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ON A GEOMETRIC MODEL OF AN UNIVERSE

The celestial spheres were the fundamental entities of the cosmological models developed by Plato, Eudoxus, Aristotle, Ptolemy and others [1]. Our concept of the world can be considered as a modern interpretation of ideas of ancient greeks or, perhaps, of more old sources which we do not know. Some geometric model of the world (W) is considered where the manifold W is identified with a locally trivial fibre bundle W of so called crystal spheres over a manifold U called the universal time (structure of U is unknown). A sphere bundle is a fiber bundle whose fiber is a n-sphere. Given a vector bundle E with a metric (such as the tangent bundle to a Riemannian manifold) one can construct the associated unit sphere bundle for which the fiber over a point x is the set of all unit vectors in E_x. When the vector bundle is the tangent bundle $T(M)$, the unit sphere bundle is known as the unit tangent bundle, and is denoted $UT(M)$.

It is well known that a n-sphere is identified by the stereographic projection with $\mathbf{R}n\cup\{\infty\}$ where $\{\infty\}$ is a singular point.

Further, we consider only one crystal sphere $S(n)\subset W$ with a smooth triangulation considered above. We can fix some Riemannian metric g on the manifold $S(n)$ which defines the length of arc of a piecewise smooth curve and the continuous function $r(x; y)$ of the distance between two points $x, y\in S(n)$. The topology defined by the function of distance (metric) r is the same as the topology of the manifold $S(n)$, For any n-simpex dn the diameter $d(dn)$ is defined by the formula $d(dn)=\max r(x; y)$, $x, y\in dn$. The diameter of the triangulation is called the maximal value among the diameters of the n-simplexes. It seems that the diameter of the triangulation can be very small (*subatomic*).

Using a smooth triangulation above and the function of distance we consider an algorithm of extension of coordinate neighborhood (inner part of the *canonical polyhedron*). The beginning of the algorithm we call the geometric *Big Bang*. The inner part of the canonical polyhedron is painted white and the boundary of the canonical polyhedron is painted black every step, the other part of the manifold which has not been still painted assumes to be grey (*three kinds of matter* from a physical point of view). A small closed neighborhood of the boundary of the canonical polyhedron we repaint black and call a *geometric black hole* (it seems that black holes observed in astronomy are presentations of one big black object).

Main algorithm. Let δ_0^n be some simplex of the fixed triangulation of the manifold *S(n).* We paint the inner part $Int\delta_0^n$ of the simplex δ_0^n white and the boundary $\partial\delta_0^n$ of δ_0^n black. There exist coordinates on $Int\delta_0^n$ given by diffeomorphism φ_0. A subsimplex $\delta_{01}^{n-1} \subset \delta_0^n$ is defined by a black face $\delta_{01}^{n-1} \subset \delta_0^n$ and the center c_0 of δ_0^n. We connect c_0 with the center d_0 of the face δ_{01}^{n-1} and decompose the subsimplex δ_{01}^n as a set of intervals which are parallel to the interval c_0d_0. The face δ_{01}^{n-1} is a face of some simplex δ_1^n that has not been painted. We draw an interval between d_0 and the vertex v_1 of the subsimplex δ_1^n which is opposite to the face δ_{01}^{n-1} then we decompose δ_1^n as a set of intervals which are parallel to the interval d_0v_1. The set $\delta_{01}^n \cup \delta_1^n$ is a union of such broken lines every one from which consists of two intervals where the endpoint of the first interval coincides with the beginning of the second interval (in the face δ_{01}^{n-1}) the first interval belongs to δ_{01}^n and the second interval belongs to δ_1^n. We construct a homeomorphism (extension) ϕ_{01}^1: $Int\delta_{01}^n \to Int\left(\delta_{01}^n \bigcup \delta_1^n\right)$. Let us consider a point $x \in Int\delta_{01}^n$ and let x belong to a broken line consisting of two intervals the first interval is of a length of s_1 and the second interval is of a length of s_2 and let x be at a distance of s from the beginning of the first interval. Then we suppose that $\phi_{01}^1(x)$ belongs to the same broken line at a distance of $\frac{s_1+s_2}{s_1}\cdot s$ from the beginning of the first interval. It is clear that ϕ_{01}^1 is a homeomorphism giving coordinates on $Int\left(\delta_{01}^n \bigcup \delta_1^n\right)$. We paint points of $Int\left(\delta_{01}^n \bigcup \delta_1^n\right)$ white. Assuming the coordinates of points of white initial faces of subsimplex δ_{01}^n to be fixed we obtain correctly introduced coordinates on $Int\left(\delta_0^n \bigcup \delta_1^n\right)$. The set $\sigma_1 = \delta_0^n \bigcup \delta_1^n$ is called a *canonical polyhedron*. We paint faces of the boundary $\partial\sigma_1$ black.

We describe the contents of the successive step of the algorithm of extension of coordinate neighborhood. Let us have a canonical polyhedron σ_{k-1} with white inner points (they have introduced *white coordinates*) and the black boundary $\partial\sigma_{k-1}$. We look for such an *n*-simplex in σ_{k-1}, let it be δ_0^n that has such a black face, let it be δ_{01}^{n-1} that is simultaneously a face of some *n*-simplex, let it be δ_1^n, inner points of which are not painted. Then we apply the procedure described above to the pair δ_0^n, δ_1^n. As a result we have a polyhedron σ_k with one simplex more than σ_{k-1} has. Points of $Int\sigma_k$ are painted in white and the boundary $\partial\sigma_k$ is painted in black. The process is finished in the case when all the black faces of the last polyhedron border on the set of white points (the cell) from two sides.

After that all the points of the manifold *S(n)* are painted in black or white.

Further, we consider deformations of tensor fields, fiber bundles and operators (physical structures and equations) towards the black hole. These deformations are continuous and sectionally smooth and they have a very simple constructions on a white neighborhood where a parameter *t*(*l*) of the deformations of structures can be considered as a local time along every piecewise smooth broken line *l*. We have got only one black point $x_0 \in S(n)$ at the end of all considered algorithms (other part of the manifold is white). Let $B(x_0)$ be a small black closed ball with the center x_0. All the resulting parts of the deformed structures have been concentrated into $B(x_0)$. We consider an inversion (*Big Bang*) painting inner part of $B(x_0)$ white and other points of *S(n)* grey and begin again the process above where the initial simplex is a subset of $B(x_0)$. Thus, Big Bangs have a cyclical nature.

We remark that all the algorithms considered in the article are based on the mathematical methodology "step by step". From a physical point of view the processes must have explosive characters i.e. a big number of the steps of the algorithms must be produced almost simultaneously.

Conclusion. Thus we have got a geometric model of an universe where the world is identified with a fibre bundle of crystal spheres. The following mathematical notions have been considered which are close to those studied in physics.

1. Extension of white coordinate neighborhood – extension of the universe.
2. Three paintings – three kinds of matter.
3. The set of piecewise smooth broken lines – strings.
4. A parameter of deformations along a line – a local time along the line.
5. Geometric black hole – black holes (It seems that black holes observed in astronomy are presentations of one big black object).
6. Deformations of tensor fields, operators, fibre bundle towards the geometric black hole – corresponding situations in physics.
7. Geometric Big Bang – Big Bang.

REFERENCES

1. Crowe, M. J. Theories of the World from Antiquity to the Copernican Revolution / M.J. Crowe. – NY: Dover Publ., 2001. – 380 p.
2. Ermolitski, A.A. Crystal Spheres as the World // Open Journal of Modern Physics. 2014. V. 1. N. 2. P. 66–74.