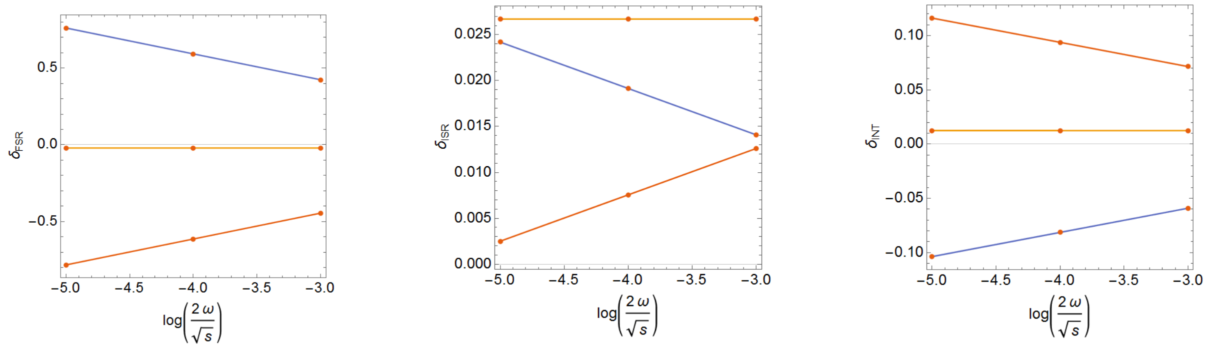

Figure 3 – Dependence of ISR, FSR and their interference of real photon softness parameter ω. One contribution on the plot is real hard photon emission. Another is virtual+soft photon emission. And their sum is flat horizontal line independent of ω

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ON P-NONINVARIANT WAVE EQUATION FOR SPIN 1/2 PARTICLE WITH ANOMALOUS MAGNETIC MOMENT, INTERACTION WITH EXTERNAL FIELDS

Within the theory of relativistic wave equations with extended sets of Lorentz group representations, a new P-noninvariant 20-component wave equation for spin 1/2 particle is proposed. The presence of external electromagnetic field and Riemannian space-time background are taken into account. Due to internal structure of the particle, additional interaction terms appear, they correspond to additional characteristic-anomalous magnetic moment.

1. Gel'fand-Yaglom Basis

The goal of the paper is to construct a new *P*-noninvariant wave equation for a massive spin 1/2 particle. We apply general theory of relativistic wave equations with extended sets of representation of the Lorentz group [1-4]. We start with the following set of irreducible representations (this is 20-component theory)

$$T = (0,1/2) \oplus (1/2,0) \oplus (0,1/2)' \oplus (1/2,0)' \oplus (1,1/2) \oplus (1/2,1),$$

where the "prime" serves to distinguish repeated representations of the Lorentz group. The matrix Γ_4 of corresponding wave equation has the following structure $\Gamma_4 = (C^{(1/2)} \otimes I_2) \oplus (C^{(3/2)} \otimes I_4)$, where $C^{(1/2)}$, $C^{(3/2)}$ represent spin-blocks related to spins 1/2 and 3/2. With the use of the numeration of irreducible components

$$(0,1/2) \sim 1, \quad (0,1/2)' \sim 2, \quad (1,1/2) \sim 3,$$
$$(1/2,0) \sim 4, \quad (1/2,0)' \sim 5, \quad (1/2,1) \sim 6,$$

