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Field Emission in Silicon Vacuum Nanostructure

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Transmission coefficient and field emission current in a silicon vacuum nanostructure with a pyramidal cathode were calculated as a function of applied voltage, size of the cathode and distance between the anode and cathode by the phase function method. The field emission current density in the range of 1–10 A/cm^2 was found to be achieved by varying the distance between the anode and cathode in the range of 15–25 nm and the applied voltage in the range of 1.2–2.3 V.

Keywords: Vacuum nanostructure; field emission; phase function; pyramidal cathode; electron tunneling; transmission coefficient; amplification coefficient.

1. Introduction

Silicon vacuum nanoelectronics is currently an actively developing field. It uses achievements of silicon technology and advantages of electron motion in vacuum.[1,2] Vacuum silicon nanoelectronic devices need a perfect design of silicon cathodes.[3] Silicon is a suitable material for field emission cathodes as its work function in the range of 4.0–4.6 eV is comparable to that of metals. In addition, fabrication of sharp silicon cathodes with a tip of an atomic dimension (less than 1 nm) has been already developed thus allowing significant reduction of the operating voltage.[4]

However, silicon cathodes provide current densities which are still lower than metallic ones. An increased current density in conventional silicon cathodes inevitably brings their rapid degradation.[2,3] It could be minimized by the cathode geometry and electrical regimes of operation. In this paper, we present the relationships between the most important factors.

2. Model

The cross-section of the considered field emission nanostructure is depicted in the Fig. 1. It includes a pointed silicon cathode in the form of a pyramid of the height h with a rounded tip of radius r. The flat anode is located from the silicon substrate at a distance d. The gates are used to control the cathode current.

The above nanostructure is considered as a basic unit of vacuum nanotriodes and other devices of vacuum nanoelectronics. The pointed cathode is used to form increased electric field strength at the cathode tip to obtain the largest effective electron emission.

Numerical simulation of the electron transmission and the emission current in the nanostructure

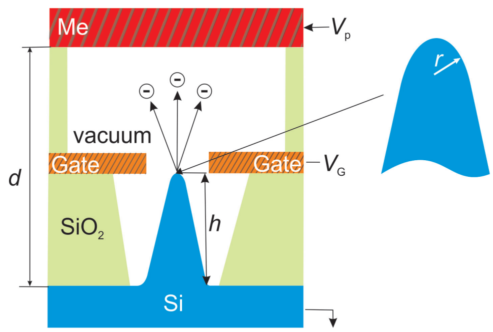

Fig. 1. Cross-section of the vacuum nanostructure.

can be done by using the Schrödinger equation or applying the Ventzel–Kramers–Brillouin (WKB) approximation. In the first case, in order to determine the transmission coefficient, it is necessary to approximate the potential barrier between the cathode and the anode by a set of narrow rectangular potentials and then combine together the solutions of the Schrödinger equation obtained for each region.[5] For potentials of a rather complex form, this procedure is very time consuming. Moreover, an assessment of the accuracy of the results obtained is difficult.

The WKB approximation is not always applicable for the calculation of the transmission coefficient due to the restriction applied to the shape of the potential barrier and a significant change of the potential at the de Broglie wavelength of an electron.

In this paper, the phase function method[6,7] developed for quantum systems is used since accounting for a significant change of the potential at the de Broglie wavelength of an electron, the Fowler-Nordheim theory[8] is no more valid.

The fundamental principle of the phase function method implies that only a change of the wave function of a quantum system is estimated but not the wave function itself. The simplicity of the phase equation is that it is an ordinary first-order differential equation (Riccati equation). There is also a possibility to analyze potential barriers of various types, including those depending on the electron momentum. The physical sense of the phase function is that it is the phase of scattering at the corresponding potential relief.

The equation for the electron reflection function $B(z)$ from the potential barrier $U(z)$ has the form[7]:

$$\frac{dB(z)}{dz} = -\frac{U(z)}{2ik}[\exp(ikz) + B(z)\exp(-ikz)]^2, \quad (1)$$

where z is the coordinate in the direction of tunneling, $k = (2mE/\hbar^2)^{1/2}$ is the wave number of the tunneling electron. The effective potential is written as

$$U(z) = (2m/\hbar^2)[U_0 - qV(z) + q\varphi(z)], \quad (2)$$

where q is the electron charge, m, E is the effective mass and the energy of the tunneling electron correspondingly, $\hbar$ is the normalized Planck constant, U_0 is the work function of the cathode, $V(z) = Fz$ is the potential of the external electric field, $\varphi(z) = -q^2/4\varepsilon\varepsilon_0 z$ is the force potential of the mirror image, F is the external field strength. The magnitude of the square of the reflection function modulus is interpreted as a coefficient of reflection from the potential barrier: $R(z) = |B(z)|^2$. Assuming that $B(z) = a(z) + ib(z)$ and expanding $\exp(\pm ikz)$, we obtain the following system of equations for determination of the components of the reflection function

$$\begin{aligned} \frac{da(z)}{dz} &= \frac{U(z)}{2k}[-\sin(2kz) - 2b \\ &\quad + (a^2 - b^2)\sin(2kz) - 2ab\cos(2kz)], \\ \frac{db(z)}{dz} &= \frac{U(z)}{2k}[\cos(2kz) + 2a \\ &\quad + (a^2 - b^2)\cos(2kz) - 2ab\sin(2kz)]. \end{aligned} \quad (3)$$

The transmission coefficient for the barrier is equal to:

$$T(k) = \exp\left[\frac{1}{k}\int_0^{z_0} U(z)[b(z)\cos(2kz), -a(z)\sin(2kz)]dz\right], \quad (4)$$

where z_0 is the width of the tunneling barrier at the Fermi level of silicon. The emitted electron current density can be described by the following equation:

$$J = q\int N(k)T(k)dk, \quad (5)$$

with $N(k)$ being the supply function which describes the electron flux to the potential barrier and the transmission coefficient $T(k)$ which represents a chance of tunneling.[9]

The system of Eqs. (3)– (5) allows calculations of the dependence of the transmission coefficient as well as the emission current density on the wave number k for the barrier described by the effective potential $U(z)$.

