Fermion with Three Mass Parameters, P-Asymmetric Model, Interaction with External Fields

O.N. Bianko*
Secondary school,Hoiniki, Belarus

Ya. A. Voynova† and V. M. Red'kov‡
B. I. Stepanov Institute of Physics of NAS of Belarus,
68-2 Nezavisimosti Ave., 220072 Minsk, BELARUS

E.M. Ovsiyuk§
Mozyr State Pedagogical University, Mozyr State Pedagogical University named after I.P. Shamyakin

V.A. Pletyukhov¶
Brest State University named after A.S. Pushkin

V.V. Kisel**
State University of Informatics and Radio-electronics, Minsk

The theory of a P-asymmetric spin 1/2 fermion with three mass parameters is developed. It is based on 20-component representation of the Lorentz group. Starting with the Gel'fand - Yaglom approach, we derive a spin-tensor form of a generalized wave equation. In absence of external electromagnetic fields, the system of equation can be transformed to the form of three separate P-asymmetric Dirac-like equations for bispinor wave functions with different masses. The values of masses values vary within some freedom, determined by parameters of the model. In presence of external electromagnetic fields, three bispinors turn out to be linked into the unified system of equations. If the scalar curvature of the space differs from zero, then between three bispinor components there arises additional terms of geometrical interaction. Extension of the theory to General relativity is performed, at this there arises additional interaction term through Ricci scalar function. The model for P-asymmetric fermion fermion with three mass parameters allows for restriction to Majorana case.

The next step should be the development of a model accounting for both P-symmetric and P-asymmetric.

I. INTRODUCTION

In the present time, existence of small but not vanishing masses for neutrinos is an the experimentally established fact, this is proved by neutrino oscillation effect. Historically, in the time when a postulation basis for the relativistic wave equations theory was created (for more detail see in [1–3]), it was believed that any correct wave equation must be invariant with respect to both proper Lorentz group and spacial reflection (so-called P-invariance was assumed as obligatory. However, later it was established that in processes with neutrinos, for instance in β-decay, the conservation low for P-parity may not be valid. Therefore, we can expect that when describing neutrinos within conventional theory the requirement of P-invariance may be ignored. In any case, it should be helpful to have studied various P-noninvariant models In the present paper, we will study a possibility within the general Gel'fand–Yaglom approach to construct a relativistic wave equation which describes a particle with single value of spin, $S = 1/2$, and spectrum of masses (three mass parameters). This complicated wave equation is considered as relevant to describing a triple generation of neutrinos, being considered as a unified physical object. This equation is asymmetric with respect to the operation of spacial P-reflection. Also see some related studies in [4–15].

*Electronic address: olka-proc@mail.ru
†Electronic address: voinovayanina@mail.ru
‡Electronic address: v.redkov@ifanbel.bas-net.by
§Electronic address: e.ovsiyuk@mail.ru
¶Electronic address: pletyukhov@yandex.by
**Electronic address: vasiliy$_{-}$bspu@mail.ru

II. GEL'FAND – YAGLOM FORMALISM

In order to develop an P-asymmetric model for a spin 1/2 particle with three mass parameters, we start with the following set of irreducible representations of the Lorentz group (see notations in [...])

$$T = (0,\frac{1}{2}) \oplus (\frac{1}{2},0) \oplus (0,\frac{1}{2})' \oplus (\frac{1}{2},0)' \oplus (1,\frac{1}{2}) \oplus (\frac{1}{2},1)\,, \tag{II.1}$$

or numerating representations by Arabic numerals $1,...,6$: $T = 1 \oplus 2 \oplus 3 \oplus 4 \oplus 5 \oplus 6$,

$$1 = (0,\frac{1}{2}), \quad 2 = (\frac{1}{2},0), \quad 3 = (0,\frac{1}{2})', \quad 4 = (\frac{1}{2},0)', \quad 6 = (1,\frac{1}{2}), \quad 5 = (\frac{1}{2},1)\,. \tag{II.2}$$

The corresponding linking scheme has the structure

$$\begin{array}{ccc} 1 & - & 2 \\ | & & | \\ 6 & - & 5 \\ | & & | \\ 3 & - & 4 \end{array}\;. \tag{II.3}$$

Basis vectors of the space are divided in five groups (see in [...])

$$\begin{array}{c} (\epsilon^{1}_{\frac{1}{2},\frac{1}{2}}, \epsilon^{1}_{\frac{1}{2},-\frac{1}{2}}, \epsilon^{2}_{\frac{1}{2},\frac{1}{2}}, \epsilon^{2}_{\frac{1}{2},-\frac{1}{2}}), \quad (\epsilon^{3}_{\frac{1}{2},\frac{1}{2}}, \epsilon^{3}_{\frac{1}{2},-\frac{1}{2}}, \epsilon^{4}_{\frac{1}{2},\frac{1}{2}}, \epsilon^{4}_{\frac{1}{2},-\frac{1}{2}}), \quad (\epsilon^{5}_{\frac{1}{2},\frac{1}{2}}, \epsilon^{5}_{\frac{1}{2},-\frac{1}{2}}, \epsilon^{6}_{\frac{1}{2},\frac{1}{2}}, \epsilon^{6}_{\frac{1}{2},-\frac{1}{2}})\,, \\ (\epsilon^{5}_{\frac{3}{2},\frac{3}{2}}, \epsilon^{5}_{\frac{3}{2},-\frac{3}{2}}, \epsilon^{6}_{\frac{3}{2},\frac{3}{2}}, \epsilon^{6}_{\frac{3}{2},-\frac{3}{2}}), \quad (\epsilon^{5}_{\frac{3}{2},\frac{1}{2}}, \epsilon^{5}_{\frac{3}{2},-\frac{1}{2}}, \epsilon^{6}_{\frac{3}{2},\frac{1}{2}}, \epsilon^{6}_{\frac{3}{2},-\frac{1}{2}})\,. \end{array} \tag{II.4}$$

The matrix Γ_4 of the relevant 1-st order wave equation may have the structure

$$\Gamma_4 = \left| \begin{array}{cc} C_4^{(1/2)} & 0 \\ 0 & C_4^{(3/2)} \end{array} \right|, \tag{II.5}$$

where the possible spin blocks $C_4^{(1/2)}$ and $C_4^{(3/2)}$ may read as follows

$$C^{(1/2)} = \left| \begin{array}{cccccccccccc}
0 & 0 & c^{1/2}_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{1/2}_{16} & 0 \\
0 & 0 & 0 & c^{1/2}_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{1/2}_{16} \\
c^{1/2}_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{1/2}_{25} & 0 & 0 & 0 \\
0 & c^{1/2}_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{1/2}_{25} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & c^{1/2}_{34} & 0 & 0 & 0 & c^{1/2}_{36} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{1/2}_{34} & 0 & 0 & 0 & c^{1/2}_{36} \\
0 & 0 & 0 & 0 & c^{1/2}_{43} & 0 & 0 & 0 & c^{1/2}_{45} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c^{1/2}_{43} & 0 & 0 & 0 & c^{1/2}_{45} & 0 & 0 \\
0 & 0 & c^{1/2}_{52} & 0 & 0 & 0 & c^{1/2}_{54} & 0 & 0 & 0 & c^{1/2}_{56} & 0 \\
0 & 0 & 0 & c^{1/2}_{52} & 0 & 0 & 0 & c^{1/2}_{54} & 0 & 0 & 0 & c^{1/2}_{56} \\
c^{1/2}_{61} & 0 & 0 & 0 & c^{1/2}_{63} & 0 & 0 & 0 & c^{1/2}_{65} & 0 & 0 & 0 \\
0 & c^{1/2}_{61} & 0 & 0 & 0 & c^{1/2}_{63} & 0 & 0 & 0 & c^{1/2}_{65} & 0 & 0
\end{array} \right|, \tag{II.6}$$

$$C^{(3/2)} = \left| \begin{array}{cccccccc}
0 & 0 & c^{3/2}_{56} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c^{3/2}_{56} & 0 & 0 & 0 & 0 \\
c^{3/2}_{56} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & c^{3/2}_{56} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & c^{3/2}_{56} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c^{3/2}_{56} \\
0 & 0 & 0 & 0 & c^{3/2}_{56} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c^{3/2}_{56} & 0 & 0
\end{array} \right| = I_2 \otimes \left| \begin{array}{cccc}
0 & 0 & c^{(3/2)}_{56} & 0 \\
0 & 0 & 0 & c^{(3/2)}_{56} \\
c^{(3/2)}_{65} & 0 & 0 & 0 \\
0 & c^{(3/2)}_{65} & 0 & 0
\end{array} \right|.$$

(II.7)

???? From general theory also follow two restrictions $c_{56}^{3/2} = 2c_{56}^{1/2}$, $c_{65}^{3/2} = 2c_{65}^{1/2}$, therefore the block $C^{(3/2)}$ reads

$$C^{(3/2)} = 2I_2 \otimes \begin{vmatrix} 0 & 0 & c_{56}^{1/2} & 0 \\ 0 & 0 & 0 & c_{56}^{1/2} \\ c_{65}^{1/2} & 0 & 0 & 0 \\ 0 & c_{65}^{1/2} & 0 & 0 \end{vmatrix} .$$

Exclusion spin 3/2 from the model leads to restrictions

$$c_{56}^{(3/2)} = c_{65}^{(3/2)} = 0 \quad \Longrightarrow \quad c_{56}^{(1/2)} = c_{65}^{(1/2)} = 0 , \quad C^{(3/2)} = 0 .$$

Correspondingly, the block $C^{(1/2)}$ takes on the form (it is convenient to decompose it into the sum of two terms, in fact separating P-symmetric and P-asymmetric parts)

$$C^{(1/2)} = \frac{1}{2} \begin{vmatrix} c_{12}^{(1/2)} + c_{21}^{(1/2)} & 0 & c_{16}^{(1/2)} + c_{25}^{(1/2)} \\ 0 & c_{34}^{(1/2)} + c_{43}^{(1/2)} & c_{36}^{(1/2)} + c_{45}^{(1/2)} \\ c_{52}^{(1/2)} + c_{61}^{(1/2)} & c_{54}^{(1/2)} + c_{63}^{(1/2)} & 0 \end{vmatrix} \otimes \gamma_4 +$$
$$\frac{1}{2} \begin{vmatrix} c_{12}^{(1/2)} - c_{21}^{(1/2)} & 0 & c_{16}^{(1/2)} - c_{25}^{(1/2)} \\ 0 & c_{34}^{(1/2)} - c_{43}^{(1/2)} & c_{36}^{(1/2)} - c_{45}^{(1/2)} \\ c_{52}^{(1/2)} - c_{61}^{(1/2)} & c_{54}^{(1/2)} - c_{63}^{(1/2)} & 0 \end{vmatrix} \otimes \gamma_5\gamma_4 . \tag{II.8}$$

Further, by simplicity reason, we exclude completely the P-symmetric components, so obtaining

$$C^{(1/2)} = -i \begin{vmatrix} c_{12}^{(1/2)} & 0 & c_{16}^{(1/2)} \\ 0 & c_{34}^{(1/2)} & c_{36}^{(1/2)} \\ c_{52}^{(1/2)} & c_{54}^{(1/2)} & 0 \end{vmatrix} \otimes (i\gamma_5\gamma_4) , \quad (i\gamma_5\gamma_4)^2 = I. \tag{II.9}$$

We will apply the following representation of the Dirac matrices

$$\gamma_1 = \begin{vmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{vmatrix}, \gamma_3 = \begin{vmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{vmatrix}, \gamma_2 = \begin{vmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{vmatrix}, \gamma_4 = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}, \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} .$$

It will be convenient to use the shortening presentation of the block (II.9):

$$C^{(1/2)} = \begin{vmatrix} b_1 & 0 & b_2 \\ 0 & b_3 & b_4 \\ b_5 & b_6 & 0 \end{vmatrix} \otimes (i\gamma_5\gamma_4) , \quad (i\gamma_5\gamma_4)^2 = I . \tag{II.10}$$

Let us find the characteristic equation for block $C^{(1/2)}$, that is

$$\lambda^3 - \lambda^2(b_1 + b_3) + \lambda(b_1b_3 - b_2b_5 - b_4b_6) - (b_2b_3b_5 + b_1b_4b_6) = 0 . \tag{II.11}$$

Evidently, we have three constraints for the roots:

$$b_2b_3b_5 + b_1b_4b_6 = -\lambda_1\lambda_2\lambda_3, \quad b_1 + b_3 = \lambda_1 + \lambda_2 + \lambda_3 ,$$
$$b_1b_3 - b_2b_5 - b_4b_6 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 . \tag{II.12}$$

Now, we are to get restrictions related to existence of a matrix of the invariant bilinear form:

$$\eta^{(1/2)} = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} \otimes i\gamma_5\gamma_4 \, ; . \tag{II.13}$$

where a, b, c may take the values (± 1). The basic constraint on the matrix $\eta^{(1/2)}$ reads

$$\eta^{(1/2)} C^{(1/2)} = [\eta^{(1/2)} C^{(1/2)}]^{+} \, ,$$

whence it follows

$$\left| \begin{array}{ccc} ab_1 & 0 & ab_2 \\ 0 & bb_3 & bb_4 \\ cb_5 & cb_6 & 0 \end{array} \right| = \left| \begin{array}{ccc} ab_1^* & 0 & cb_5^* \\ 0 & bb_3^* & cb_6^* \\ ab_2^* & bb_4^* & 0 \end{array} \right| ,$$

so we get possible solutions:

$$\begin{array}{rcl} ab_1 = ab_1^*, \; bb_3 = bb_3^* & \Longrightarrow & b_1 = b_1^*, \; b_3 = b_3^*; \\ ab_2 = cb_5^*, \; cb_5 = ab_2^* & \Longrightarrow & b_5 = f b_2^*, \quad f = \pm 1 \; ; \\ bb_4 = cb_6^*, cb_6 = bb_4^* & \Longrightarrow & b_6 = g b_4^*, \quad g = \pm 1 \; ; \end{array} \tag{II.14}$$

in general, parameters b_2, b_5, b_4, b_6 may be real or complex

III. RELATIONS BETWEEN CANONICAL, SPINOR, AND GEL'FAND – YAGLOM BASES

As shown above in Gel'fand – Yaglom basis, the (20×20)-matrix $\Gamma_4^{(GY)}$ of the generalized equation has the structure

$$\Gamma_4^{(GY)} = \left| \begin{array}{ccccc} B_1 & 0 & B_2 & 0 & 0 \\ 0 & B_3 & B_4 & 0 & 0 \\ B_5 & B_6 & 0 & & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right| , \; B_i = \left| \begin{array}{cccc} 0 & 0 & b_i & 0 \\ 0 & 0 & 0 & b_i \\ -b_i & 0 & 0 & 0 \\ 0 & -b_i & 0 & 0 \end{array} \right| , i = 1, 2, 3, 4, 5, 6 \tag{III.1}$$