the blocks $C^{(1/2)}$ and $C^{(3/2)}$ are given by the formulas

$$C^{(\frac{1}{2})} = \begin{vmatrix} 0 & 0 & 0 & c_{14}^{(\frac{1}{2})} & c_{15}^{(\frac{1}{2})} & c_{16}^{(\frac{1}{2})} \\ 0 & 0 & 0 & c_{24}^{(\frac{1}{2})} & c_{25}^{(\frac{1}{2})} & c_{26}^{(\frac{1}{2})} \\ 0 & 0 & 0 & c_{34}^{(\frac{1}{2})} & c_{35}^{(\frac{1}{2})} & c_{36}^{(\frac{1}{2})} \\ c_{41}^{(\frac{1}{2})} & c_{42}^{(\frac{1}{2})} & c_{43}^{(\frac{1}{2})} & 0 & 0 & 0 \\ c_{51}^{(\frac{1}{2})} & c_{52}^{(\frac{1}{2})} & c_{53}^{(\frac{1}{2})} & 0 & 0 & 0 \\ c_{61}^{(\frac{1}{2})} & c_{62}^{(\frac{1}{2})} & c_{63}^{(\frac{1}{2})} & 0 & 0 & 0 \end{vmatrix}, \quad C^{(\frac{3}{2})} = \begin{vmatrix} 0 & c_{36}^{(\frac{3}{2})} \\ c_{63}^{(\frac{3}{2})} & 0 \end{vmatrix}. \qquad (1)$$

From invariance of the equation under proper Lorentz group follow the constraints

$$c_{36}^{(\frac{3}{2})} = 2c_{36}^{(\frac{1}{2})}, \quad c_{63}^{(\frac{3}{2})} = 2c_{63}^{(\frac{1}{2})}.$$

Besides, without loss of generality, the links between repeated components may be broken:

$$c_{15}^{(\frac{1}{2})} = c_{51}^{(\frac{1}{2})} = c_{24}^{(\frac{1}{2})} = c_{42}^{(\frac{1}{2})} = 0.$$

Because, we wish to construct the model of a particle with single spin $1/2$, we require that eigenvalues of the block $C^{(3/2)}$ be equal to zero. Therefore, we set $c_{36}^{(\frac{3}{2})} = c_{63}^{(\frac{3}{2})} = 0$, whence it follows $c_{36}^{(\frac{1}{2})} = c_{63}^{(\frac{1}{2})} = 0$.

2. Modified Gel'fand-Yaglom basis

Let us find the form of the matrix Γ_4 in so-called modified Gel'fand-Yaglom basis. Here the listing of basis elements of the complete wave function ψ is slightly different. We find the following decomposition for spin block $C^{(1/2)}$:

$$C^{(1/2)} = \frac{1}{2}\begin{vmatrix} (c_{14}^{(\frac{1}{2})} + c_{41}^{(\frac{1}{2})}) & 0 & (c_{16}^{(\frac{1}{2})} + c_{43}^{(\frac{1}{2})}) \\ 0 & (c_{25}^{(\frac{1}{2})} + c_{52}^{(\frac{1}{2})}) & (c_{26}^{(\frac{1}{2})} + c_{53}^{(\frac{1}{2})}) \\ (c_{34}^{(\frac{1}{2})} + c_{61}^{(\frac{1}{2})}) & (c_{35}^{(\frac{1}{2})} + c_{62}^{(\frac{1}{2})}) & 0 \end{vmatrix} \otimes \gamma_4 +$$

$$+\frac{1}{2}\begin{vmatrix} (c_{14}^{(\frac{1}{2})} - c_{41}^{(\frac{1}{2})}) & 0 & (c_{16}^{(\frac{1}{2})} - c_{43}^{(\frac{1}{2})}) \\ 0 & (c_{25}^{(\frac{1}{2})} - c_{52}^{(\frac{1}{2})}) & (c_{26}^{(\frac{1}{2})} - c_{53}^{(\frac{1}{2})}) \\ (c_{34}^{(\frac{1}{2})} - c_{61}^{(\frac{1}{2})}) & (c_{35}^{(\frac{1}{2})} - c_{62}^{(\frac{1}{2})}) & 0 \end{vmatrix} \otimes \gamma_5\gamma_4; \quad (2)$$

here the first term corresponds to purely P-invariant model, the second term relates to purely P-noninvariant model. We will we restrict ourselves to the second variant of the theory.

