЛЕМЫ РАСПОЛОЖЕНИЯ СЕНСОРОВ

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В работе рассматривается приложение теории графов для построения оптимальных и субоптимальных решений задачи расположения сенсоров. Эта задача называется проблемой расположения сенсоров (SLP) для графа. Для построения решений задачи SLP для графа мы представляем псевдокоды алгоритмов для нахождения дуговых потоков на ненаблюдаемой части сети. В псевдокоде алгоритма 1 определяются сенсорные конфигурации субоптимального решения и дуговые потоки на ненаблюдаемой части сети.

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