Isolation of two-dimensional E-fields in the excitation equations for hollow waveguides

A. A. Kuraev [1], A. A. Koronovskii [2], S. A. Kurkin [1,2], A. O. Rak [1],
I. S. Rempen [1], A. A. Badarin [1,2], and A. E. Hramov [1,2]

[1] *Belarusian State University of Informatics and Radioelectronics,*
6, P. Browki Str., Minsk, 220013, Belarus
[2] *Saratov State University,*
83, Astrakhanskaya Str., Saratov, 410012, Russian Federation
[3] *Saratov State Technical University,*
77, Politehnicheskaya Str., Saratov, 410056, Russian Federation
hramovae@gmail.com

Abstract: *The traditional excitation equations of the Vainshtein waveguide excitation theory based on presenting the field excited by extraneous sources as a sum of TE and TM eigenwaves of the hollow waveguide, which transversal components form the complete basis in the class of the transversal vectors of a waveguide section. However, in the excitation sources area (longitudinal sonde, electron beam) the waveguide is not hollow and the potential transverse electric field appears in this area. The transversal electric field completes the basis of TE and TM waves. This is not taken into account in the traditional excitation equations. In this paper, the necessary correction of the excitation equations in the sources area is obtained.*

Выделение двумерного электрического поля в уравнениях возбуждения для полых волноводов [1]

[1] Кураев А. А., [2] Короновский А. А., [1,2] Куркин С. А., [1] Рак А. О.,
[1] Ремпен И. С., [1,2] Бадарин А. А., [1,2] Храмов А. Е.

[1] *Белорусский государственный университет информатики и радиоэлектроники*
ул. П. Бровки, 6, Минск, 220013, Беларусь
[2] *Саратовский государственный университет*
ул. Астраханская, 83, Саратов, 410012, Российская Федерация
[3] *Саратовский государственный технический университет*
ул. Политехническая, 77, Саратов, 410056, Российская Федерация
hramovae@gmail.com

Аннотация: *Хорошо известно, что уравнения возбуждения волноводов Л. А. Вайнштейна основываются на возбуждении сторонними источниками тока электромагнитного поля в виде суммы ТЕ и ТМ собственных волн полого волновода, при этом поперечные компоненты образуют полный базис в классе поперечных векторов секции волновода. Тем не менее, в области источников возбуждения (продольный зонд, электронный пучок) волновод не является полым и в этой области возникает потенциальное поперечное электрическое поле. Поперечное электрическое поле дополняет структуру основных ТЕ и ТМ волн. Это не учитывается в традиционных уравнениях возбуждения. В настоящей работе произведена необходимая корректировка уравнений возбуждения в области источников.*

Ключевые слова: *теория волноводов Вайнштейна, уравнения возбуждения, электронный пучок.*

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[1] Статья является расширенной версией доклада, представленного на 24-й Международной Крымской конференции «СВЧ-техника и телекоммуникационные технологии» — КрыМиКо'2014 (Севастополь, РФ, 7—13 сентября 2014 г.).

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1. Introduction

The development and application of effective semi-analytical or numerical methods for the analysis and investigation of waveguides, resonators and other microwave components in the presence of some sources is an important and actual problem of the modern microwave theory [1–5]. One of such methods is the waveguide excitation theory. The excitation equations of the Vainshtein waveguide excitation theory for regular waveguides [6–8] are widely used in microwave electronics [7–12]. There are a number of problems in this theory. One of them is a representation of static electric and magnetic fields induced by electron beam in a waveguide in the form of solutions to equations of the Vainshtein excitation theory. Other problem solved in this work is the need of several corrections of the waveguide excitation equations in the sources areas. Let us note that the equations of the waveguide excitation theory are based on the representation of the excited field as a sum of TE (E) and TM (H) waves of a hollow waveguide. Their transversal components form a full system in the class of the transversal vectors of a hollow waveguide section. Nevertheless, in the area of a source (a longitudinal sonde, an electron beam) the waveguide contains not a simply connected area of the cross-section but a doubly or multiply connected area, which supposes the existence of the transversal potential electric field. Thus, the basis of TE and TM waves is not complete, and it is necessary to take into account the TEM-wave, since its propagation constant $\Gamma_T = k = \omega/c$ is not equal to Γ_ν^E or Γ_ν^H of TM or TE waves ($\Gamma_\nu^{E,H} = \sqrt{k^2 - \left(k_{\perp\nu}^{E,H}\right)^2}$) and it is orthogonal to all TE and TM fields of a hollow waveguide [6–8]. In other words, it cannot be represented using the TM and TE basis of a hollow waveguide and requires a separate calculation. The separated quasi-static field ($\omega = 0$) determines the main part of the source electric field – 2D transversal field ($\Gamma_T = \omega/c = 0$). The last is especially important for the theory of microwave O- and M-type devices [13], gyroresonance devices [14, 15], semiconductor superlattices in a resonator [16] and so on.

