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Kisel V.V., Ovsiyuk E.M., Voynova Ya.A., Pletyukhov V. A., Gilewsky V.V., Red’kov V.M.

Solutions with Spherical Symmetry of the Wave Equation for a Spin 3/2 Particle

The wave equation for a spin 3/2 particle, described by 16-component vector-bispinor , is investigated in spherical coordinates . In the frame of Pauli-Fierz approach, the complete equation is split into the main equation, and two additional constraints, algebraic and differential ones. There are constructed solutions on which4 operators are diaginalized: energy, square and third projection of the total angular momentum, and spacial reflection, they correspond to quantum numbers {ϵ, j, m, P}. After separating the variables, we derive the main system of 8 radial frst-order equations and additional 2 algebraic and 2 differential constraints. Solutions of the radial equations are constructed as linear combinations of Bessel functions. With the use of the known properties of the Bessel functions, the system of differential equations is transformed to the form of purely algebraic equations with respect to three quantities a_1, a_2, a_3 . Its solutions may be chosen in various ways by resolving the simple linear condition $A_1a_1 + A_2a_2 + A_3a_3 = 0$, where coefficients A_i are expressed trough the quantum numbers ϵ, j. Any two linearly independent sets a1, a2, a3 determine quantum states (1) and (2) of the spin 3/2 particle. Two most simple and symmetric solutions $a_i^{(1)}$ and $a_i^{(2)}$ have been chosen. Thus, at fixed quantum numbers {ϵ, j, m, P} there exists double-degeneration of the quantum states. Explicit form of the operator associated with such a degeneration is not found.

Keywords: spin 3/2 particle, degrees of freedom, spherical symmetry, exact solutions, Bessel functions, degeneration of quantum state.