We will need the listing of all components of the wave function in three bases, Gel'fand – Yaglom,canonical, and spinor ones:

$$\Psi^{GY} = \left| \begin{array}{c} \psi^{(1)}_{1/2,+1/2} \\ \psi^{(1)}_{1/2,-1/2} \\ \psi^{(4)}_{1/2,+1/2} \\ \psi^{(4)}_{1/2,-1/2} \\ \\ \psi^{(2)}_{1/2,+1/2} \\ \psi^{(2)}_{1/2,-1/2} \\ \psi^{(5)}_{1/2,+1/2} \\ \psi^{(5)}_{1/2,-1/2} \\ \\ \psi^{(3)}_{1/2,+1/2} \\ \psi^{(3)}_{1/2,-1/2} \\ \psi^{(6)}_{1/2,+1/2} \\ \psi^{(6)}_{1/2,-1/2} \\ \\ \psi^{(3)}_{3/2,+3/2} \\ \psi^{(3)}_{3/2,-3/2} \\ \psi^{(6)}_{3/2,+3/2} \\ \psi^{(6)}_{3/2,-3/2} \\ \\ \psi^{(3)}_{3/2,+1/2} \\ \psi^{(3)}_{3/2,-1/2} \\ \psi^{(6)}_{3/2,+1/2} \\ \psi^{(6)}_{3/2,-1/2} \end{array} \right| , \quad \Psi^{canon} = \left| \begin{array}{c} \psi^{(1)}_{(0,+1/2)} \\ \psi^{(1)}_{(0,-1/2)} \\ \psi^{(4)}_{(+1/2,0)} \\ \psi^{(4)}_{(-1/2,0)} \\ \\ \psi^{(2)}_{(0,+1/2)} \\ \psi^{(2)}_{(0,-1/2)} \\ \psi^{(5)}_{(+1/2,0)} \\ \psi^{(5)}_{(-1/2,0)} \\ \\ \psi^{(3)}_{(+1,+1/2)} \\ \psi^{(3)}_{(0,+1/2)} \\ \psi^{(3)}_{(-1,+1/2)} \\ \psi^{(3)}_{(+1,-1/2)} \\ \psi^{(3)}_{(0,-1/2)} \\ \psi^{(3)}_{(-1,-1/2)} \\ \\ \psi^{(6)}_{(+1/2,+1)} \\ \psi^{(6)}_{(+1/2,0)} \\ \psi^{(6)}_{(+1/2,-1)} \\ \psi^{(6)}_{(-1/2,+1)} \\ \psi^{(6)}_{(-1/2,0)} \\ \psi^{(6)}_{(-1/2,-1)} \end{array} \right| , \quad \Psi^{spin} = \left| \begin{array}{c} (\psi^{(1)})^{\dot{1}} \\ (\psi^{(1)})^{\dot{2}} \\ (\psi^{(4)})_{1} \\ (\psi^{(4)})_{2} \\ \\ (\psi^{(2)})^{\dot{1}} \\ (\psi^{(2)})^{\dot{2}} \\ (\psi^{(5)})_{1} \\ (\psi^{(5)})_{2} \\ \\ (\psi^{(3)})^{\dot{1}}_{(11)} \\ (\psi^{(3)})^{\dot{1}}_{(12)} \\ (\psi^{(3)})^{\dot{1}}_{(22)} \\ (\psi^{(3)})^{\dot{2}}_{(11)} \\ \\ (\psi^{(3)})^{\dot{2}}_{(12)} \\ (\psi^{(3)})^{\dot{2}}_{(22)} \\ (\psi^{(5)})^{\dot{1}\dot{1}}_{1} \\ (\psi^{(5)})^{\dot{1}\dot{2}}_{1} \\ \\ (\psi^{(5)})^{\dot{2}\dot{2}}_{1} \\ (\psi^{(5)})^{\dot{1}\dot{1}}_{2} \\ (\psi^{(5)})^{\dot{1}\dot{2}}_{2} \\ (\psi^{(5)})^{\dot{2}\dot{2}}_{2} \end{array} \right| \tag{III.2}$$

The first two are related by the following linear transformation $\Psi^{GY} = B\Psi^{canon}$, $\Psi^{canon} = B^{+}\Psi^{GY}$,

$$
B = \left|\begin{array}{ccccc}
I_4 & 0 & 0 & 0 & 0 \\
0 & I_4 & 0 & 0 & 0 \\
0 & 0 & \begin{array}{cccc} 0 & -\sqrt{\frac{1}{3}} & 0 & \sqrt{\frac{2}{3}} \\ 0 & 0 & -\sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & 0 & 0 & 0 \\ \sqrt{\frac{1}{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\frac{1}{3}} \\ 0 & 0 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\sqrt{\frac{2}{3}} & 0 & 0 \\ \sqrt{\frac{2}{3}} & 0 & -\sqrt{\frac{1}{3}} & 0 \end{array} \\
0 & 0 & \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \\
0 & 0 & \begin{array}{cccc} 1 & \sqrt{\frac{2}{3}} & 0 & \sqrt{\frac{1}{3}} \\ 0 & 0 & \sqrt{\frac{1}{3}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & 0 & 0 & 0 \\ \sqrt{\frac{2}{3}} & 1 & 0 & 0 \\ 0 & 0 & 1 & \sqrt{\frac{2}{3}} \\ 0 & 0 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{1}{3}} & 0 & 0 \\ \sqrt{\frac{1}{3}} & 0 & \sqrt{\frac{2}{3}} & 1 \end{array}
\end{array}\right|
$$

Relation between the second two bases is given by $\Psi^{canon} = A\Psi^{spin}$, $\quad \Psi^{spin} = A^{-1}\Psi^{canon}$, where

$$
A = \left|\begin{array}{ccccc}
I_4 & 0 & 0 & 0 & 0 \\
0 & I_4 & 0 & 0 & 0 \\
0 & 0 & \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} & 0 & 0 \\
0 & 0 & 0 & \begin{array}{cccc} \sqrt{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{array} & 0 \\
0 & 0 & 0 & 0 & \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{array}
\end{array}\right|
$$

After performing needed calculation we get for the matrix Γ_4^{spin} the following form (the matrix is written down by columns, 2+3):

$$
\Gamma_4^{spin} = i\left|\begin{array}{cc}
\begin{array}{cccc} 0 & 0 & b_1 & 0 \\ 0 & 0 & 0 & b_1 \\ -b_1 & 0 & 0 & 0 \\ 0 & -b_1 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \\
\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & b_3 \\ -b_3 & 0 & 0 & 0 \\ 0 & -b_3 & 0 & 0 \end{array} \\
\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & -b_5/\sqrt{6} & 00 \\ 0 & 0 & -b_5\sqrt{2/3} & 0 \\ 0 & 0 & b_5\sqrt{2/3} & 0 \end{array} & \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & -b_6/\sqrt{6} & 00 \\ 0 & 0 & -b_6\sqrt{2/3} & 0 \\ 0 & 0 & b_6\sqrt{2/3} & 0 \end{array} \\
\begin{array}{cccc} 0 & 0 & 0 & b_5/\sqrt{6} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -b_5/\sqrt{6} & 0 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & 0 & 0 & b_6/\sqrt{6} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -b_6/\sqrt{6} & 0 & 0 & 0 \end{array} \\
\begin{array}{cccc} 0 & -b_5\sqrt{2/3}0 & 0 & \\ b_5\sqrt{2/3} & 0 & 0 & 0 \\ 0 & b_5/\sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} & \begin{array}{cccc} 0 & -b_6\sqrt{2/3}0 & 0 & \\ b_6\sqrt{2/3} & 0 & 0 & 0 \\ 0 & b_6/\sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}
\end{array}\right|
$$

$$
\left.\begin{array}{ccc}
\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_2\sqrt{\frac{2}{3}} & 0 & -b_2\sqrt{\frac{2}{3}} \\ 0 & 0 & b_2\sqrt{\frac{2}{3}} & 0 \end{matrix} &
\begin{matrix} 0 & 0 & 0 & b_2\sqrt{\frac{2}{3}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -b_2\sqrt{\frac{2}{3}} & 0 & 0 & 0 \end{matrix} &
\begin{matrix} 0 & -b_2\sqrt{\frac{2}{3}} & 0 & 0 \\ b_2\sqrt{\frac{2}{3}} & 0 & -b_2\sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \\
\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_4\sqrt{\frac{2}{3}} & 0 & -b_4\sqrt{\frac{2}{3}} \\ 0 & 0 & b_4\sqrt{\frac{2}{3}} & 0 \end{matrix} &
\begin{matrix} 0 & 0 & 0 & b_4\sqrt{\frac{2}{3}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -b_4\sqrt{\frac{2}{3}} & 0 & 0 & 0 \end{matrix} &
\begin{matrix} 0 & -b_4\sqrt{\frac{2}{3}} & 0 & 0 \\ b_4\sqrt{\frac{2}{3}} & 0 & -b_4\sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right|
$$

Then we find expression for $\Gamma_4\Psi$ (the label of *spin basis* is omitted)

$$
\Gamma_4\Psi = i\left|\begin{array}{c}
\left[b_1\frac{1}{i}(\sigma^4)^{\dot a b}\psi_b+\sqrt{\frac{2}{3}}b_2\frac{1}{i}(\sigma^4)^c_{\dot b}\psi_c^{(\dot a\dot b)}\right] \\
-\left[b_1\frac{1}{i}(\sigma^4)_{a\dot b}\psi^{\dot b}+\sqrt{\frac{2}{3}}b_2\frac{1}{i}(\sigma^4)^b_{\dot c}\psi^{\dot c}_{(ab)}\right] \\
\left[b_3\frac{1}{i}(\sigma^4)^{\dot a b}\psi'_b+\sqrt{\frac{2}{3}}b_4\frac{1}{i}(\sigma^4)^c_{\dot b}\psi_c^{(\dot a\dot b)}\right] \\
-\left[b_3\frac{1}{i}(\sigma^4)_{a\dot b}\psi'^{\dot b}+\sqrt{\frac{2}{3}}b_4\frac{1}{i}(\sigma^4)^b_{\dot c}\psi^{\dot c}_{(ab)}\right] \\
-\frac{1}{\sqrt{6}}\left[b_5\frac{1}{i}\left((\sigma^4)^{\dot a}_b\psi_c+(\sigma^4)^{\dot a}_c\psi_b\right)+b_6\frac{1}{i}\left((\sigma^4)^{\dot a}_b\psi'_c+(\sigma^4)^{\dot a}_c\psi'_b\right)\right] \\
\frac{1}{\sqrt{6}}\left[b_5\frac{1}{i}\left((\sigma^4)^{\dot b}_a\psi^{\dot c}+(\sigma^4)^{\dot c}_a\psi^{\dot b}\right)+b_6\frac{1}{i}\left((\sigma^4)^{\dot b}_a\psi'^{\dot c}+(\sigma^4)^{\dot c}_a\psi'^{\dot b}\right)\right]
\end{array}\right|. \tag{III.3}
$$

IV. SYSTEM OF SPINOR EQUATIONS

In accordance with results of previous section, we have the following system of equations in spinor form:

$$ i\left(b_1\partial^{\dot a b}\psi_b+\sqrt{\frac{2}{3}}b_2\partial^c_{\dot b}\psi_c^{(\dot a\dot b)}\right)+m\psi^{\dot a}=0, \tag{IV.1} $$

$$ -i\left(b_1\partial_{a\dot b}\psi^{\dot b}+\sqrt{\frac{2}{3}}b_2\partial^b_{\dot c}\psi^{\dot c}_{(ab)}\right)+m\psi_a=0, \tag{IV.2} $$

$$ i\left(b_3\partial^{\dot a b}\psi'_b+\sqrt{\frac{2}{3}}b_4\partial^c_{\dot b}\psi_c^{(\dot a\dot b)}\right)+M\psi'^{\dot a}=0, \tag{IV.3} $$

$$ -i\left(b_3\partial_{a\dot b}\psi'^{\dot b}+\sqrt{\frac{2}{3}}b_4\partial^b_{\dot c}\psi'^{\dot c}_{(ab)}\right)+m\psi'_a=0, \tag{IV.4} $$

$$ -\frac{i}{\sqrt{6}}\left[b_5\left(\partial^{\dot a}_b\psi_c+\partial^{\dot a}_c\psi_b\right)+b_6\left(\partial^{\dot a}_b\psi'_c+\partial^{\dot a}_c\psi'_b\right)\right]+M\psi^{\dot a}_{(bc)}=0, \tag{IV.5} $$

$$ \frac{i}{\sqrt{6}}\left[b_5\left(\partial^{\dot b}_a\psi^{\dot c}+\partial^{\dot c}_a\psi^{\dot b}\right)+b_6\left(\partial^{\dot b}_a\psi'^{\dot c}+\partial^{\dot c}_a\psi'^{\dot b}\right)\right]+M\psi_a^{(\dot b\dot c)}=0. \tag{IV.6} $$

Let us introduce spin-vectors, $\psi_{\mu b}$ and $\psi^{\dot b}_\mu$:

$$ \psi^{\dot a}=\sigma^{\mu\dot a b}\psi_{\mu b}, \quad \psi_a=\sigma^\mu_{z\dot b}\psi^{\dot b}_\mu, \quad \psi_c^{(\dot a\dot b)}=\frac{1}{2}(\sigma^{\mu\dot a}_c\psi^{\dot b}_m+\sigma^{\mu\dot b}_c\psi^{\dot a}_m), \quad \psi^{\dot c}_{(ab)}=\frac{1}{2}(\sigma^{\mu\dot c}_a\psi_{b\mu}+\sigma^{\mu\dot c}_b\psi_{a\mu}), \tag{IV.7} $$

below instead of the prime we will use the symbol of *zero*: $\psi'^{\dot{a}} = \psi_0^{\dot{a}}$, $\psi'_a = \psi_{a0}$. Taking in mind (IV.7), eq. (IV.1) can be re-written as

$$i\left[b_1\partial^{\dot{a}b}\sigma^{\mu}_{b\dot{c}}\psi^{\dot{c}}_{\mu} + \frac{1}{2}\sqrt{\frac{2}{3}}b_2\partial^{c}_{\dot{b}}\left(\sigma^{\mu\dot{a}}_{c}\psi^{\dot{b}}_{m} + \sigma^{\mu\dot{b}}_{c}\psi^{\dot{a}}_{m}\right)\right] + m\sigma^{\mu\dot{a}b}\psi_{\mu b} = 0\,,$$

or

$$i\left[b_1\partial^{\dot{a}b}\sigma^{\mu}_{b\dot{c}}\psi^{\dot{c}}_{\mu} + \sqrt{\frac{1}{6}}b_2\left(-\partial^{\dot{b}c}\sigma^{\mu}_{c\dot{b}}\psi^{\dot{a}}_{\mu} - \sigma^{\mu\dot{a}c}\partial_{c\dot{b}}\psi^{\dot{b}}_{\mu}\right)\right] + m\sigma^{\mu\dot{a}b}\psi_{\mu b} = 0\,.$$

Allowing for the identity

$$-\partial^{\dot{b}c}\sigma^{\mu}_{c\dot{b}}\psi^{\dot{a}}_{\mu} = -\partial_{\nu}\sigma^{\dot{b}c}_{\nu}\sigma^{\mu}_{c\dot{b}}\psi^{\dot{a}}_{\mu} = \frac{2}{i}\partial_{\mu}\psi^{\dot{a}}_{\mu}$$

the last equation is presented as

$$i\left[b_1\partial^{\dot{a}b}\sigma^{\mu}_{b\dot{c}}\psi^{\dot{c}}_{\mu} + \sqrt{\frac{1}{6}}b_2\left(\frac{2}{i}\partial_{\mu}\psi^{\dot{a}}_{\mu} - \sigma^{\mu\dot{a}c}\partial_{c\dot{b}}\psi^{\dot{b}}_{\mu}\right)\right] + m\sigma^{\mu\dot{a}b}\psi_{\mu b} = 0\,. \qquad \text{(IV.8)}$$