In the present paper, we make the important transformation of the excitation equations for hollow waveguides, in which the transversal electric field of the sources is taken into account.

2. Regular waveguide

Let us set the task of the stimulation of regular waveguide by an off-site electric source as following. Firstly, it is necessary to solve Maxwell's equations:

$$\left.\begin{array}{r}\operatorname{rot}\vec{H}=\varepsilon_a\dfrac{\partial\vec{E}}{\partial t}+\vec{\delta}_{es},\\[2mm]\operatorname{rot}\vec{E}=-\mu_a\dfrac{\partial\vec{H}}{\partial t},\\[2mm]\operatorname{div}\left(\varepsilon_a\vec{E}\right)=\rho_{es},\\[2mm]\operatorname{div}\left(\mu_a\right)=0\end{array}\right\}\qquad(1)$$

with the following boundary condition on the waveguide walls:

$$[\vec{n}\vec{E}]\big|_S=0.\qquad(2)$$

Here $\vec{E}$ and $\vec{H}$ are the electric and magnetic components of the waveguide field; ε_a and μ_a are the dielectric and magnetic permittivities of the waveguide filling; $\vec{\delta}_{es}$ and ρ_{es} are the vector of current density and the space charge density of the off-site electric sources; $\vec{n}$ is the waveguide wall normal. Condition (2) fits the perfectly conducting walls; it is not hard to make generalization for the occurrence of losses [8].

To find the solution of Eqs. (1) and (2), let us represent the sought field $\vec{E}$ as two components:

$$\vec{E}=\vec{E}'+\vec{E}'',\qquad(3)$$

while $\operatorname{rot}\vec{E}''=0$. Dividing the field $\vec{E}$ into $\vec{E}'$ and $\vec{E}''$ in equation (3) means that $\operatorname{div}\vec{E}'\neq0$ and $\operatorname{rot}\vec{E}'\neq0$. At the same time, the two-dimensional field $\vec{E}''$ conform to TEM field.

Thereafter the system (1) is divided into two ones:

$$\left.\begin{array}{r}\operatorname{rot}\vec{H}=\varepsilon_a\dfrac{\partial\overrightarrow{E'}}{\partial t}+\vec{\delta}'_{es},\\[2mm]\operatorname{rot}\overrightarrow{E'}=-\mu_a\dfrac{\partial\vec{H}}{\partial t},\end{array}\right\}\qquad(4)$$

$$\operatorname{div}\vec{E}''=\rho_{es}/\varepsilon_a-\operatorname{div}\vec{E}'$$

or

$$\begin{array}{l}\nabla^2\Phi=-\rho'/\varepsilon_a,\\[2mm]\vec{E}''=-\operatorname{grad}\Phi,\end{array}\qquad(5)$$

where $\vec{\delta}'_{es}=\vec{\delta}_{es}+\varepsilon_a\dfrac{\partial\overrightarrow{E''}}{\partial t}=\vec{\delta}_{es}-\varepsilon_a\dfrac{\partial}{\partial t}(grad\ \Phi)$, $\rho'=\rho-\varepsilon_a\operatorname{div}\overrightarrow{E'}$, ε_a and μ_a are constants.