It is convenient to employ shortening notations, then the spin block $C^{(1/2)}$ reads

$$C^{(1/2)} = \begin{vmatrix} a_1 & 0 & a_2 \\ 0 & a_3 & a_4 \\ a_5 & a_6 & 0 \end{vmatrix} \otimes \gamma_4 + \begin{vmatrix} ib_1 & 0 & ib_2 \\ 0 & ib_3 & ib_4 \\ ib_5 & ib_5 & 0 \end{vmatrix} \otimes \gamma_5\gamma_4. \quad (3)$$

For purely P-noninvariant model it becomes simpler

$$C^{(1/2)} = i\begin{vmatrix} b_1 & 0 & b_2 \\ 0 & b_3 & b_4 \\ b_5 & b_5 & 0 \end{vmatrix} \otimes \gamma_5\gamma_4. \quad (4)$$

We create the model for particle with one mass, so the matrix

$$\begin{vmatrix} b_1 & 0 & b_2 \\ 0 & b_3 & b_4 \\ b_5 & b_5 & 0 \end{vmatrix}$$

must have only one non-vanishing eigenvalue. In accordance with this, parameters b_i obey restrictions

$$b_1 + b_3 = 1, \quad b_1 b_3 - b_2 b_5 - b_4 b_6 = 0, \quad b_2 b_3 b_5 + b_1 b_4 b_6 = 0. \quad (5)$$

3. Spinor form of the wave equation

After some technical manipulations we derive the set of spinor equations (where $\partial_{a\dot{b}} = \frac{1}{i}\partial_\mu \sigma^\mu_{a\dot{b}}$)

$$i\left\{ b_1 \partial^{\dot{a}b} \Psi_b + \sqrt{\frac{2}{3}} b_2 \partial^b_{\dot{c}} \Psi^{(\dot{a}\dot{c})}_b \right\} + M \Psi^{\dot{a}} = 0,$$

$$-i\left\{ b_1 \partial_{a\dot{b}} \Psi^{\dot{b}} + \sqrt{\frac{2}{3}} b_2 \partial^c_{\dot{b}} \Psi^{\dot{b}}_{(ac)} \right\} + M \Psi_a = 0,$$

$$i\left\{ b_3 \partial^{\dot{a}b} \Psi'_b + \sqrt{\frac{2}{3}} b_4 \partial^b_{\dot{c}} \Psi^{(\dot{a}\dot{c})}_b \right\} + M \Psi'^{\dot{a}} = 0,$$

$$-i\left\{ b_3 \partial_{a\dot{b}} \Psi'^{\dot{b}} + \sqrt{\frac{2}{3}} b_4 \partial^c_{\dot{b}} \Psi^{\dot{b}}_{(ac)} \right\} + M \Psi'_a = 0,$$

$$-\frac{i}{\sqrt{6}} b_5 (\partial^{\dot{c}}_a \Psi_b + \partial^{\dot{c}}_b \Psi_a) - \frac{i}{\sqrt{6}} b_6 (\partial^{\dot{c}}_a \Psi'_b + \partial^{\dot{c}}_b \Psi'_a) + M \Psi^{\dot{c}}_{(ab)} = 0,$$

$$\frac{i}{\sqrt{6}} b_5 (\partial^{\dot{a}}_c \Psi^{\dot{b}} + \partial^{\dot{b}}_c \Psi^{\dot{a}}) + \frac{i}{\sqrt{6}} b_6 (\partial^{\dot{a}}_c \Psi'^{\dot{b}} + \partial^{\dot{b}}_c \Psi'^{\dot{a}}) + M \Psi^{(\dot{a}\dot{b})}_c = 0. \quad (6)$$

4. Equations in spin-tensor form

After additional work we arrive at the spin-tensor system (where $\hat{\partial} = \partial_\mu \gamma_\mu$)

$$i\gamma_5 \left\{ b_1 \hat{\partial}(\gamma_\mu \Psi_\mu) - \frac{4b_2}{\sqrt{6}} \left[-\frac{1}{4} \hat{\partial}(\gamma_\mu \Psi_\mu) + (\partial_\mu \Psi_\mu) \right] \right\} + M(\gamma_\mu \Psi_\mu) = 0,$$

$$i\gamma_5 \left\{ b_3 \hat{\partial} \Psi_0 - i\frac{4b_4}{\sqrt{6}} \left[(\partial_\mu \Psi_\mu) - \frac{1}{4} \hat{\partial}(\gamma_\mu \Psi_\mu) \right] \right\} + M \Psi_0 = 0,$$