In similar manner, from eq. (IV.2) it follows

$$-i\left[b_1\partial_{a\dot{b}}\sigma^{\mu\dot{b}c}\psi_{\mu c} + \sqrt{\frac{1}{6}}b_2\left(\frac{2}{i}\partial_{\mu}\psi_{\mu a} - \sigma^{\mu}_{a\dot{c}}\partial^{\dot{c}b}\psi_{\mu b}\right)\right] + m\sigma^{\mu}_{a\dot{b}}\psi^{\dot{b}}_{\mu} = 0\,. \qquad \text{(IV.9)}$$

In the same manner, from eqs. (IV.3) and (IV.4) we derive

$$i\left[b_3\partial^{\dot{a}b}\psi_{b0} + \sqrt{\frac{1}{6}}b_4\left(\frac{2}{i}\partial_{\mu}\psi^{\dot{a}}_{\mu} - \sigma^{\mu\dot{a}c}\partial_{c\dot{b}}\psi^{\dot{b}}_{\mu}\right)\right] + M\psi^{\dot{a}}_{0} = 0\,, \qquad \text{(IV.10)}$$

$$-i\left[b_3\partial_{a\dot{b}}\psi^{\dot{b}}_{0} + \sqrt{\frac{1}{6}}b_4\left(\frac{2}{i}\partial_{\mu}\psi_{a\mu} - \sigma^{\mu}_{a\dot{c}}\partial^{\dot{c}b}\psi_{\mu b}\right)\right] + M\psi_{a0} = 0\,. \qquad \text{(IV.11)}$$

Now let us turn to eq. (IV.6), re-written in the form

$$\frac{i}{\sqrt{6}}\left[b_5\left(\partial^{\dot{b}}_{a}\psi^{\dot{c}} + \partial^{\dot{c}}_{a}\psi^{\dot{b}}\right) + b_6\left(\partial^{\dot{b}}_{a}\psi'^{\dot{c}} + \partial^{\dot{c}}_{a}\psi'^{\dot{b}}\right)\right] + m\psi^{(\dot{b}\dot{c})}_{a} = 0\,,$$

whence taking in mind (IV.7) we get

$$\frac{i}{\sqrt{6}}\left[b_5\left(\partial^{\dot{b}}_{a}\sigma^{\mu\dot{c}n}\psi_{n\mu} + \partial^{\dot{c}}_{a}\sigma^{\mu\dot{b}n}\psi_{n\mu}\right) + b_6\left(\partial^{\dot{b}}_{a}\psi^{\dot{c}}_{0} + \partial^{\dot{c}}_{a}\psi^{\dot{b}}_{0}\right)\right] + \frac{M}{2}\left(\sigma^{\mu\dot{b}}_{a}\psi^{\dot{c}}_{\mu} + \sigma^{\mu\dot{c}}_{a}\psi^{\dot{b}}_{\mu}\right) = 0$$

Let us multiply this equation by $\sigma^{\lambda a}_{\dot{b}}$:

$$\frac{i}{\sqrt{6}}\left[b_5\left(\sigma^{\lambda a}_{\dot{b}}\partial^{\dot{b}}_{a}\sigma^{\mu\dot{c}n}\psi_{n\mu} + \sigma^{\lambda a}_{\dot{b}}\partial^{\dot{c}}_{a}\sigma^{\mu\dot{b}n}\psi_{n\mu}\right) + b_6\left(\sigma^{\lambda a}_{\dot{b}}\partial^{\dot{b}}_{a}\psi^{\dot{c}}_{0} + \sigma^{\lambda a}_{\dot{b}}\partial^{\dot{c}}_{a}\psi^{\dot{b}}_{0}\right)\right] + \frac{M}{2}\left(\sigma^{\lambda a}_{\dot{b}}\sigma^{\mu\dot{b}}_{a}\psi^{\dot{c}}_{\mu} + \sigma^{\lambda a}_{\dot{b}}\sigma^{\mu\dot{c}}_{a}\psi^{\dot{b}}_{\mu}\right) = 0\,,$$

then, by changing positions of spinor indexes, we obtain

$$\frac{i}{\sqrt{6}}\left\{b_5\left[-(\sigma^{\lambda\dot{b}a}\partial_{a\dot{b}})\sigma^{\mu\dot{c}n}\psi_{n\mu} - \partial_{\dot{c}a}\sigma^{\lambda}_{a\dot{b}}\sigma^{\mu\dot{b}n}\psi_{n\mu}\right] + b_6\left[-(\sigma^{\lambda\dot{b}a}\partial_{a\dot{b}})\psi^{\dot{c}}_{0} - \partial^{\dot{c}a}\sigma^{\lambda}_{a\dot{b}}\psi^{\dot{b}}_{0}\right]\right\} +$$

$$+\frac{M}{2}\left(-(\sigma^{\lambda\dot{b}a}\sigma^{\mu}_{a\dot{b}})\psi^{\dot{c}}_{\mu} - \sigma^{\mu\dot{c}a}\sigma^{\lambda}_{a\dot{b}}\psi^{\dot{b}}_{\mu}\right) = 0\,,$$

and further we get the equation

$$\frac{i}{\sqrt{6}}\left\{b_5\left[\frac{2}{i}\partial_{\lambda}\sigma^{\mu\dot{c}n}\psi_{n\mu} - \partial_{\dot{c}a}\sigma_{\lambda a\dot{b}}\sigma^{\mu\dot{b}n}\psi_{n\mu}\right] + b_6\left[\frac{2}{i}\partial_{\lambda}\psi^{\dot{c}}_{0} - \partial^{\dot{c}a}\sigma_{\lambda a\dot{b}}\psi^{\dot{b}}_{0}\right]\right\} + \frac{M}{2}\left(2\psi^{\dot{c}}_{\lambda} - \sigma^{\mu\dot{c}a}\sigma_{\lambda a\dot{b}}\psi^{\dot{b}}_{\mu}\right) = 0\,. \qquad \text{(IV.12)}$$

In similar manner, from eq. (IV.5) it follows (note that position of vector index, up or down, does not matter)

$$-\frac{i}{\sqrt{6}}\left\{b_5\left[\frac{2}{i}\partial_{\lambda}\sigma^{\mu}_{c\dot{n}}\psi^{\dot{n}}_{\mu} - \partial_{c\dot{a}}\sigma^{\lambda\dot{a}b}\sigma^{\mu}_{b\dot{n}}\psi^{\dot{n}}_{\mu}\right] + b_6\left[\frac{2}{i}\partial_{\lambda}\psi_{c0} - \partial_{c\dot{a}}\sigma^{\lambda\dot{a}b}\psi_{b0}\right]\right\} + \frac{M}{2}\left(2\psi_{\lambda c} - \sigma^{\mu}_{c\dot{a}}\sigma^{\lambda\dot{a}b}\psi_{\mu b}\right) = 0\,. \qquad \text{(IV.13)}$$

V. SPIN-TENSOR EQUATIONS

Let us recall spinor notations for elements of Dirac matrices:

$$\gamma_\mu = \frac{1}{i}\begin{vmatrix} 0 & \sigma^{\mu\dot{a}b} \\ \sigma^\mu_{a\dot{b}} & 0 \end{vmatrix}, \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{vmatrix} I & 0 \\ 0 & -I \end{vmatrix}, \sigma^1_{\dot{a}b} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \sigma^2_{\dot{a}b} = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}, \sigma^3_{\dot{a}b} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}, \sigma^4_{\dot{a}b} = \begin{vmatrix} i & 0 \\ 0 & i \end{vmatrix}.$$

Equations (IV.8) and (IV.9)

$$i\left[b_1\partial^{\dot{a}b}\sigma^\mu_{b\dot{c}}\psi^{\dot{c}}_\mu + \sqrt{\frac{1}{6}}b_2\left(\frac{2}{i}\partial_\mu\psi^{\dot{a}}_\mu - \sigma^{\mu\dot{a}c}\partial_{c\dot{b}}\psi^{\dot{b}}_\mu\right)\right] + m\sigma^{\mu\dot{a}b}\psi_{\mu b} = 0\,,$$

$$-i\left[b_1\partial_{a\dot{b}}\sigma^{\mu\dot{b}c}\psi_{\mu c} + \sqrt{\frac{1}{6}}b_2\left(\frac{2}{i}\partial_\mu\psi_{\mu a} - \sigma^\mu_{a\dot{c}}\partial^{\dot{c}b}\psi_{\mu b}\right)\right] + m\sigma^\mu_{a\dot{b}}\psi^{\dot{b}}_\mu = 0\,,$$

may be joint into one:

$$i\begin{vmatrix} I & 0 \\ 0 & -I \end{vmatrix}\left\{b_1\begin{vmatrix} \partial^{\dot{a}b}\sigma^\mu_{b\dot{c}} & 0 \\ 0 & \partial_{a\dot{b}}\sigma^{\mu\dot{b}c} \end{vmatrix}\begin{vmatrix} \psi^{\dot{c}}_\mu \\ \psi_{c\mu} \end{vmatrix} + \right.$$

$$\left. + \frac{b_2}{\sqrt{6}}\left[\frac{2}{i}\partial_\mu\begin{vmatrix} \psi^{\dot{a}}_\mu \\ \psi_{a\mu} \end{vmatrix} - \begin{vmatrix} \partial^{\dot{a}b}\sigma^\mu_{b\dot{c}} & 0 \\ 0 & \partial_{a\dot{b}}\sigma^{\mu\dot{b}c} \end{vmatrix}\begin{vmatrix} \psi^{\dot{c}}_\mu \\ \psi_{c\mu} \end{vmatrix}\right]\right\} + M\begin{vmatrix} 0 & \sigma^{\mu\dot{a}b} \\ \sigma^\mu_{a\dot{b}} & 0 \end{vmatrix}\begin{vmatrix} \psi^{\dot{b}}_\mu \\ \psi_{b\mu} \end{vmatrix} = 0\,,$$

that is

$$\gamma_5\left\{ib_1\hat{\partial}\gamma_\mu\Psi_\mu + \frac{b_2}{\sqrt{6}}\left[-2i\partial_\mu\Psi_\mu - i\gamma_\mu\hat{\partial}\Psi_\mu\right]\right\} + M\gamma_\mu\Psi_\mu = 0\,.$$

taking into account the identity

$$-i\gamma_\mu\hat{\partial}\Psi_\mu = -i\gamma_\mu\gamma_\nu\partial_\nu\Psi_\mu = -i(2\delta_{\mu\nu} - \gamma_\nu\gamma_\mu)\partial_\nu\Psi_\mu = -2i\partial_\mu\Psi_\mu + i\hat{\partial}(\gamma_\mu\Psi_\mu)$$

we can re-write the previous equation as follows

$$\gamma_5\left\{ib_1\hat{\partial}\gamma_\mu\Psi_\mu - \frac{4ib_2}{\sqrt{6}}\left[(\partial_\mu\Psi_\mu) - \frac{1}{4}\hat{\partial}(\gamma_\mu\Psi_\mu)\right]\right\} + M(\gamma_\mu\Psi_\mu) = 0\,. \tag{V.1}$$

In similar manner, equations (IV.10) and (IV.11)

$$i\left[b_3\partial^{\dot{a}b}\psi_{b0} + \sqrt{\frac{1}{6}}b_4\left(\frac{2}{i}\partial_\mu\psi^{\dot{a}}_\mu - \sigma^{\mu\dot{a}c}\partial_{c\dot{b}}\psi^{\dot{b}}_\mu\right)\right] + M\psi^{\dot{a}}_0 = 0\,,$$

$$-i\left[b_3\partial_{a\dot{b}}\psi^{\dot{b}}_0 + \sqrt{\frac{1}{6}}b_4\left(\frac{2}{i}\partial_\mu\psi_{a\mu} - \sigma^\mu_{a\dot{c}}\partial^{cb}\psi_{\mu\dot{b}}\right)\right] + M\psi_{a0} = 0$$

may be joint into the following one

$$\gamma_5\left\{ib_3\hat{\partial}\Psi_0 - \frac{4ib_4}{\sqrt{6}}\left[(\partial_\mu\Psi_\mu) - \frac{1}{4}\hat{\partial}(\gamma_\mu\Psi_\mu)\right]\right\} + M\Psi_0 = 0\,. \tag{V.2}$$

Equations (IV.12) and (IV.13)

$$\frac{i}{\sqrt{6}}\left\{b_5\left[\frac{2}{i}\partial_\lambda\sigma^{\mu\dot{c}n}\psi_{n\mu} - \partial_{\dot{c}a}\sigma^\lambda_{a\dot{b}}\sigma^{\mu\dot{b}n}\psi_{n\mu}\right] + b_6\left[\frac{2}{i}\partial_\lambda\psi^{\dot{c}}_0 - \partial^{\dot{c}a}\sigma^\lambda_{a\dot{b}}\psi^{\dot{b}}_0\right]\right\} + \frac{M}{2}\left(2\psi^{\dot{c}}_\lambda - \sigma^{\mu\dot{c}a}\sigma^\lambda_{a\dot{b}}\psi^{\dot{b}}_\mu\right) = 0\,.$$

$$-\frac{i}{\sqrt{6}}\left\{b_5\left[\frac{2}{i}\partial_\lambda\sigma^\mu_{c\dot{n}}\psi^{\dot{n}}_\mu - \partial_{c\dot{a}}\sigma^{\lambda\dot{a}b}\sigma^\mu_{b\dot{n}}\psi^{\dot{n}}_\mu\right] + b_6\left[\frac{2}{i}\partial_\lambda\psi_{c0} - \partial_{c\dot{a}}\sigma^{\lambda\dot{a}b}\psi_{b0}\right]\right\} + \frac{M}{2}\left(2\psi_{\lambda c} - \sigma^\mu_{c\dot{a}}\sigma^{\lambda\dot{a}b}\psi_{\mu b}\right) = 0$$

are joint in the one as follows

$$\frac{i}{\sqrt{6}}\begin{vmatrix} I & 0 \\ 0 & -I \end{vmatrix} \left\{ b_5 \left[\frac{2}{i}\partial_\lambda \begin{vmatrix} 0 & \sigma^{\mu\dot{c}n} \\ \sigma^{\mu}_{c\dot{n}} & 0 \end{vmatrix} \begin{vmatrix} \psi^{\dot{n}}_{\mu} \\ \psi_{n\mu} \end{vmatrix} - \begin{vmatrix} 0 & \partial^{\dot{c}a}\sigma^{\lambda}_{a\dot{b}}\sigma^{\mu\dot{b}n} \\ \partial_{c\dot{a}}\sigma^{\lambda\dot{a}b}\sigma^{\mu}_{b\dot{n}} & 0 \end{vmatrix} \begin{vmatrix} \psi^{\dot{n}}_{\mu} \\ \psi_{n\mu} \end{vmatrix} \right] + \right.$$

$$\left. + b_6 \left[\frac{2}{i}\partial_\lambda \begin{vmatrix} \psi^{\dot{c}}_{0} \\ \psi_{c0} \end{vmatrix} - \begin{vmatrix} \partial^{\dot{c}a}\sigma^{\lambda}_{a\dot{b}} & 0 \\ 0 & \partial_{c\dot{a}}\sigma^{\lambda\dot{a}b} \end{vmatrix} \begin{vmatrix} \psi^{\dot{b}}_{0} \\ \psi_{b0} \end{vmatrix} \right] \right\} + \frac{M}{2}\left[2 \begin{vmatrix} \psi^{\dot{c}}_{\lambda} \\ \psi_{c\lambda} \end{vmatrix} - \begin{vmatrix} 0 & \sigma^{\mu\dot{c}a}\sigma^{\lambda}_{a\dot{b}} \\ \sigma^{\mu}_{c\dot{a}}\sigma^{\lambda\dot{a}b} & 0 \end{vmatrix} \begin{vmatrix} \psi^{\dot{b}}_{\mu} \\ \psi_{b\mu} \end{vmatrix} \right] = 0\,,$$

that is

$$\frac{i}{\sqrt{6}}\gamma_5 \left\{ b_5 \left[2\partial_\lambda(\gamma_\mu\Psi_\mu) + \hat{\partial}\gamma_\lambda(\gamma_\mu\Psi_\mu) \right] + b_6 \left[\frac{2}{i}\partial_\lambda\Psi_0 - i\hat{\partial}\gamma_\lambda\Psi_0 \right] \right\} + \frac{M}{2}\left[2\Psi_\lambda + \gamma_\mu\gamma_\lambda\Psi_\mu \right] = 0\,.$$