3. Results and Discussion

We assume that work function $U_0 = 4.03\,\mathrm{eV}$. The parameters of the nanostructure with silicon cathode and the amplification coefficient

$$\beta = F/V = 2(h/r)(1/d)/[\ln(4h/r) - 2] \qquad (6)$$

are given in Table 1. Three variants indicated as a, b and c were analyzed.

Figure 2 presents the calculated transmission coefficient $T(k')$ for various external voltages V_p and the nanostructure parameters corresponding to the variant a in Table 1. The dimensionless wave number $k' = z_0 k$.

Figure 3 shows current–voltage characteristics of the nanostructure with different sizes of its elements. An increase of the current density is achieved by means of an increase of the amplification coefficient β and, accordingly, increase of the field strength at the tip of the cathode.

The calculations performed have shown that the transmission coefficient changes significantly with the wave number. It rises sharply upon a certain value of the wave number corresponding to the certain energy E. Valuable currents of the emitted electrons arise in the conditions where $T(k') > 0.1$. For such cases, z_0 depends on the external voltage V_p, and the energy of electrons is calculated as $E = [(k'\hbar)^2/2m](1/z_0)^2$.

The emission current density in the range from $1\,\mathrm{A/cm^2}$ to $10\,\mathrm{A/cm^2}$ was calculated to be achieved in the analyzed nanostructure at external voltages of 1–2.5 V. They look attractive for practical purposes.

Table 1. Parameters of the nanostructure used in the calculations.

Parameter	Value a	Value b	Value c
d, nm	20	20	25
h, nm	10	15	20
r, nm	1	1	0.5
β, 1/m	5.92×10^8	7.16×10^8	1.04×10^9

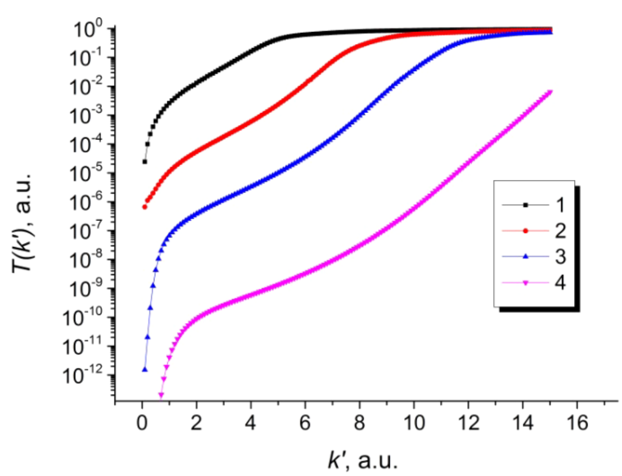

Fig. 2. Transmission coefficient T as a function of the wave number k' at the external bias $V_p = 5$ V (1), 3 V (2), 2.2 V (3) and 1.5 V (4).

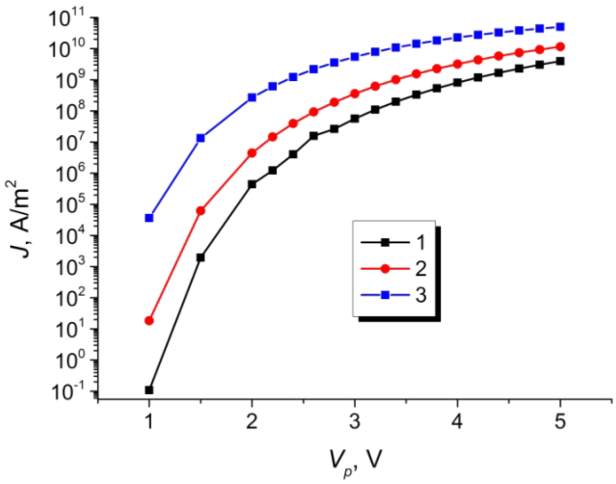

Fig. 3. Current–voltage characteristics of the nanostructure with the sizes of its elements corresponding to the variants a (1), b (2) and c (3) in Table 1.

4. Conclusion

The phase function method was demonstrated to be useful for the numerical simulation of the transmission coefficient, emission current density and current–voltage characteristics of a vacuum silicon field emission nanostructure with a pyramidal cathode. It accounts for a change of the potential energy in the structure at the de Broglie electron wavelength. It is sensitive to the external voltage applied to the structure, the cathode size and the distance between the anode and cathode. The increase of the amplification coefficient of the field strength at the cathode tip of about 2 times results in up to 2 orders of magnitude increase of the emitted electron current density. It has been found that the emission current density of $1\,\mathrm{A/cm^2}$ can be produced at an external voltage in the range of 1.2–2.0 V by varying the anode-to-cathode distance from 15 nm to 25 nm, while the current density of $10\,\mathrm{A/cm^2}$ can be achieved at 1.4–2.3 V. These are sufficient for a practical application in new generation of silicon-based vacuum field emission nanostructures.

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