$$\frac{2i}{\sqrt{6}} \gamma_5 \left\{ b_5 \left[\partial_\lambda (\gamma_\mu \Psi_\mu) - \frac{1}{4} \gamma_\lambda \hat{\partial}(\gamma_\mu \Psi_\mu) \right] - \right.$$

$$\left. -ib_6 \left[\partial_\lambda \Psi_0 - \frac{1}{4} \gamma_\lambda \hat{\partial} \Psi_0 \right] + M\{\Psi_\lambda - \frac{1}{4} \gamma_\lambda (\gamma_\mu \Psi_\mu) \right\} = 0. \quad (7)$$

5. Reducing the system to minimal equation

In absence of external fields, the main component of the wave function is

$$\Phi(x) = b_5 \gamma_\mu \Psi_\mu(x) - ib_6 \Psi_0(x). \quad (8)$$

The main bispinor $\Phi(x)$ satisfies modified Dirac–like P-noninvariant equation

$$\left\{ i\gamma_5(\gamma_\mu \partial_\nu) + M \right\} \Phi(x) = 0. \quad (9)$$

Concomitant bispinors may be constructed by the rules

$$\gamma_\mu \Psi_\mu(x) = \frac{b_1^2}{b_5(b_1^2 - b_3^2)} \Phi(x), \quad \Psi_0(x) = \frac{-ib_3^2}{b_6(b_1^2 - b_3^2)} \Phi(x). \quad (10)$$

6. The minimal equation in presence of electromagnetic field

In presence of electromagnetic fields, the main component is the same

$$\Phi(x) = b_5 \gamma_\mu \Psi_\mu(x) - ib_6 \Psi_0(x).$$

This component obeys the following P-noninvariant equation for a particle with anomalous magnetic moment

$$\left\{ i\gamma^5 \gamma_\mu (\partial_\mu + ieA_\nu) - \frac{4b_1 b_3}{M} ieF_{\mu\nu} \frac{\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu}{4} + M \right\} \Psi = 0. \quad (11)$$

Expressions for $\Psi = 0$ and $(\gamma_\mu \Psi_\mu)$ are

$$(\gamma_\mu \Psi_\mu) = \frac{b_1^2}{b_5(b_1^2 - b_3^2)} \left\{ 1 + \frac{4}{3} \left(\frac{b_1 b_3}{M} \right)^2 ieF_{\mu\nu} \frac{\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu}{4} \right\} \Phi,$$

$$\Phi_0 = -i \frac{b_3^2}{b_6(b_1^2 - b_3^2)} \left\{ 1 + \frac{4}{3} \left(\frac{b_1 b_3}{M} \right)^2 ieF_{\mu\nu} \frac{\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu}{4} \right\} \Phi. \quad (12)$$

7. Extension of the model to General relativity

In order to follow extension of the model from flat Minkowski space to any Riemannian space-time we should turn back and make several simple modifications.

1. In Riemannian space we use the metric $g_{\alpha\beta}(x)$, related to signature $(+,-,-,-)$, we must make the change:

$$M \text{-->} iM. \quad (13)$$

2. Now Dirac matrices in spinor basis are

$$\gamma^0 = \begin{vmatrix} 0 & I \\ I & 0 \end{vmatrix}, \quad \gamma^i = \begin{vmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{vmatrix}. \quad (14)$$

3. Derivatives are modified according to the rules

$$D_\alpha(x) = \nabla_\alpha + \Gamma_\alpha(x) + ieA_\alpha(x), \ \hat{D} = \gamma^\alpha(x) D_\alpha(x),$$

where $\Gamma_\alpha(x)$ is bispinor connection, and $\gamma^\alpha(x) = \gamma^a e^\alpha_{(a)}(x)$.