After simple transformation it reads

$$\frac{2i}{\sqrt{6}}\gamma_5 \left\{ b_5 \left[\partial_\lambda(\gamma_\mu\Psi_\mu) - \frac{1}{4}\gamma_\lambda\hat{\partial}(\gamma_\mu\Psi_\mu) \right] - ib_6 \left[\partial_\lambda\Psi_0 - \frac{1}{4}\gamma_\lambda\hat{\partial}\Psi_0 \right] \right\} + M\left[\Psi_\lambda - \frac{1}{4}\gamma_\lambda(\gamma_\mu\Psi_\mu) \right] = 0\,. \tag{V.3}$$

VI. FREE PARTICLE, THREE MASS STATES

Let us re-write three spin-vector equations differently (multiplying them by $-i\gamma_5$):

$$b_1\hat{\partial}(\gamma_\mu\Psi_\mu) - \frac{4b_2}{\sqrt{6}}\left[(\partial_\mu\Psi_\mu) - \frac{1}{4}\hat{\partial}(\gamma_\mu\Psi_\mu) \right] - i\gamma_5 M(\gamma_\mu\Psi_\mu) = 0\,, \tag{VI.1}$$

$$\left\{ b_3\hat{\partial}\Psi_0 - i\frac{4b_4}{\sqrt{6}}\left[(\partial_\mu\Psi_\mu) - \frac{1}{4}\hat{\partial}(\gamma_\mu\Psi_\mu) \right] \right\} - i\gamma_5 M\Psi_0 = 0\,, \tag{VI.2}$$

$$\frac{2}{\sqrt{6}}\left\{ b_5 \left[\partial_\lambda(\gamma_\mu\Psi_\mu) - \frac{1}{4}\gamma_\lambda\hat{\partial}(\gamma_\mu\Psi_\mu) \right] - \right.$$
$$\left. -ib_6 \left[\partial_\lambda\Psi_0 - \frac{1}{4}\gamma_\lambda\hat{\partial}\Psi_0) \right] \right\} - i\gamma_5 M\left[\Psi_\lambda - \frac{1}{4}\gamma_\lambda(\gamma_\mu\Psi_\mu) \right] = 0. \tag{VI.3}$$

Let us multiply (VI.1) by $(-\frac{1}{4}\gamma_\lambda)$,

$$-b_1\frac{1}{4}\gamma_\lambda\hat{\partial}(\gamma_\mu\Psi_\mu) + \frac{1}{4}\gamma_\lambda\frac{4b_2}{\sqrt{6}}\left[(\partial_\mu\Psi_\mu) - \frac{1}{4}\hat{\partial}(\gamma_\mu\Psi_\mu) \right] + i\gamma_\lambda\gamma_5\frac{1}{4}M(\gamma_\mu\Psi_\mu) = 0\,, \tag{VI.4}$$

then sum eqs. (VI.3) and (VI.4):

$$\frac{2}{\sqrt{6}}\left\{ b_5 \left[\partial_\lambda(\gamma_\mu\Psi_\mu) - \frac{1}{4}\gamma_\lambda\hat{\partial}(\gamma_\mu\Psi_\mu) \right] - ib_6 \left[\partial_\lambda\Psi_0 - \frac{1}{4}\gamma_\lambda\hat{\partial}\Psi_0) \right] \right\} - i\gamma_5 M\left[\Psi_\lambda - \frac{1}{4}\gamma_\lambda(\gamma_\mu\Psi_\mu) \right] -$$
$$-b_1\frac{1}{4}\gamma_\lambda\hat{\partial}(\gamma_\mu\Psi_\mu) + \frac{1}{4}\gamma_\lambda\frac{4b_2}{\sqrt{6}}\left[(\partial_\mu\Psi_\mu) - \frac{1}{4}\hat{\partial}(\gamma_\mu\Psi_\mu) \right] - i\gamma_5\gamma_\lambda\frac{1}{4}M(\gamma_\mu\Psi_\mu) = 0\,,$$

or

$$\frac{2b_5}{\sqrt{6}}\partial_\lambda(\gamma_\mu\Psi_\mu) - \frac{1}{4}\frac{2b_5}{\sqrt{6}}\gamma_\lambda\hat{\partial}(\gamma_\mu\Psi_\mu) - i\frac{2b_6}{\sqrt{6}}\left[\partial_\lambda\Psi_0 - \frac{1}{4}\gamma_\lambda\hat{\partial}\Psi_0) \right] -$$
$$-b_1\frac{1}{4}\gamma_\lambda\hat{\partial}(\gamma_\mu\Psi_\mu) + \frac{b_2}{\sqrt{6}}\gamma_\lambda(\partial_\mu\Psi_\mu) - \frac{b_2}{\sqrt{6}}\frac{1}{4}\gamma_\lambda\hat{\partial}(\gamma_\mu\Psi_\mu) - i\gamma_5 M\Psi_\lambda. = 0$$

Whence we derive

$$-i\frac{2b_6}{\sqrt{6}}\left[\partial_\lambda\Psi_0 - \frac{1}{4}\gamma_\lambda\hat{\partial}\Psi_0) \right] + \frac{2b_5}{\sqrt{6}}\partial_\lambda(\gamma_\mu\Psi_\mu) +$$
$$+\frac{1}{4}\left[-b_1 - \frac{b_2}{\sqrt{6}} - \frac{2b_5}{\sqrt{6}} \right]\gamma_\lambda\hat{\partial}(\gamma_\mu\Psi_\mu) + \frac{b_2}{\sqrt{6}}\gamma_\lambda(\partial_\mu\Psi_\mu) - i\gamma_5 M\Psi_\lambda = 0\,. \tag{VI.5}$$

Now, let us act on eq. (VI.5) by operator ∂_λ, this resulys in (let $\Box = \partial_\lambda \partial_\lambda$):

$$-i\frac{2b_6}{\sqrt{6}}\frac{3}{4}\Box\Psi_0 + \frac{2b_5}{\sqrt{6}}\Box(\gamma_\mu\Psi_\mu) + \frac{1}{4}\left[-b_1 - \frac{b_2}{\sqrt{6}} - \frac{2b_5}{\sqrt{6}}\right]\Box(\gamma_\mu\Psi_\mu) + \frac{b_2}{\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu) - i\gamma_5 M(\partial_\mu\Psi_\mu) = 0\,,$$

which leads to

$$-i\frac{\sqrt{6}b_6}{4}\Box\Psi_0 - \frac{1}{4}\left[(b_1 + \frac{b_2}{\sqrt{6}}) - \sqrt{6}b_5\right]\Box(\gamma_\mu\Psi_\mu) + \frac{b_2}{\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu) - i\gamma_5 M(\partial_\mu\Psi_\mu) = 0\,. \tag{VI.6}$$

Now, let us act on eq. (VI.1) by operator $\hat{\partial} = \gamma_\mu\partial_\mu$:

$$b_1\Box(\gamma_\mu\Psi_\mu) - \frac{4b_2}{\sqrt{6}}\left[\hat{\partial}(\partial_\mu\Psi_\mu) - \frac{1}{4}\Box(\gamma_\mu\Psi_\mu)\right] + i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu) = 0\,,$$

this leads to

$$\frac{1}{4}(b_1 + \frac{b_2}{\sqrt{6}})\Box(\gamma_\mu\Psi_\mu) = \frac{b_2}{\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu) - i\frac{1}{4}\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu)\,. \tag{VI.7}$$

Allowing for eq. (VI.7) in (VI.6), we obtain

$$-i\frac{\sqrt{6}b_6}{4}\Box\Psi_0 - \frac{b_2}{\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu) + i\frac{1}{4}\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu) + \frac{\sqrt{6}}{4}b_5\Box(\gamma_\mu\Psi_\mu) + \frac{b_2}{\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu) - i\gamma_5 M(\partial_\mu\Psi_\mu) = 0\,,$$

or

$$-i\frac{\sqrt{6}b_6}{4}\Box\Psi_0 + \frac{\sqrt{6}}{4}b_5\Box(\gamma_\mu\Psi_\mu) - i\gamma_5 M(\partial_\mu\Psi_\mu) + \frac{i}{4}\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu) = 0\,. \tag{VI.8}$$

In this equation, one can transform the term $\Box(\gamma_\mu\Psi_\mu)$ with the help of relation (VI.7):

$$-i\frac{\sqrt{6}b_6}{4}\Box\Psi_0 + \frac{\sqrt{6}}{4}b_5\frac{1}{b_1 + b_2/\sqrt{6}}\left[\frac{4b_2}{\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu) - i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu)\right] - i\gamma_5 M(\partial_\mu\Psi_\mu) + \frac{i}{4}\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu) = 0$$

or

$$-i\frac{\sqrt{6}b_6}{4}\Box\Psi_0 + \frac{b_2 b_5}{b_1 + b_2/\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu) - \frac{\sqrt{6}}{4}\frac{b_5}{b_1 + b_2/\sqrt{6}}i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu) - i\gamma_5 M(\partial_\mu\Psi_\mu) + \frac{i}{4}\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu) = 0 \tag{VI.9}$$

The next idea is to exclude the term with 2-nd order operator $\Box$. To this end, led us act on eq. (VI.2) by operator $\hat{\partial}$ (taking in mind the identity $\hat{\partial}\hat{\partial} = \Box$):

$$b_3\Box\Psi_0 - i\frac{4}{\sqrt{6}}b_4\hat{\partial}(\partial_\mu\Psi_\mu) + i\frac{1}{\sqrt{6}}b_4\Box(\gamma_\mu\Psi_\mu) + i\gamma_5 M\hat{\partial}\Psi_0 = 0\,;$$

in this relation we van exclude the term $\Box(\gamma_\mu\Psi_\mu)$ with the help of (VI.7):

$$b_3\Box\Psi_0 - i\frac{4}{\sqrt{6}}b_4\hat{\partial}(\partial_\mu\Psi_\mu) + i\frac{1}{\sqrt{6}}b_4\frac{1}{b_1 + b_2/\sqrt{6}}\left[\frac{4b_2}{\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu) - i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu)\right] + i\gamma_5 M\hat{\partial}\Psi_0 = 0\,;$$

or

$$b_3\Box\Psi_0 - i\frac{4}{\sqrt{6}}b_4\hat{\partial}(\partial_\mu\Psi_\mu) + i\frac{2}{3}\frac{b_2 b_4}{b_1 + b_2/\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu) - i\frac{1}{\sqrt{6}}\frac{b_4}{b_1 + b_2/\sqrt{6}}i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu) + i\gamma_5 M\hat{\partial}\Psi_0 = 0\,;$$

Whence one can express $\Box\Psi_0$:

$$\Box\Psi_0 = \frac{4i}{\sqrt{6}}\frac{b_4}{b_3}\hat{\partial}(\partial_\mu\Psi_\mu) - \frac{2i}{3}\frac{b_2 b_4}{b_3(b_1 + b_2/\sqrt{6})}\hat{\partial}(\partial_\mu\Psi_\mu) + \frac{i}{\sqrt{6}}\frac{b_4}{b_3(b_1 + b_2/\sqrt{6})}i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu) - \frac{1}{b_3}iM\gamma_5\hat{\partial}\Psi_0\,; \tag{VI.10}$$

now let us take into account (VI.10) in eq. (VI.9):

$$-i\frac{\sqrt{6}b_6}{4}\left\{\frac{4i}{\sqrt{6}}\frac{b_4}{b_3}\hat{\partial}(\partial_\mu\Psi_\mu) - \frac{2i}{3}\frac{b_2 b_4}{b_3(b_1 + b_2/\sqrt{6})}\hat{\partial}(\partial_\mu\Psi_\mu) + \frac{i}{\sqrt{6}}\frac{b_4}{b_3(b_1 + b_2/\sqrt{6})}i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu) - \frac{1}{b_3}iM\gamma_5\hat{\partial}\Psi_0\right\} +$$

$$+\frac{b_2 b_5}{b_1+b_2/\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu)-\frac{\sqrt{6}}{4}\frac{b_5}{b_1+b_2/\sqrt{6}}i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu)-i\gamma_5 M(\partial_\mu\Psi_\mu)+\frac{i}{4}\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu)=0\,,$$

or

$$\frac{b_4 b_6}{b_3}\hat{\partial}(\partial_\mu\Psi_\mu)-\frac{1}{\sqrt{6}}\frac{b_2 b_4 b_6}{b_3(b_1+b_2/\sqrt{6})}\hat{\partial}(\partial_\mu\Psi_\mu)+\frac{1}{4}\frac{b_4 b_6}{b_3(b_1+b_2/\sqrt{6})}i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu)+i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}iM\gamma_5\hat{\partial}\Psi_0+$$

$$+\frac{b_2 b_5}{b_1+b_2/\sqrt{6}}\hat{\partial}(\partial_\mu\Psi_\mu)-\frac{\sqrt{6}}{4}\frac{b_5}{b_1+b_2/\sqrt{6}}i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu)-i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}iM\gamma_5\hat{\partial}\Psi_0-iM\gamma_5(\partial_\mu\Psi_\mu)=0\,.$$

Whence after regrouping the terms we derive

$$\left[\frac{b_4 b_6}{b_3}-\frac{1}{\sqrt{6}}\frac{b_2 b_4 b_6}{b_3(b_1+b_2/\sqrt{6})}+\frac{b_2 b_5}{b_1+b_2/\sqrt{6}}\right]\hat{\partial}(\partial_\mu\Psi_\mu)+$$

$$+\frac{1}{4}\left[\frac{b_4 b_6}{b_3(b_1+b_2/\sqrt{6})}-\sqrt{6}\frac{b_5}{b_1+b_2/\sqrt{6}}+1\right]i\gamma_5 M\hat{\partial}(\gamma_\mu\Psi_\mu)+i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}iM\gamma_5\hat{\partial}\Psi_0-iM\gamma_5(\partial_\mu\Psi_\mu)=0\,.$$

and further arrive at

$$\frac{b_1 b_4 b_6+b_2 b_3 b_5}{b_3(b_1+b_2/\sqrt{6})}\;\hat{\partial}(\partial_\mu\Psi_\mu)+$$

$$+\frac{1}{4}\frac{b_4 b_6-\sqrt{6}b_3 b_5+b_3(b_1+b_2/\sqrt{6})}{b_3(b_1+b_2/\sqrt{6})}\;iM\gamma_5\hat{\partial}(\gamma_\mu\Psi_\mu)+i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}iM\gamma_5\hat{\partial}\Psi_0-iM\gamma_5(\partial_\mu\Psi_\mu)=0\,. \qquad \text{(VI.11)}$$