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Let us suppose that the waveguide excitation is a stationary process, i.e. the field sources change periodically in time. Then the sought fields $\vec{E}$, $\vec{H}$ can be represented as Fourier series:

$$\vec{E}' = Re \sum_{n=1}^{\infty} \vec{E}^n e^{jn\omega t}, \qquad \vec{H} = Re \sum_{n=1}^{\infty} \vec{H}^n e^{jn\omega t}. \tag{6}$$

Solutions for $\vec{E}^n$ and $\vec{H}^n$ can be written, using the results obtained in Ref. [8], replacing $\vec{\delta}_{es}$ by $\vec{\delta}'_{es}$:

$$\left. \begin{array}{c} \vec{E}^n = \dot{C}_S^n \dot{\vec{E}}_S^n + \dot{C}_{-S}^n \dot{\vec{E}}_{-S}^n - \dfrac{(\vec{\delta}_z')^n}{jn\omega\varepsilon_a}, \\ \vec{H}^n = \dot{C}_S^n \dot{\vec{H}}_S^n + \dot{C}_{-S}^n \dot{\vec{H}}_{-S}^n \end{array} \right\} \tag{7}$$

Here $\dot{\vec{E}}_{\pm S}^n$ and $\dot{\vec{H}}_{\pm S}^n$ are the fields of unidirectional ($+S$) and counter-propagating ($-S$) inherent waves of the waveguide (without sources) on the frequency $n\omega$, $(\vec{\delta}_z')^n$ is the harmonic of z-component of current density $\vec{\delta}_z'$ on the frequency $n\omega$, $n > 0$ is the harmonic number.

The coefficients $\dot{C}_{\pm S}^n$ can be found from the following differential equations:

$$\frac{d\dot{C}_{\pm S}^n}{dz} = \frac{1}{\pi N_S^n} \int_0^{2\pi} \int_{\perp} \vec{\delta}' \dot{\vec{E}}_{\mp S}^n \, dS_{\perp} e^{-jn\omega t} d\omega t, \tag{8}$$

where

$$N_S^n = \int_{S_{\perp}} \left(\left[\dot{\vec{E}}_{\mp S}^n, \dot{\vec{H}}_{-S}^n \right] - \left[\dot{\vec{E}}_{-S}^n, \dot{\vec{H}}_{\mp S}^n \right] \right) dS_{\perp} \tag{9}$$

is the norm of the wave with S number on the frequency $n\omega$,

$$(\vec{\delta}_z')^n = (\vec{\delta}_z)^n - jn\omega\varepsilon_a \frac{\partial \Phi^n}{\partial z}. \tag{10}$$

Now let us discuss the solution of Eq. (5) representing (6):

$$\dot{\Phi} = Re \sum \Phi^n e^{jn\omega t}. \tag{11}$$

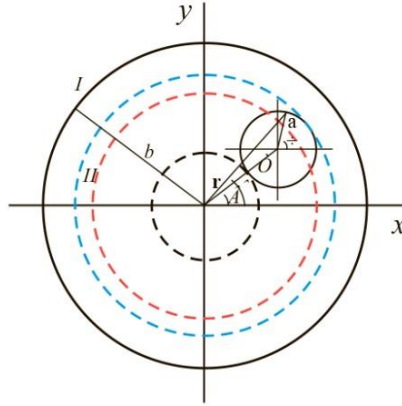Hence, using Eqs. (7) and (8) we obtain the following for Φ^n:

Fig. 1. Isophase tube forming in an electron beam for TE_{01}
electromagnetic wave in the gyroresonance device.