4. Note important commutation rules

$$\hat{D}(x) = \gamma^\rho(x) D_\beta = D_\beta \gamma^\rho(x), \quad D_\sigma(x) g_{\alpha\beta}(x) = g_{\alpha\beta}(x) D_\sigma(x),$$

$$\hat{D}\hat{D} = D^\alpha D_\alpha - \Sigma(x), \quad \Sigma(x) = -ieF_{\alpha\beta}\sigma^{\alpha\beta}(x) + \frac{R}{4},$$

where $R(x)$ is the Ricci scalar.

5. Note the notations

$$\gamma^5(x) = \frac{i}{4!}\varepsilon_{\alpha\beta\rho\sigma}(x)\gamma^\alpha(x)\gamma^\beta(x)\gamma^\rho(x)\gamma^\sigma(x),$$

$$\varepsilon^{\alpha\beta\rho\sigma}(x) = \varepsilon^{abcd} \ e^\alpha_{(a)}(x)\, e^\beta_{(b)}(x)\, e^\rho_{(c)}(x)\, e^\sigma_{(d)}(x), \quad \varepsilon_{0123} = -1.$$

Levi-Civita object $\varepsilon^{\alpha\beta\rho\sigma}(x)$ changes under tetrad transformations as follows

$$\varepsilon'^{\alpha\beta\rho\sigma}(x) = \det[L_a^{\ b}(x)]\, \varepsilon^{\alpha\beta\rho\sigma}(x).$$

In particular, at the tetrad P–reflection, it transforms as a tetrad pseudoscalar

$$\varepsilon^{(p)\alpha\beta\rho\sigma}(x) = (-1)\, \varepsilon^{\alpha\beta\rho\sigma}(x).$$

The above analysis for the generally covariant system remains in fact the same. We can write down final result without repeating the calculation.

$$\Phi = b_5(\gamma_\mu \Psi_\mu) - ib_6 \Psi_0,$$

$$\left\{ i\gamma^5(x)\hat{D}(x) - \frac{4b_1b_3}{M}\left[-ieF_{\mu\nu}\sigma^{\mu\nu}(x) + \frac{R(x)}{4} \right] + iM \right\}\Phi = 0. \quad (15)$$

Expressions for concomitant components are given the formulas

$$\gamma^\mu(x)\Psi_\mu(x) = \frac{b_1^2}{b_5(b_1^2 - b_3^2)}\left\{ 1 - \frac{4}{3}\left(\frac{b_1b_3}{iM}\right)^2 \left(-ieF_{\mu\nu}\sigma^{\mu\nu} + \frac{R(x)}{4} \right) \right\}\Phi,$$

$$\Psi_0(x) = -i\frac{b_3^2}{b_6(b_1^2 - b_3^2)}\left\{ 1 - \frac{4}{3}\left(\frac{b_1b_3}{iM}\right)^2 \left(-ieF_{\mu\nu}\sigma^{\mu\nu} + \frac{R(x)}{4} \right) \right\}\Phi. \quad (16)$$

Conclusions

This theory gives a P-noninvariant model for spin 1/2 particle with anomalous magnetic moment.

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THE ENERGY-MOMENTUM TENSOR FOR A SPIN 1/2 PATICLE TAKING INTO ACCOUNT POLARIZABILITIES

Introduction

Interaction of the electromagnetic field with structural particles in the electrodynamic of hadrons is based on main principles of the relativistic quantum field theory. In the model conceptions where basically the diagram technique is used a number of features for interaction of photons with hadrons have been determined [1, 2]. However, the diagram technique is mainly employed for a description of electromagnetic processes on simplest quark systems. In the case of interaction for the electromagnetic field with complex quark–gluon systems in the low-energy region perturbative methods of QCD are nonapplicable. That is why the low-energy theorems and sum rules are widely used lately [3, 4, 5, 6].

In the present time the low-energy electromagnetic characteristics which connect with hadron structure, such as formfactor and polarizabilities, it is possible to obtain from nonrelativistic theory [5]. Passing on from the nonrelativistic electrodynamics to the relativistic field theory one can make use the correspondence principle. But it is necessary step by step to investigate a transition from the covariant Lagrangian formalism to the Hamiltonian one [7, 8, 9].