Let us express from this equation the term $iM\gamma_5(\partial_\mu\Psi_\mu)$:

$$iM\gamma_5(\partial_\mu\Psi_\mu)=\frac{b_1 b_4 b_6+b_2 b_3 b_5}{b_3(b_1+b_2/\sqrt{6})}\;\hat{\partial}(\partial_\mu\Psi_\mu)+$$

$$+\frac{1}{4}\frac{b_4 b_6-\sqrt{6}b_3 b_5+b_3(b_1+b_2/\sqrt{6})}{b_3(b_1+b_2/\sqrt{6})}\;iM\gamma_5\hat{\partial}(\gamma_\mu\Psi_\mu)+i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}iM\gamma_5\hat{\partial}\Psi_0\,,$$

which after multiplying by $-i\gamma_5/M$ gives

$$(\partial_\mu\Psi_\mu)=-\frac{1}{M}\frac{b_1 b_4 b_6+b_2 b_3 b_5}{b_3(b_1+b_2/\sqrt{6})}\;i\gamma_5\hat{\partial}(\partial_\mu\Psi_\mu)+\frac{1}{4}\frac{b_4 b_6-\sqrt{6}b_3 b_5+b_3(b_1+b_2/\sqrt{6})}{b_3(b_1+b_2/\sqrt{6})}\;\hat{\partial}(\gamma_\mu\Psi_\mu)+i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}\hat{\partial}\Psi_0. \qquad \text{(VI.12)}$$

Now, let us substitute (VI.12) into (VI.1):

$$b_1\hat{\partial}(\gamma_\mu\Psi_\mu)-\frac{4b_2}{\sqrt{6}}\left\{-\frac{1}{M}\frac{b_1 b_4 b_6+b_2 b_3 b_5}{b_3(b_1+b_2/\sqrt{6})}\;i\gamma_5\hat{\partial}(\partial_\mu\Psi_\mu)+\right.$$

$$\left.+\frac{1}{4}\frac{b_4 b_6-\sqrt{6}b_3 b_5+b_3(b_1+b_2/\sqrt{6})}{b_3(b_1+b_2/\sqrt{6})}\;\hat{\partial}(\gamma_\mu\Psi_\mu)+i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}\hat{\partial}\Psi_0\right\}+\frac{b_2}{\sqrt{6}}\hat{\partial}(\gamma_\mu\Psi_\mu)-i\gamma_5 M(\gamma_\mu\Psi_\mu)=0\,,$$

which after re-gr0uping the terms takes the form

$$\left[b_1-\frac{b_2}{\sqrt{6}}\frac{b_4 b_6-\sqrt{6}b_3 b_5+b_3(b_1+b_2/\sqrt{6})}{b_3(b_1+b_2/\sqrt{6})}+\frac{b_2}{\sqrt{6}}\right]\hat{\partial}(\gamma_\mu\Psi_\mu)+$$

$$+\frac{1}{M}\frac{4b_2}{\sqrt{6}}\frac{b_1 b_4 b_6+b_2 b_3 b_5}{b_3(b_1+b_2/\sqrt{6})}\;i\gamma_5\hat{\partial}(\partial_\mu\Psi_\mu)-i\frac{b_2 b_6}{b_3}\hat{\partial}\Psi_0-iM\gamma_5(\gamma_\mu\Psi_\mu)=0\,,$$

or after some simplifying we get

$$\left[b_1 - \frac{b_2}{\sqrt{6}}\,\frac{b_4b_6 - \sqrt{6}b_3b_5}{b_3(b_1 + b_2/\sqrt{6})}\right]\hat{\partial}(\gamma_\mu\Psi_\mu) + \frac{1}{M}\,\frac{4b_2}{\sqrt{6}}\,\frac{b_1b_4b_6 + b_2b_3b_5}{b_3(b_1 + b_2/\sqrt{6})}\; i\gamma_5\hat{\partial}(\partial_\mu\Psi_\mu) - i\frac{b_2b_6}{b_3}\hat{\partial}\Psi_0 - iM\gamma_5(\gamma_\mu\Psi_\mu) = 0\,.$$

or

$$\left[\frac{\sqrt{6}b_1b_3(b_1 + b_2/\sqrt{6}) - b_2(b_4b_6 - \sqrt{6}b_3b_5)}{\sqrt{6}b_3(b_1 + b_2/\sqrt{6})}\right]\hat{\partial}(\gamma_\mu\Psi_\mu) +$$

$$+\frac{1}{M}\,\frac{4b_2(b_1b_4b_6 + b_2b_3b_5)}{\sqrt{6}b_3(b_1 + b_2/\sqrt{6})}\; i\gamma_5\hat{\partial}(\partial_\mu\Psi_\mu) - i\frac{b_2b_6}{b_3}\hat{\partial}\Psi_0 - iM\gamma_5(\gamma_\mu\Psi_\mu) = 0\,. \qquad \text{(VI.13)}$$

Finally, let us substitute (VI.12) into (VI.2):

$$b_3\hat{\partial}\Psi_0 - i\frac{4b_4}{\sqrt{6}}\left\{-\frac{1}{M}\,\frac{b_1b_4b_6 + b_2b_3b_5}{b_3(b_1 + b_2/\sqrt{6})}\; i\gamma_5\hat{\partial}(\partial_\mu\Psi_\mu) + \right.$$

$$\left. +\frac{1}{4}\,\frac{b_4b_6 - \sqrt{6}b_3b_5 + b_3(b_1 + b_2/\sqrt{6})}{b_3(b_1 + b_2/\sqrt{6})}\;\hat{\partial}(\gamma_\mu\Psi_\mu) + i\frac{\sqrt{6}}{4}\,\frac{b_6}{b_3}\hat{\partial}\Psi_0\right\} + i\frac{b_4}{\sqrt{6}}\hat{\partial}(\gamma_\mu\Psi_\mu) - i\gamma_5 M\Psi_0 = 0$$

or

$$\frac{b_3^2 + b_4b_6}{b_3}\hat{\partial}\Psi_0 + \frac{i}{M}\,\frac{4}{\sqrt{6}}\,\frac{b_4(b_1b_4b_6 + b_2b_3b_5)}{b_3(b_1 + b_2/\sqrt{6})}\; i\gamma_5\hat{\partial}(\partial_\mu\Psi_\mu) - i\,\frac{(b_4b_6 - \sqrt{6}b_3b_5)b_4}{\sqrt{6}b_3(b_1 + b_2/\sqrt{6})}\;\hat{\partial}(\gamma_\mu\Psi_\mu) - iM\gamma_5\Psi_0 = 0\,. \qquad \text{(VI.14)}$$

Thus, wa have derived three equations, (VI.13), (VI.14), (VI.11):

$$\left[\frac{\sqrt{6}b_1b_3(b_1 + b_2/\sqrt{6}) - b_2(b_4b_6 - \sqrt{6}b_3b_5)}{\sqrt{6}b_3(b_1 + b_2/\sqrt{6})}\right] i\gamma_5\hat{\partial}(\gamma_\mu\Psi_\mu) -$$

$$-\frac{1}{M}\,\frac{4b_2(b_1b_4b_6 + b_2b_3b_5)}{\sqrt{6}b_3(b_1 + b_2/\sqrt{6})}\hat{\partial}(\partial_\mu\Psi_\mu) - i\frac{b_2b_6}{b_3}i\gamma_5\hat{\partial}\Psi_0 + M(\gamma_\mu\Psi_\mu) = 0\,,$$

$$\frac{b_3^2 + b_4b_6}{b_3}i\gamma_5\hat{\partial}\Psi_0 - \frac{i}{M}\,\frac{4}{\sqrt{6}}\,\frac{b_4(b_1b_4b_6 + b_2b_3b_5)}{b_3(b_1 + b_2/\sqrt{6})}\hat{\partial}(\partial_\mu\Psi_\mu) - i\frac{(b_4b_6 - \sqrt{6}b_3b_5)b_4}{\sqrt{6}b_3(b_1 + b_2/\sqrt{6})}\; i\gamma_5\hat{\partial}(\gamma_\mu\Psi_\mu) + M\Psi_0 = 0\,,$$

$$\frac{b_1b_4b_6 + b_2b_3b_5}{b_3(b_1 + b_2/\sqrt{6})}\; i\gamma_5\hat{\partial}(\partial_\mu\Psi_\mu) - \frac{M}{4}\,\frac{b_4b_6 - \sqrt{6}b_3b_5 + b_3(b_1 + b_2/\sqrt{6})}{b_3(b_1 + b_2/\sqrt{6})}\;\hat{\partial}(\gamma_\mu\Psi_\mu) - i\frac{\sqrt{6}}{4}\,\frac{b_6}{b_3}M\hat{\partial}\Psi_0 + M(\partial_\mu\Psi_\mu) = 0\,.$$

Let us introduce 12-component function

$$\Psi = \begin{vmatrix} \gamma_\mu\Psi_\mu \\ \Psi_0 \\ \gamma_5\partial_\mu\Psi_\mu \end{vmatrix}\,. \qquad \text{(VI.15)}$$

Correspondingly, the previous system is re-written as follows:

$$\frac{\sqrt{6}b_1b_3(b_1 + b_2/\sqrt{6}) - b_2(b_4b_6 - \sqrt{6}b_3b_5)}{\sqrt{6}b_3(b_1 + b_2/\sqrt{6})}\cdot i\gamma_5\hat{\partial}(\gamma_\mu\Psi_\mu) -$$

$$-i\frac{b_2b_6}{b_3}\cdot i\gamma_5\hat{\partial}\Psi_0 - i\frac{1}{M}\,\frac{4b_2(b_1b_4b_6 + b_2b_3b_5)}{\sqrt{6}b_3(b_1 + b_2/\sqrt{6})}\cdot i\gamma_5\hat{\partial}(\gamma_5\partial_\mu\Psi_\mu) + M(\gamma_\mu\Psi_\mu) = 0\,, \qquad \text{(VI.16)}$$

$$-i\,\frac{(b_4b_6 - \sqrt{6}b_3b_5)b_4}{\sqrt{6}b_3(b_1 + b_2/\sqrt{6})}\cdot i\gamma_5\hat{\partial}(\gamma_\mu\Psi_\mu) + \frac{b_3^2 + b_4b_6}{b_3}\cdot i\gamma_5\hat{\partial}\Psi_0 +$$

$$\frac{1}{M}\,\frac{4}{\sqrt{6}}\,\frac{b_4(b_1b_4b_6 + b_2b_3b_5)}{b_3(b_1 + b_2/\sqrt{6})}\cdot i\gamma_5\hat{\partial}(\gamma_5\partial_\mu\Psi_\mu) + +M\Psi_0 = 0\,, \qquad \text{(VI.17)}$$

$$i\frac{M}{4}\frac{b_4b_6-\sqrt{6}b_3b_5+b_3(b_1+b_2/\sqrt{6})}{b_3(b_1+b_2/\sqrt{6})}\cdot i\gamma_5\hat{\partial}(\gamma_\mu\Psi_\mu)-\frac{\sqrt{6}}{4}\frac{b_6}{b_3}M\cdot i\gamma_5\hat{\partial}\Psi_0-$$
$$-\frac{b_1b_4b_6+b_2b_3b_5}{b_3(b_1+b_2/\sqrt{6})}\cdot i\gamma_5\hat{\partial}(\gamma_5\partial_\mu\Psi_\mu)+M(\gamma_5\partial_\mu\Psi_\mu)=0\,. \qquad \text{(VI.18)}$$

It may be presented in the matrix form:

$$(K\ i\gamma_5\hat{\partial}+M)\Psi=0,\quad K=\begin{vmatrix} A_1 & B_1 & R_1\\ A_2 & B_2 & R_2\\ A_3 & B_3 & R_3\end{vmatrix},\quad \Psi=\begin{vmatrix}\gamma_\mu\Psi_\mu\\ \Psi_0\\ \gamma_5\partial_\mu\Psi_\mu\end{vmatrix}, \qquad \text{(VI.19)}$$

where we use notations

$$A_1=\frac{\sqrt{6}b_1b_3(b_1+b_2/\sqrt{6})-b_2(b_4b_6-\sqrt{6}b_3b_5)}{\sqrt{6}b_3(b_1+b_2/\sqrt{6})},\quad B_1=-i\frac{b_2b_6}{b_3},\quad R_1=-\frac{i}{M}\frac{4}{\sqrt{6}}\frac{b_2(b_1b_4b_6+b_2b_3b_5)}{b_3(b_1+b_2/\sqrt{6})},$$
$$A_2=-\frac{i}{\sqrt{6}}\frac{(b_4b_6-\sqrt{6}b_3b_5)b_4}{b_3(b_1+b_2/\sqrt{6})},\quad B_2=\frac{b_3^2+b_4b_6}{b_3},\quad R_2=\frac{1}{M}\frac{4}{\sqrt{6}}\frac{b_4(b_1b_4b_6+b_2b_3b_5)}{b_3(b_1+b_2/\sqrt{6})}, \qquad \text{(VI.20)}$$
$$A_3=\frac{iM}{4}\frac{b_4b_6-\sqrt{6}b_3b_5+b_3(b_1+b_2/\sqrt{6})}{b_3(b_1+b_2/\sqrt{6})},\quad B_3=-\frac{\sqrt{6}}{4}\frac{b_6}{b_3}M,\quad R_3=-\frac{b_1b_4b_6+b_2b_3b_5}{b_3(b_1+b_2/\sqrt{6})}.$$

The matrix K explicitly reads

$$K=\begin{vmatrix} \frac{\sqrt{6}b_1b_3(b_1+b_2/\sqrt{6})-b_2(b_4b_6-\sqrt{6}b_3b_5)}{\sqrt{6}b_3(b_1+b_2/\sqrt{6})} & -i\frac{b_2b_6}{b_3} & -\frac{i}{M}\frac{4}{\sqrt{6}}\frac{b_2(b_1b_4b_6+b_2b_3b_5)}{b_3(b_1+b_2/\sqrt{6})}\\ -\frac{i}{\sqrt{6}}\frac{(b_4b_6-\sqrt{6}b_3b_5)b_4}{b_3(b_1+b_2/\sqrt{6})} & \frac{b_3^2+b_4b_6}{b_3} & \frac{1}{M}\frac{4}{\sqrt{6}}\frac{b_4(b_1b_4b_6+b_2b_3b_5)}{b_3(b_1+b_2/\sqrt{6})}\\ \frac{iM}{4}\frac{b_4b_6-\sqrt{6}b_3b_5+b_3(b_1+b_2/\sqrt{6})}{b_3(b_1+b_2/\sqrt{6})} & -\frac{\sqrt{6}}{4}\frac{b_6}{b_3}M & -\frac{b_1b_4b_6+b_2b_3b_5}{b_3(b_1+b_2/\sqrt{6})}\end{vmatrix}.$$

Allowing for restrictions (II.14)

$$b_1=b_1^*,\ b_3=b_3^*,\ b_5=fb_2^*,\ b_6=gb_4^*,\quad f=\pm1,\ g=\pm1 \qquad \text{(VI.21)}$$