Рис. 1. Формирование гирорезонансной изофазной трубки
в электронном пучке для электромагнитной волны TE_{01}

$$\nabla^2 \Phi^n = -(\rho')^n/\varepsilon_a = -\rho^n/\varepsilon_a + \operatorname{div} \vec{E}^n =$$

$$-\rho^n/\varepsilon_a + \sum_S \left(\frac{d\dot{C}_S^n}{dz} \dot{E}_{zS}^n + \frac{d\dot{C}_{-S}^n}{dz} \dot{E}_{z-S}^n \right) - \frac{\partial \delta_z^n}{\partial z} / jn\omega\varepsilon_a + \frac{\partial^2 \Phi}{\partial z^2},$$

or

$$\nabla^2 \Phi^n = -\rho^n/\varepsilon_a + \sum_S \left(\frac{d\dot{C}_S^n}{dz} \dot{E}_{zS}^n + \frac{d\dot{C}_{-S}^n}{dz} \dot{E}_{z-S}^n \right) - \frac{1}{jn\omega\varepsilon_a} \frac{\partial \delta_z^n}{\partial z}. \quad (12)$$

It is obvious from Eq. (12) that $(\vec{E}'')^n = -\operatorname{grad} \Phi^n$, i.e. it is the electric field component that is missed in the excitation equations [6–8].

We notice that according to the theory stated in the monograph [17, Chapter 6] the components of natural fields of the regular waveguide $\vec{E}_S^n, \vec{E}_{-S}^n$, $\vec{H}_S^n$ and $\vec{H}_{-S}^n$ have the following form:

$$\vec{E}_S^n = \vec{E}_{St}^n(\vec{r_\perp})e^{-jh_S^n z}, \quad \vec{H}_S^n = \vec{H}_{S0}^n(\vec{r_\perp})e^{-jh_S^n z}, \quad (13)$$

$$\vec{E}_{-S}^n = \vec{E}_{-S0}^n(\vec{r_\perp})e^{-jh_S^n z}, \quad \vec{H}_{-S}^n = \vec{H}_{-S0}^n(\vec{r_\perp})e^{-jh_S^n z}, \quad (14)$$

where $h_S^n = \sqrt{\left(\frac{n\omega}{c}\right)^2 - \chi_S^2}$ is the propagation constant of S-type waves on the frequency $n\omega$, $\vec{r_\perp}$ means the coordinates transversal to z, c is the velocity of light in free space, χ_S is the eigenvalue for the S-type wave mode.

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For TM-type waves we have the following ratio:

$$\vec{E}^n_{-S0t}(\vec{r_\perp}) = -\vec{E}^n_{-S0t}(\vec{r_\perp}),$$
$$\vec{E}^n_{-S0z}(\vec{r_\perp}) = -\vec{E}^n_{-S0z}(\vec{r_\perp}), \quad (15)$$
$$\vec{H}^n_{-S0t}(\vec{r_\perp}) = \vec{H}^n_{-S0t}(\vec{r_\perp}),$$

and for TE-type waves:

$$\vec{H}^n_{-S0t}(\vec{r_\perp}) = -\vec{H}^n_{-S0t}(\vec{r_\perp}),$$
$$\vec{H}^n_{-S0z}(\vec{r_\perp}) = \vec{H}^n_{-S0z}(\vec{r_\perp}), \quad (16)$$
$$\vec{E}^n_{-S0t}(\vec{r_\perp}) = -\vec{E}^n_{-S0t}(\vec{r_\perp}).$$

Here the index "t" means the transversal component of the vector.

Taking into account Eqs. (8), (15) and (16), we can ascertain the following equations:

$$\frac{d\dot{C}^n_S}{dz}\dot{E}^n_{zS} + \frac{d\dot{C}^n_{-S}}{dz}\dot{E}^n_{-zS} = \frac{1}{N^n_S}\frac{2}{\pi}\int_0^{2\pi}\int_{S_\perp}\vec{\delta'}\vec{E}^n_{S0t}dS_\perp e^{-jn\omega t}d\omega t \cdot E_{S0z}(\vec{r_\perp}),$$

and

$$\text{div}\left(\dot{C}^n_S\vec{H}^n_S + \dot{C}^n_{-S}\vec{H}^n_{-S}\right) = \frac{d\dot{C}^n_S}{dz}H^n_{zS} + \frac{d\dot{C}^n_{-S}}{dz}H^n_{-zS} = 0.$$

3. Examples of problems with isolation of two-dimensional E-fields

Let us discuss partial cases that illustrate the application of the general theory expounded in Section 1. Such type of microwave electronics problems needs extraction of 2D quasi-static space charge fields.