(recall that b_2,b_4 may be complex or real), we reduce the matrix K to the form

$$K=\begin{vmatrix} \frac{b_1b_3(b_1+b_2/\sqrt{6})-\frac{1}{\sqrt{6}}gb_2|b_4|^2+fb_3|b_2|^2}{b_3(b_1+b_2/\sqrt{6})} & -ig\frac{b_2b_4^*}{b_3} & -\frac{i}{M}\frac{4}{\sqrt{6}}\frac{b_2(gb_1|b_4|^2+fb_3|b_2|^2)}{b_3(b_1+b_2/\sqrt{6})}\\ -\frac{i}{\sqrt{6}}\frac{b_4(g|b_4|^2-\sqrt{6}fb_2^*b_3)}{b_3(b_1+b_2/\sqrt{6})} & \frac{b_3^2+g|b_4|^2}{b_3} & \frac{1}{M}\frac{4}{\sqrt{6}}\frac{b_4(gb_1|b_4|^2+fb_3|b_2|^2)}{b_3(b_1+b_2/\sqrt{6})}\\ \frac{iM}{4}\frac{g|b_4|^2-\sqrt{6}fb_2^*b_3+b_3(b_1+b_2/\sqrt{6})}{b_3(b_1+b_2/\sqrt{6})} & -M\frac{\sqrt{6}}{4}\frac{gb_4^*}{b_3} & -\frac{gb_1|b_4|^2+fb_3|b_2|^2}{b_3(b_1+b_2/\sqrt{6})}\end{vmatrix}. \qquad \text{(VI.22)}$$

Next idea is to find linear transformation $\Psi'=S\Psi$ which would diagonalize the matrix K:

$$S(K\ i\gamma_5\hat{\partial}+M)S^{-1}\Psi'=0,\quad S=\begin{vmatrix} a_1 & a_2 & a_3\\ c_1 & c_2 & c_3\\ r_1 & r_2 & r_3\end{vmatrix},\quad SKS^{-1}=K_0=\begin{vmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{vmatrix}. \qquad \text{(VI.23)}$$

Characteristic equation for matrix K: $\det(K-\lambda I)=0$ after needed calculation reduces to 3-rd order equation

$$\lambda^3-\lambda^2(b_1+b_3)+\lambda\left(b_1b_3-g|b_4|^2-f|b_2|^2\right)+\left(gb_1|b_4|^2+fb_3|b_2|^2\right)=0\,; \qquad \text{(VI.24)}$$

and it coincides with that fpr spin block

$$\beta^{(1/2)}=\begin{vmatrix} b_1 & 0 & b_2\\ 0 & b_3 & b_4\\ fb_2^* & gb_4^* & 0\end{vmatrix}. \qquad \text{(VI.25)}$$

After diagonalyzing the matrix K according to (VI.23), the system reduces to three P-asymmetric separate equations with different masses:

$$\left(i\gamma_5\hat{\partial}+\frac{M}{\lambda_1}\right)\Phi_1=0\,,\quad \left(i\gamma_5\hat{\partial}+\frac{M}{\lambda_2}\right)\Phi_2=0\,,\quad \left(i\gamma_5\hat{\partial}+\frac{M}{\lambda_3}\right)\Phi_3=0\,, \tag{VI.26}$$

The transformation matrix S obeys the following equation

$$\left|\begin{array}{ccc} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ r_1 & r_2 & r_3 \end{array}\right| \left|\begin{array}{ccc} A_1 & B_1 & R_1 \\ A_2 & B_2 & R_2 \\ A_3 & B_3 & R_3 \end{array}\right| = \left|\begin{array}{ccc} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{array}\right| \left|\begin{array}{ccc} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ r_1 & r_2 & r_3 \end{array}\right|,$$

whence if follow three linear sub-systems

$$a_1A_1+a_2A_2+a_3A_3=\lambda_1 a_1\,, a_1B_1+a_2B_2+a_3B_3=\lambda_1 a_2\,, a_1R_1+a_2R_2+a_3R_3=\lambda_1 a_3\,; \tag{VI.27}$$

$$c_1A_1+c_2A_2+c_3A_3=\lambda_2 c_1\,, c_1B_1+c_2B_2+c_3B_3=\lambda_2 c_2, c_1R_1+c_2R_2+c_3R_3=\lambda_2 c_3\,; \tag{VI.28}$$

$$r_1A_1+r_2A_2+r_3A_3=\lambda_3 r_1\,, r_1B_1+r_2B_2+r_3B_3=\lambda_3 r_2, r_1R_1+r_2R_2+r_3R_3=\lambda_3 r_3\,. \tag{VI.29}$$

The have the similar structure, in each we may exclude one equation (let it be the first)

$$\begin{array}{c} a_2(B_2-\lambda_1)+a_3B_3=-a_1B_1\,,\quad a_2R_2+a_3(R_3-\lambda_1)=-a_1R_1\,; \\ c_2(B_2-\lambda_2)+c_3B_3=-c_1B_1\,,\quad c_2R_2+c_3(R_3-\lambda_2)=-c_1R_1\,; \\ r_2(B_2-\lambda_3)+r_3B_3=-r_1B_1\,,\quad r_2R_2+r_3(R_3-\lambda_3)=-r_1R_1\,. \end{array} \tag{VI.30}$$

Their solutions are given by the following formulas:

$$\begin{array}{c} a_2=-a_1\,\dfrac{ig\lambda_1 b_2 b_4^*(b_1+b_2/\sqrt{6})}{\lambda_1(\lambda_1-b_3)b_3(b_1+b_2/\sqrt{6})+(\lambda_1-b_3)[gb_1|b_4|^2+fb_3|b_2|^2]-\lambda_1 g(b_1+b_2/\sqrt{6})|b_4|^2}\,, \\ a_3=-a_1\dfrac{4}{\sqrt{6}}\dfrac{1}{M}\,\dfrac{ib_2(\lambda_1-b_3)[gb_1|b_4|^2+fb_3|b_2|^2]}{\lambda_1(\lambda_1-b_3)b_3(b_1+b_2/\sqrt{6})+(\lambda_1-b_3)[gb_1|b_4|^2+fb_3|b_2|^2]-\lambda_1 g(b_1+b_2/\sqrt{6})|b_4|^2}\,; \end{array} \tag{VI.31}$$

Their solutions are given by the following formulas:

$$\begin{array}{c} c_2=-c_1\,\dfrac{ig\lambda_2 b_2 b_4^*(b_1+b_2/\sqrt{6})}{\lambda_2(\lambda_2-b_3)b_3(b_1+b_2/\sqrt{6})+(\lambda_2-b_3)[gb_1|b_4|^2+fb_3|b_2|^2]-\lambda_2 g(b_1+b_2/\sqrt{6})|b_4|^2}\,, \\ c_3=-c_1\dfrac{4}{\sqrt{6}}\dfrac{1}{M}\,\dfrac{ib_2(\lambda_2-b_3)[gb_1|b_4|^2+fb_3|b_2|^2]}{\lambda_2(\lambda_2-b_3)b_3(b_1+b_2/\sqrt{6})+(\lambda_2-b_3)[gb_1|b_4|^2+fb_3|b_2|^2]-\lambda_2 g(b_1+b_2/\sqrt{6})|b_4|^2}\,; \end{array} \tag{VI.32}$$

Their solutions are given by the following formulas:

$$\begin{array}{c} r_2=-r_1\,\dfrac{ig\lambda_3 b_2 b_4^*(b_1+b_2/\sqrt{6})}{\lambda_3(\lambda_3-b_3)b_3(b_1+b_2/\sqrt{6})+(\lambda_3-b_3)[gb_1|b_4|^2+fb_3|b_2|^2]-\lambda_3 g(b_1+b_2/\sqrt{6})|b_4|^2}\,, \\ r_3=-r_1\dfrac{4}{\sqrt{6}}\dfrac{1}{M}\,\dfrac{ib_2(\lambda_3-b_3)[gb_1|b_4|^2+fb_3|b_2|^2]}{\lambda_3(\lambda_3-b_3)b_3(b_1+b_2/\sqrt{6})+(\lambda_3-b_3)[gb_1|b_4|^2+fb_3|b_2|^2]-\lambda_3 g(b_1+b_2/\sqrt{6})|b_4|^2}\,. \end{array} \tag{VI.33}$$

Now, let us additionally detail the diagonalization procedure of the system

$$\begin{array}{c} i\gamma_5\hat{\partial}\,[\,A_1\,(\gamma_\mu\Psi_\mu)+B_1\,\Psi_0+R_1\,(\gamma_5\partial_\mu\Psi_\mu)\,]+M(\gamma_\mu\Psi_\mu)=0\,, \\ i\gamma_5\hat{\partial}\,[\,A_2\,(\gamma_\mu\Psi_\mu)+B_2\,\Psi_0+R_2\,(\gamma_5\partial_\mu\Psi_\mu)\,]+M\Psi_0=0\,, \\ i\gamma_5\hat{\partial}\,[\,A_3\,(\gamma_\mu\Psi_\mu)+B_3\,\Psi_0+R_3\,(\gamma_5\partial_\mu\Psi_\mu)\,]+M(\gamma_5\partial_\mu\Psi_\mu)=0\,. \end{array} \tag{VI.34}$$

Multiply the first equation by a_1,the second – by a_2, the third – by a_3, and then sum the results, in this way we obtain

$$\Phi_1=a_1(\gamma_\mu\Psi_\mu)+a_2\Psi_0+a_3(\gamma_5\partial_\mu\Psi_\mu),\quad \left(i\gamma_5\hat{\partial}+\frac{M}{\lambda_1}\right)\Phi_1=0\,. \tag{VI.35}$$

Multiply the first equation by c_1,the second – by c_2, the third – by c_3, and then sum the results, this leads to

$$\Phi_2 = c_1(\gamma_\mu \Psi_\mu) + c_2 \Psi_0 + c_3(\gamma_5 \partial_\mu \Psi_\mu), \quad \left(i\gamma_5 \hat{\partial} + \frac{M}{\lambda_2} \right) \Phi_2 = 0 \,. \tag{VI.36}$$

Multiply the first equation by r_1,the second – by r_2, the third – by r_3, and then sum the results, this gives

$$\Phi_3 = r_1(\gamma_\mu \Psi_\mu) + r_2 \Psi_0 + r_3(\gamma_5 \partial_\mu \Psi_\mu), \quad \left(i\gamma_5 \hat{\partial} + \frac{M}{\lambda_3} \right) \Phi_3 = 0 \,. \tag{VI.37}$$

VII. TAKING INTO ACCOUNT THE PRESENCE OF ELECTROMAGNETIC FIELDS

In presence of external electromagnetic field we should math the change $\partial_\mu \Longrightarrow D_\mu = \partial_\mu - ieA_\mu,$, then from (VI.1)–(VI.3) obtain the modified system of spin-tensor equations

$$\left\{ b_1 \hat{D}(\gamma_\mu \Psi_\mu) - \frac{4b_2}{\sqrt{6}} \left[(D_\mu \Psi_\mu) - \frac{1}{4} \hat{D}(\gamma_\mu \Psi_\mu) \right] \right\} - i\gamma_5 M(\gamma_\mu \Psi_\mu) = 0 \,, \tag{VII.1}$$

$$\left\{ b_3 \hat{D} \Psi_0 - i\frac{4b_4}{\sqrt{6}} \left[(D_\mu \Psi_\mu) - \frac{1}{4} \hat{D}(\gamma_\mu \Psi_\mu) \right] \right\} - i\gamma_5 M \Psi_0 = 0 \,, \tag{VII.2}$$

$$\begin{gathered} \frac{2}{\sqrt{6}} \left\{ b_5 \left[D_\lambda(\gamma_\mu \Psi_\mu) - \frac{1}{4}\gamma_\lambda \hat{D}(\gamma_\mu \Psi_\mu) \right] - \right. \\ \left. -ib_6 \left[D_\lambda \Psi_0 - \frac{1}{4}\gamma_\lambda \hat{D}\Psi_0) \right] \right\} - i\gamma_5 M \left[\Psi_\lambda - \frac{1}{4}\gamma_\lambda(\gamma_\mu \Psi_\mu) \right] = 0 \,. \end{gathered} \tag{VII.3}$$

Recall the constraints

$$\begin{gathered} b_2 b_3 b_5 + b_1 b_4 b_6 = -\lambda_1 \lambda_2 \lambda_3 \,, \quad b_1 + b_3 = \lambda_1 + \lambda_2 + \lambda_3, \\ b_1 b_3 - b_2 b_5 - b_4 b_6 = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3, \end{gathered} \tag{VII.4}$$

which with the formulas

$$b_1^* = b_1, \; db_3^* = b_3, \; b_5 = fb_2^*, \; db_6 = gb_4^*, \quad f, g = \pm 1 \tag{VII.5}$$

read

$$\begin{gathered} fb_3|b_2|^2 + gb_1|b_4|^2 = -\lambda_1 \lambda_2 \lambda_3 \,, \quad b_1 + b_3 = \lambda_1 + \lambda_2 + \lambda_3, \\ b_1 b_3 - f|b_2|^2 - g|b_4|^2 = \lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3 \,. \end{gathered} \tag{VII.6}$$

Now we should repeat with some modification the all calculation from previous section.

Let us multiply eq. (VII.1) by $(-\frac{1}{4}\gamma_\lambda)$, and sum the result with (VII.3), so obtaining

$$\begin{gathered} \frac{2}{\sqrt{6}} b_5 \left[D_\lambda(\gamma_\mu \Psi_\mu) - \frac{1}{4}\gamma_\lambda \hat{D}(\gamma_\mu \Psi_\mu) \right] - i\frac{2}{\sqrt{6}} b_6 \left[D_\lambda \Psi_0 - \frac{1}{4}\gamma_\lambda \hat{D}\Psi_0) \right] + \\ -\frac{1}{4} b_1 \gamma_\lambda \hat{D}(\gamma_\mu \Psi_\mu) + \frac{b_2}{\sqrt{6}} \left[\gamma_\lambda (D_\mu \Psi_\mu) - \frac{1}{4}\gamma_\lambda \hat{D}(\gamma_\mu \Psi_\mu) \right] - i\gamma_5 M \Psi_\lambda = 0 \,. \end{gathered} \tag{VII.7}$$

Now, let us act on eq. (VII.7) by operator D_λ:

$$\begin{gathered} \frac{2}{\sqrt{6}} b_5 \left[D^2(\gamma_\mu \Psi_\mu) - \frac{1}{4} \hat{D}\hat{D}(\gamma_\mu \Psi_\mu) \right] - i\frac{2}{\sqrt{6}} b_6 \left[D^2 \Psi_0 - \frac{1}{4} \hat{D}\hat{D}\Psi_0) \right] + \\ -\frac{b_1}{4} \hat{D}\hat{D}(\gamma_\mu \Psi_\mu) + \frac{b_2}{\sqrt{6}} \hat{D} \left[(D_\mu \Psi_\mu) - \frac{1}{4} \hat{D}(\gamma_\mu \Psi_\mu) \right] - i\gamma_5 M(D_\mu \Psi_\mu) = 0 \,, \end{gathered} \tag{VII.8}$$

where $D^2 = D_\mu D_\mu, \; \hat{D} = \gamma_\mu D_\mu$. Below we will take into account identities

$$\hat{D}\hat{D} = D_\mu D_\nu \gamma_\mu \gamma_\nu = D^2 + \frac{1}{2} D_\mu D_\nu (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) = D^2 + (D_\mu D_\nu - D_\nu D_\mu) \frac{1}{4} (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu),$$

and

$$\hat{D}\hat{D} = D^2 - ieF_{\mu\nu}\sigma_{\mu\nu}, \qquad D_\mu D_\nu - D_\nu D_\mu = -ieF_{\mu\nu}\,, \quad \sigma_{\mu\nu} = \frac{\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu}{4}\,. \tag{VII.9}$$

Therefore, eq. (VII.8) may be re-written as

$$\frac{2}{\sqrt{6}}b_5\left[D^2(\gamma_\mu\Psi_\mu) - \frac{1}{4}D^2(\gamma_\mu\Psi_\mu) + \frac{ie}{4}F_{\lambda\rho}\sigma_{\lambda\rho}(\gamma_\mu\Psi_\mu)\right] - i\frac{2}{\sqrt{6}}b_6\left[D^2\Psi_0 - \frac{1}{4}D^2\Psi_0 + \frac{ie}{4}F_{\mu\nu}\sigma_{\mu\nu}\Psi_0\right] +$$

$$-\frac{b_1}{4}[\,D^2 - ieF_{\lambda\rho}\sigma_{\lambda\rho}\,]\,(\gamma_\mu\Psi_\mu) + \frac{b_2}{\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) - \frac{b_2}{4\sqrt{6}}[\,D^2 - ieF_{\lambda\rho}\sigma_{\lambda\rho}\,]\,(\gamma_\mu\Psi_\mu) - i\gamma_5 M(D_\mu\Psi_\mu) = 0\,,$$

or differently

$$\frac{2}{\sqrt{6}}b_5\left[\frac{3}{4}D^2(\gamma_\mu\Psi_\mu) + \frac{ie}{4}F_{\lambda\rho}\sigma_{\lambda\rho}(\gamma_\mu\Psi_\mu)\right] - i\frac{2}{\sqrt{6}}b_6\left[\frac{3}{4}D^2\Psi_0 + \frac{ie}{4}F_{\mu\nu}\sigma_{\mu\nu}\Psi_0\right] +$$

$$-\frac{b_1}{4}[\,D^2 - ieF_{\lambda\rho}\sigma_{\lambda\rho}\,]\,(\gamma_\mu\Psi_\mu) + \frac{b_2}{\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) - \frac{b_2}{4\sqrt{6}}[\,D^2 - ieF_{\lambda\rho}\sigma_{\lambda\rho}\,]\,(\gamma_\mu\Psi_\mu) - i\gamma_5 M(D_\mu\Psi_\mu) = 0\,.$$

Finally, we arrive at the equation

$$\begin{aligned} &\frac{\sqrt{6}}{4}b_5 D^2(\gamma_\mu\Psi_\mu) - i\frac{\sqrt{6}}{4}b_6 D^2\Psi_0 - \\ -\frac{1}{4}(b_1 + \frac{b_2}{\sqrt{6}})D^2(\gamma_\mu\Psi_\mu) &+ \frac{b_2}{\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) + \frac{e}{2\sqrt{6}}b_6 F_{\lambda\rho}\sigma_{\lambda\rho}\Psi_0 + \\ +\frac{ie}{4}(b_1 + \frac{2}{\sqrt{6}}b_5 &+ \frac{b_2}{\sqrt{6}})F_{\lambda\rho}\sigma_{\lambda\rho}(\gamma_\mu\Psi_\mu) - iM\gamma_5(D_\mu\Psi_\mu) = 0\,. \end{aligned} \tag{VII.10}$$

Now, let us act on eq. (VII.1) by operator $\hat{D}$:

$$b_1\hat{D}\hat{D}(\gamma_\mu\Psi_\mu) - \frac{4b_2}{\sqrt{6}}\left[\hat{D}(D_\mu\Psi_\mu) - \frac{1}{4}\hat{D}\hat{D}(\gamma_\mu\Psi_\mu)\right] - i\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu) = 0\,,$$

or

$$(b_1 + \frac{b_2}{\sqrt{6}})\hat{D}\hat{D}(\gamma_\mu\Psi_\mu) - \frac{4}{\sqrt{6}}b_2\hat{D}(D_\mu\Psi_\mu)(\gamma_\mu\Psi_\mu) - i\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu) = 0\,,$$

that is

$$(b_1 + \frac{b_2}{\sqrt{6}})[D^2 - ieF_{\lambda\rho}\sigma_{\lambda\rho}](\gamma_\mu\Psi_\mu) = \frac{4}{\sqrt{6}}b_2\hat{D}(D_\mu\Psi_\mu) + i\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu)\,,$$

whence it follows expression for term $D^2(\gamma_\mu\Psi_\mu)$:

$$D^2(\gamma_\mu\Psi_\mu) = ieF_{\lambda\rho}\sigma_{\lambda\rho}(\gamma_\mu\Psi_\mu) + \frac{4}{\sqrt{6}}\frac{b_2}{b_1 + b_2/\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) - \frac{1}{b_1 + b_2/\sqrt{6}}i\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu)\,. \tag{VII.11}$$

With (VII.11) in mind, relation (VII.10) takes the form

$$\frac{\sqrt{6}}{4}b_5 D^2(\gamma_\mu\Psi_\mu) - i\frac{\sqrt{6}}{4}b_6 D^2\Psi_0 - \frac{ie}{4}(b_1 + \frac{b_2}{\sqrt{6}})F_{\lambda\rho}\sigma_{\lambda\rho}(\gamma_\mu\Psi_\mu) - \frac{1}{\sqrt{6}}b_2\hat{D}(D_\mu\Psi_\mu) + \frac{i}{4}\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu)$$

$$+\frac{b_2}{\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) + \frac{e}{2\sqrt{6}}b_6 F_{\lambda\rho}\sigma_{\lambda\rho}\Psi_0 + \frac{ie}{4}(b_1 + \frac{2}{\sqrt{6}}b_5 + \frac{b_2}{\sqrt{6}})F_{\lambda\rho}\sigma_{\lambda\rho}(\gamma_\mu\Psi_\mu) - iM\gamma_5(D_\mu\Psi_\mu) = 0\,,$$

which after some re-grouping the term reads

$$\frac{\sqrt{6}}{4}b_5 D^2(\gamma_\mu\Psi_\mu) - i\frac{\sqrt{6}}{4}b_6 D^2\Psi_0 + \frac{e}{2\sqrt{6}}b_6 F_{\lambda\rho}\sigma_{\lambda\rho}\Psi_0 + ie\frac{1}{2\sqrt{6}}b_5 F_{\lambda\rho}\sigma_{\lambda\rho}(\gamma_\mu\Psi_\mu) + \frac{i}{4}M\gamma_5\hat{D}(\gamma_\mu\Psi_\mu) - iM\gamma_5(D_\mu\Psi_\mu) = 0\,.$$

Having used again relation (VII.11), we obtain

$$\frac{\sqrt{6}}{4}b_5\left\{ieF_{\lambda\rho}\sigma_{\lambda\rho}(\gamma_\mu\Psi_\mu) + \frac{4}{\sqrt{6}}\frac{b_2}{b_1+b_2/\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) - \frac{1}{b_1+b_2/\sqrt{6}}i\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu)\right\} -$$

$$-i\frac{\sqrt{6}}{4}b_6 D^2\Psi_0 + \frac{e}{2\sqrt{6}}b_6 F_{\lambda\rho}\sigma_{\lambda\rho}\Psi_0 + ie\frac{1}{2\sqrt{6}}b_5 F_{\lambda\rho}\sigma_{\lambda\rho}(\gamma_\mu\Psi_\mu) + \frac{i}{4}M\gamma_5\hat{D}(\gamma_\mu\Psi_\mu) - iM\gamma_5(D_\mu\Psi_\mu) = 0\,.$$

or (the notation $\Sigma = F_{\lambda\rho}\sigma_{\lambda\rho}$ is used)

$$-i\frac{\sqrt{6}}{4}b_6 D^2\Psi_0 + \frac{e}{2\sqrt{6}}b_6\Sigma\Psi_0 + \frac{\sqrt{6}}{3}b_5 ie\Sigma(\gamma_\mu\Psi_\mu) + \frac{b_2 b_5}{b_1+b_2/\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) +$$
$$+\frac{1}{4}\left(1 - \frac{\sqrt{6}b_5}{b_1+b_2/\sqrt{6}}\right)i\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu) - iM\gamma_5(D_\mu\Psi_\mu) = 0\,. \qquad \text{(VII.12)}$$

Now, we act on eq. (VII.2) by operator $\hat{D}$:

$$b_3\hat{D}\hat{D}\Psi_0 + i\frac{b_4}{\sqrt{6}}\hat{D}\hat{D}(\gamma_\mu\Psi_\mu) - i\frac{4b_4}{\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) + i\gamma_5 M\hat{D}\Psi_0 = 0\,,$$

whence using identity $\hat{D}\hat{D} = D^2 - ie\Sigma$, we obtain

$$b_3 D^2\Psi_0 - b_3 ie\Sigma\Psi_0 + i\frac{b_4}{\sqrt{6}}D^2(\gamma_\mu\Psi_\mu) + \frac{b_4}{\sqrt{6}}e\Sigma(\gamma_\mu\Psi_\mu) - i\frac{4b_4}{\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) + i\gamma_5 M\hat{D}\Psi_0 = 0\,.$$

Transforming the term $D^2(\gamma_\mu\Psi_\mu)$ with the help of we get (VII.11):

$$b_3 D^2\Psi_0 - b_3 ie\Sigma\Psi_0 + i\frac{b_4}{\sqrt{6}}\left\{ie\Sigma(\gamma_\mu\Psi_\mu) + \frac{4}{\sqrt{6}}\frac{b_2}{b_1+b_2/\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) - \frac{1}{b_1+b_2/\sqrt{6}}i\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu)\right\} +$$

$$+\frac{b_4}{\sqrt{6}}e\Sigma(\gamma_\mu\Psi_\mu) - i\frac{4b_4}{\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) + i\gamma_5 M\hat{D}\Psi_0 = 0$$

which after some regrouping the terms gives

$$D^2\Psi_0 = ie\Sigma\Psi_0 - i\frac{2}{3}\frac{b_2 b_4}{b_3(b_1+b_2/\sqrt{6})}\hat{D}(D_\mu\Psi_\mu) +$$
$$+\frac{i}{\sqrt{6}}\frac{b_4}{b_3(b_1+b_2/\sqrt{6})}\,i\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu) + i\frac{4}{\sqrt{6}}\frac{b_4}{b_3}\hat{D}(D_\mu\Psi_\mu) - \frac{1}{\sqrt{6}}i\gamma_5 M\hat{D}\Psi_0\,. \qquad \text{(VII.13)}$$

Substituting (VII.12) into (VII.13):

$$-i\frac{\sqrt{6}}{4}b_6\left\{ie\Sigma\Psi_0 - i\frac{2}{3}\frac{b_2 b_4}{b_3(b_1+b_2/\sqrt{6})}\hat{D}(D_\mu\Psi_\mu) +\right.$$

$$\left.+\frac{i}{\sqrt{6}}\frac{b_4}{b_3(b_1+b_2/\sqrt{6})}\,i\gamma_5 M\hat{D}(\gamma_\mu\Psi_\mu) + i\frac{4}{\sqrt{6}}\frac{b_4}{b_3}\hat{D}(D_\mu\Psi_\mu) - \frac{1}{\sqrt{6}}iM\gamma_5 M\hat{D}\Psi_0\right\} +$$

$$+\frac{e}{2\sqrt{6}}b_6\Sigma\Psi_0 + \frac{\sqrt{6}}{3}b_5 ie\Sigma(\gamma_\mu\Psi_\mu) + \frac{b_2 b_5}{b_1+b_2/\sqrt{6}}\hat{D}(D_\mu\Psi_\mu) + \frac{1}{4}\left(1 - \frac{\sqrt{6}b_5}{b_1+b_2/\sqrt{6}}\right)iM\gamma_5\hat{D}(\gamma_\mu\Psi_\mu) - iM\gamma_5(D_\mu\Psi_\mu) = 0\,.$$

Whence we find expression for the term $(D_\mu \Psi_\mu)$:

$$(D_\mu\Psi_\mu) = -\frac{1}{M}\frac{b_2b_3b_5 + b_1b_4b_6}{b_3(b_1+b_2/\sqrt{6})} i\gamma_5 \hat{D}(D_\mu\Psi_\mu) + \frac{1}{4}\frac{b_4b_6 + b_3(b_1+b_2/\sqrt{6}) - \sqrt{6}b_3b_5}{b_3(b_1+b_2/\sqrt{6})}\hat{D}(\gamma_\mu\Psi_\mu) +$$
$$+ i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}\hat{D}\Psi_0 - \frac{1}{M}\frac{2b_6}{\sqrt{6}} e\Sigma i\gamma_5\Psi_0 + \frac{1}{M}\frac{2b_5}{2\sqrt{6}} e\Sigma\gamma_5(\gamma_\mu\Psi_\mu) \,. \qquad \text{(VII.14)}$$

It remains to take into account (VII.14) in eq. (VII.1):

$$b_1\hat{D}(\gamma_\mu\Psi_\mu) - \frac{4b_2}{\sqrt{6}}\left[(D_\mu\Psi_\mu) - \frac{1}{4}\hat{D}(\gamma_\mu\Psi_\mu)\right] - i\gamma_5 M(\gamma_\mu\Psi_\mu) = 0\,;$$

which leads to

$$\frac{\sqrt{6}b_1b_3(b_1+b_2/\sqrt{6}) - b_2(b_4b_6 - \sqrt{6}b_3b_5)}{\sqrt{6}b_3(b_1+b_2/\sqrt{6})}\,\hat{D}\,(\gamma_\mu\Psi_\mu) - i\frac{b_2b_6}{b_3}\,\hat{D}\Psi_0 +$$

$$+\frac{4}{\sqrt{6}}\frac{1}{M}\frac{b_2}{b_3}\frac{b_2b_3b_5 + b_1b_4b_6}{b_3(b_1+b_2/\sqrt{6})}\, i\gamma_5\hat{D}\,(D_\mu\Psi_\mu) +$$
$$+\frac{4}{3M}\,b_2b_6\, i\gamma_5\, e\Sigma\,\Psi_0 - \frac{4}{3M}\,b_2b_5\,\gamma_5\, e\Sigma\,(\gamma_\mu\Psi_\mu) - \underline{iM\gamma_5\,(\gamma_\mu\Psi_\mu)} = 0\,. \qquad \text{(VII.15)}$$

It should emphasized that this equation has a needed structure, including the terms

$$\hat{D}\,(\gamma_\mu\Psi_\mu), \quad \hat{D}\Psi_0, \quad \hat{D}\,(D_\mu\Psi_\mu), \quad M(\gamma_\mu\Psi_\mu)\,.$$

Now, in eq. (VII.2):

$$b_3\hat{D}\Psi_0 - i\frac{4b_4}{\sqrt{6}}(D_\mu\Psi_\mu) + i\frac{b_4}{\sqrt{6}}\hat{D}(\gamma_\mu\Psi_\mu) - i\gamma_5 M\Psi_0 = 0\,,$$

let us take into equation relation (VII.14):

$$b_3\hat{D}\Psi_0 - i\frac{4b_4}{\sqrt{6}}\left\{-\frac{1}{M}\frac{b_2b_3b_5 + b_1b_4b_6}{b_3(b_1+b_2/\sqrt{6})} i\gamma_5\hat{D}(D_\mu\Psi_\mu) + \frac{1}{4}\frac{b_4b_6 + b_3(b_1+b_2/\sqrt{6}) - \sqrt{6}b_3b_5}{b_3(b_1+b_2/\sqrt{6})}\hat{D}(\gamma_\mu\Psi_\mu) +\right.$$

$$\left. + i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}\hat{D}\Psi_0 - \frac{1}{M}\frac{2b_6}{\sqrt{6}} e\Sigma i\gamma_5\Psi_0 + \frac{1}{M}\frac{2b_5}{2\sqrt{6}} e\Sigma\gamma_5(\gamma_\mu\Psi_\mu)\right\} + i\frac{b_4}{\sqrt{6}}\hat{D}(\gamma_\mu\Psi_\mu) - i\gamma_5 M\Psi_0 = 0\,,$$

whence after re-grouping the terms we obtain

$$\frac{b_3^2 + b_4b_6}{b_3}\hat{D}\Psi_0 + \frac{4i}{\sqrt{6}M}b_4\frac{b_1b_4b_6 + b_2b_3b_5}{b_3(b_1+b_2/\sqrt{6})}\, i\gamma_5\hat{D}\,(D_\mu\Psi_\mu) - \frac{i}{\sqrt{6}}\frac{b_4(b_4b_6 - \sqrt{6}b_3b_5)}{b_3(b_1+b_2/\sqrt{6})}\,\hat{D}\,(\gamma_\mu\Psi_\mu) +$$
$$+ i\frac{4}{3M}\,b_4b_6\, i\gamma_5\, e\Sigma\,\Psi_0 - \frac{4}{3M}\,b_4b_5\, i\gamma_5\, e\Sigma\,(\gamma_\mu\Psi_\mu) - iM\gamma_5\,\Psi_0 = 0\,. \qquad \text{(VII.16)}$$