3.1. Gyroresonance devices

The authors of the works [18] have used equations like Eq. (12) and obtained the solutions for a radial quasi-static space charge field for axially symmetric gyroresonance devices on TE0i working types of waves:

$$E_r = -\frac{\tau}{8\pi^2\varepsilon_0 r}\int_0^{2\pi}\left[1 + \text{sign}\,(r - r'(\alpha_0))\right]d\alpha_0. \quad (17)$$

Here $\tau = I_0/v_0$ is the linear density of beam charge, ε_0 is the vacuum permittivity, r is the radius of the observation point, r' is the radius of an isophase electron tube, α_0 is the relative phase of rotation of electrons belonging to this tube. The scheme of forming of an isophase tube for TE$_{01}$ wave is shown in Fig. 1.

The obtained solution (17) in the works mentioned above allowed defining the main factors of the influence of space charge upon the process of interaction in the gyroresonance devices: acceleration and amplification of electrons phase bunching and the increase of modulation dispersion of electron velocities.
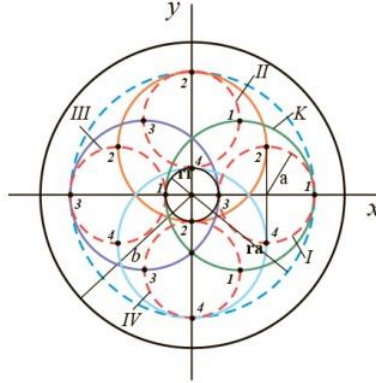

Fig. 2. Four isophase tubes forming TE_{11} electromagnetic wave in the gyroresonance device. The corresponding isophase tubes are marked with the 'I–IV' symbols.

Рис. 2. Четыре изофазных трубки, образующие в гироприборе электромагнитные волны TE_{11}. Соответствующие изофазы трубки обозначены символами «I—IV»

In the Refs. [18] the solution for gyroresonance devices with nonsymmetrical TE_{ni} working types of waves was obtained similarly. In particular, for TE_{11}-type waves the isophase tube field has the following form:

$$\dot{E}_t = E_x + jE_y = \frac{\tau}{2\pi\varepsilon_0}\left(\frac{1}{2r}\left[1 + \text{sign}\,(r - r')\right] * \left(e^{j\varphi} + \frac{a}{r}e^{2j\varphi}\right) + \frac{3}{2}\frac{a}{b^2}\right). (18)$$

Here a is the radius of Larmor orbit of electrons on this isophase tube; r, φ are the coordinates of the observation point, r is the isophase tube radius.

The scheme of forming of four isophase tubes for the working TE_{11} waveguide mode is shown in Fig. 2. Using Eqs. (17) and (18) helped making the comparison of efficiency of the optimized gyro-TWT on TE_{01} and TE_{11} modes in the Ref. [18].

3.2. Relativistic Cherenkov generators

The radial component of a quasi-static field of the beam plays the significant role in relativistic Cherenkov generators with tubular electron beams and influences upon the movement of the electrons relatively to the working surface

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of a corrugated waveguide. This field is considered in [19] as an additional component to E_r in the following form:

$$E_r^0 = -\frac{\sigma_{0i}}{2r_i b}\left(\frac{1}{\beta_{z0}} - \beta_{z0}\right). \tag{19}$$

Here $\sigma_{0i} = e|I_{0i}|/\pi\varepsilon_0 m_0 c^3$, I_{0i}, is the current of ith layer of the beam, r_i is the beam radius, b is the radius of corrugation, $\beta_{z0} = v_{z0}/c$, v_{z0} is the mean longitudinal velocity of electrons, c is the light velocity in vacuum, e, m are the electron charge and mass.

4. Conclusions

In this paper, the transformation of traditional equations of excitation of a hollow waveguide by an extraneous electric field is made, taking into account the potential electric field in the source area which appears because of the change of a waveguide structure.

Acknowledgement

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