This equation has a needed structure, including the terms

$$\hat{D}\,(\gamma_\mu\Psi_\mu), \quad \hat{D}\Psi_0, \quad \hat{D}\,(D_\mu\Psi_\mu), \quad M\Psi_0\,.$$

Now, let us turn back to eq. (VII.14)

$$\frac{1}{M}\frac{b_2b_3b_5 + b_1b_4b_6}{b_3(b_1+b_2/\sqrt{6})} i\gamma_5\hat{D}(D_\mu\Psi_\mu) - \frac{1}{4}\frac{b_4b_6 + b_3(b_1+b_2/\sqrt{6}) - \sqrt{6}b_3b_5}{b_3(b_1+b_2/\sqrt{6})}\hat{D}(\gamma_\mu\Psi_\mu) -$$

$$- i\frac{\sqrt{6}}{4}\frac{b_6}{b_3}\hat{D}\Psi_0 + \frac{1}{M}\frac{2b_6}{\sqrt{6}} e\Sigma i\gamma_5\Psi_0 - \frac{1}{M}\frac{2b_5}{\sqrt{6}} e\Sigma\gamma_5(\gamma_\mu\Psi_\mu) + (D_\mu\Psi_\mu) = 0\,,$$

after multiplying it by $M\gamma_5$ it reads

$$i\frac{b_2b_3b_5+b_1b_4b_6}{b_3(b_1+b_2/\sqrt{6})}\,\hat{D}\,(D_\mu\Psi_\mu)+i\frac{M}{4}\,\frac{b_4b_6+b_3(b_1+b_2/\sqrt{6})-\sqrt{6}b_3b_5}{b_3(b_1+b_2/\sqrt{6})}\,i\gamma_5\hat{D}\,(\gamma_\mu\Psi_\mu)-$$
$$-iM\frac{\sqrt{6}}{4}\,\frac{b_6}{b_3}\,\gamma_5\hat{D}\,\Psi_0+i\frac{2b_6}{\sqrt{6}}\,e\Sigma\,\Psi_0-\frac{2b_5}{\sqrt{6}}\,e\Sigma\,(\gamma_\mu\Psi_\mu)+M\gamma_5\,(D_\mu\Psi_\mu)=0\,. \qquad \text{(VII.17)}$$

This equation has a needed structure too, including the terms

$$\hat{D}\,(\gamma_\mu\Psi_\mu), \quad \hat{D}\Psi_0, \quad \hat{D}\,(D_\mu\Psi_\mu), \quad M(D_\mu\Psi_\mu)\,.$$

Thus, we have derived the needed form of of three equations (see. (VII.15), (VII.16), (VII.17) specifying the model under consideration in presence of external electromagnetic fields:

$$\frac{\sqrt{6}b_1b_3(b_1+b_2/\sqrt{6})-b_2(b_4b_6-\sqrt{6}b_3b_5)}{\sqrt{6}b_3(b_1+b_2/\sqrt{6})}\,\hat{D}\,(\gamma_\mu\Psi_\mu)-i\frac{b_2b_6}{b_3}\,\hat{D}\Psi_0+$$

$$+\frac{4}{\sqrt{6}}\frac{1}{M}\frac{b_2}{b_3}\frac{b_2b_3b_5+b_1b_4b_6}{b_3(b_1+b_2/\sqrt{6})}\,i\gamma_5\hat{D}\,(D_\mu\Psi_\mu)+$$
$$+\frac{4}{3M}\,b_2b_6\,i\gamma_5\,e\Sigma\,\Psi_0-\frac{4}{3M}\,b_2b_5\,\gamma_5\,e\Sigma\,(\gamma_\mu\Psi_\mu)-iM\gamma_5\,(\gamma_\mu\Psi_\mu)=0\,. \qquad \text{(VII.18)}$$

$$\frac{b_3^2+b_4b_6}{b_3}\,\hat{D}\Psi_0+\frac{4i}{\sqrt{6}M}b_4\frac{b_1b_4b_6+b_2b_3b_5}{b_3(b_1+b_2/\sqrt{6})}\,i\gamma_5\hat{D}\,(D_\mu\Psi_\mu)-\frac{i}{\sqrt{6}}\,\frac{b_4(b_4b_6-\sqrt{6}b_3b_5)}{b_3(b_1+b_2/\sqrt{6})}\,\hat{D}\,(\gamma_\mu\Psi_\mu)+$$
$$+i\frac{4}{3M}\,b_4b_6\,i\gamma_5\,e\Sigma\,\Psi_0-\frac{4}{3M}\,b_4b_5\,i\gamma_5\,e\Sigma\,(\gamma_\mu\Psi_\mu)-iM\gamma_5\,\Psi_0=0\,. \qquad \text{(VII.19)}$$

$$i\frac{b_2b_3b_5+b_1b_4b_6}{b_3(b_1+b_2/\sqrt{6})}\,\hat{D}\,(D_\mu\Psi_\mu)+i\frac{M}{4}\,\frac{b_4b_6+b_3(b_1+b_2/\sqrt{6})-\sqrt{6}b_3b_5}{b_3(b_1+b_2/\sqrt{6})}\,i\gamma_5\hat{D}\,(\gamma_\mu\Psi_\mu)-$$
$$-iM\frac{\sqrt{6}}{4}\,\frac{b_6}{b_3}\,\gamma_5\hat{D}\,\Psi_0+i\frac{2b_6}{\sqrt{6}}\,e\Sigma\,\Psi_0-\frac{2b_5}{\sqrt{6}}\,e\Sigma\,(\gamma_\mu\Psi_\mu)+M\gamma_5\,(D_\mu\Psi_\mu)=0\,. \qquad \text{(VII.20)}$$

Allowing for the constraints $f_5=fb_2^*,\ b_6=gb_4^2$, we arrive at equations

$$\begin{aligned}
&A_1i\gamma_5\hat{D}(\gamma_\mu\Psi_\mu)+B_1i\gamma_5\hat{D}\Psi_0+R_1i\gamma_5\hat{D}(\gamma_5\partial_\mu\Psi_\mu)-\\
&-i\frac{4f|b_2|^2}{3M}e\Sigma(\gamma_\mu\Psi_\mu)-\frac{4gb_4^*b_2}{3M}e\Sigma\Psi_0+M(\gamma_\mu\Psi_\mu)=0\,,\\
&A_2i\gamma_5\hat{D}(\gamma_\mu\Psi_\mu)+B_2i\gamma_5\hat{D}\Psi_0+R_2i\gamma_5\hat{D}(\gamma_5\partial_\mu\Psi_\mu)+\\
&+\frac{4fb_2^*b_4}{3M}e\Sigma(\gamma_\mu\Psi_\mu)-i\frac{4g|b_4|^2}{3M}e\Sigma\Psi_0+M\Psi_0=0\,,\\
&A_3i\gamma_5\hat{D}(\gamma_\mu\Psi_\mu)+B_3i\gamma_5\hat{D}\Psi_0+R_3i\gamma_5\hat{D}(\gamma_5\partial_\mu\Psi_\mu)-\\
&-\frac{2fb_2^*}{\sqrt{6}}e\Sigma(\gamma_\mu\Psi_\mu)+i\frac{2gb_4^*}{\sqrt{6}}e\Sigma\Psi_0+M(\gamma_5D_\mu\Psi_\mu)=0\,,
\end{aligned} \qquad \text{(VII.21)}$$

here the above determined parameters (VI.22) are used, A_i, B_i, R_i.

Let us multiply the first eq. in (VII.21) by a_1, the second – by a_2, the third – by a_3, and sum the results, it gives

$$\lambda_1 i\gamma^5\hat{D}\Phi_1+M\Phi_1+\left[-ia_1\frac{4}{3}\frac{f|b_2|^2}{M}+a_2\frac{4}{3}f\frac{b_2^*b_4}{M}-a_3\frac{2}{\sqrt{6}}fb_2^*\right]e\Sigma(\gamma_\mu\Psi_\mu)+$$
$$+\left[-a_1\frac{4}{3}\frac{gb_2b_4^*}{M}-ia_2\frac{4}{3}g\frac{|b_4|^2}{M}+ia_3\frac{2}{\sqrt{6}}gb_4^*\right]e\Sigma\Psi_0=0\,.$$

Now, let us multiply the first eq. in (VII.21) by c_1, the second – by c_2, the third – by c_3, and sum the results, this yields

$$\lambda_2 i\gamma^5 \hat{D}\Phi_2 + M\Phi_2 + \Big\{ -ic_1 \frac{4}{3}\frac{f|b_2|^2}{M} + c_2 \frac{4}{3} f \frac{b_2^* b_4}{M} - c_3 \frac{2}{\sqrt{6}} f b_2^* \Big] e\Sigma(\gamma_\mu \Psi_\mu) + \\ + \Big[-c_1 \frac{4}{3}\frac{g b_2 b_4^*}{M} - ic_2 \frac{4}{3} g \frac{|b_4|^2}{M} + ic_3 \frac{2}{\sqrt{6}} g b_4^* \Big] e\Sigma\Psi_0 = 0 \,,$$

Now, let us multiply the first eq. in (VII.21) by r_1, the second – by r_2, the third – by r_3, and sum the results, this yields

$$\lambda_3 i\gamma^5 \hat{D}\Phi_3 + M\Phi_3 + \Big[-ir_1 \frac{4}{3}\frac{f|b_2|^2}{M} + r_2 \frac{4}{3}\frac{f b_2^* b_4}{M} - r_3 \frac{2}{\sqrt{6}} f b_2^* \Big] e\Sigma(\gamma_\mu \Psi_\mu) + \\ + \Big[-r_1 \frac{4}{3}\frac{g b_2 b_4^*}{M} - ir_2 \frac{4}{3} g \frac{|b_4|^2}{M} + ir_3 \frac{2}{\sqrt{6}} g b_4^* \Big] e\Sigma\Psi_0 = 0 \,,$$

where the following combinations of bispinors are used

$$\Phi_1 = a_1(\gamma_\mu \Psi_\mu) + a_2 \Psi_0 + a_3(\gamma_5 D_\mu \Psi_\mu) \,, \\ \Phi_2 = c_1(\gamma_\mu \Psi_\mu) + c_2 \Psi_0 + c_3(\gamma_5 D_\mu \Psi_\mu) \,, \\ \Phi_3 = r_1(\gamma_\mu \Psi_\mu) + r_2 \Psi_0 + r_3(\gamma_5 D_\mu \Psi_\mu) \,. \tag{VII.22}$$

Collecting three equations together we have

$$\lambda_1 i\gamma^5 \hat{D}\Phi_1 + M\Phi_1 + \\ + \Big(-ia_1 \frac{4}{3}\frac{f|b_2|^2}{M} + a_2 \frac{4}{3} f \frac{b_2^* b_4}{M} - a_3 \frac{2}{\sqrt{6}} f b_2^* \Big) e\Sigma(\gamma_\mu \Psi_\mu) + \\ + \Big(-a_1 \frac{5}{3}\frac{g b_2 b_4^*}{M} - ia_2 \frac{4}{3} g \frac{|b_4|^2}{M} + ia_3 \frac{2}{\sqrt{6}} g b_4^* \Big) e\Sigma\Psi_0 = 0 \,. \tag{VII.23}$$

$$\lambda_2 i\gamma^5 \hat{D}\Phi_2 + M\Phi_2 + \\ + \Big(-ic_1 \frac{4}{3}\frac{f|b_2|^2}{M} + c_2 \frac{4}{3} f \frac{b_2^* b_4}{M} - c_3 \frac{2}{\sqrt{6}} f b_2^* \Big) e\Sigma(\gamma_\mu \Psi_\mu) + \\ + \Big[-c_1 \frac{4}{3}\frac{g b_2 b_4^*}{M} - ic_2 \frac{4}{3} g \frac{|b_4|^2}{M} + ic_3 \frac{2}{\sqrt{6}} g b_4^* \Big] e\Sigma\Psi_0 = 0 \,, \tag{VII.24}$$

$$\lambda_3 i\gamma^5 \hat{D}\Phi_3 + M\Phi_3 + \\ + \Big[-ir_1 \frac{4}{3}\frac{f|b_2|^2}{M} + r_2 \frac{4}{3}\frac{f b_2^* b_4}{M} - r_3 \frac{2}{\sqrt{6}} f b_2^* \Big] e\Sigma(\gamma_\mu \Psi_\mu) + \\ + \Big[-r_1 \frac{4}{3}\frac{g b_2 b_4^*}{M} - ir_2 \frac{4}{3} g \frac{|b_4|^2}{M} + ir_3 \frac{2}{\sqrt{6}} g b_4^* \Big] e\Sigma\Psi_0 = 0 \,, \tag{VII.25}$$

they may be re-written differently

$$\lambda_1 i\gamma^5 \hat{D}\Phi_1 + M\Phi_1 + e\Sigma \Big(-ia_1 \frac{4}{3}\frac{b_2}{M} + a_2 \frac{4}{3}\frac{b_4}{M} - a_3 \frac{2}{\sqrt{6}} \Big) [\, f b_2^* \; \gamma_\mu \Psi_\mu - i g b_4^* \; \Psi_0 \,] = 0 \,. \tag{VII.26}$$

$$\lambda_2 i\gamma^5 \hat{D}\Phi_2 + M\Phi_2 + e\Sigma \Big(-ic_1 \frac{4}{3}\frac{b_2}{M} + c_2 \frac{4}{3}\frac{b_4}{M} - c_3 \frac{2}{\sqrt{6}} \Big) [\, f b_2^* \; \gamma_\mu \Psi_\mu - i g b_4^* \; \Psi_0 \,] = 0 \,, \tag{VII.27}$$

$$\lambda_3 i\gamma^5 \hat{D}\Phi_3 + M\Phi_3 + e\Sigma \left(-ir_1 \frac{4}{3}\frac{b_2}{M} + r_2 \frac{4}{3}\frac{b_4}{M} - r_3 \frac{2}{\sqrt{6}} \right) [\, f b_2^* \, \gamma_\mu \Psi_\mu - i g b_4^* \, \Psi_0 \,] = 0 \,, \tag{VII.28}$$

Let us introduce notations

$$\bar{\Phi}_1 = \gamma_\mu \Psi_\mu \,, \quad \bar{\Phi}_2 = \Psi_0 \,, \quad \bar{\Phi}_3 = \gamma_5 D_\mu \Psi_\mu \,; \tag{VII.29}$$

$$\begin{aligned} Z_1 &= -ia_1 \frac{4}{3}\frac{b_2}{M} + a_2 \frac{4}{3}\frac{b_4}{M} - a_3 \frac{2}{\sqrt{6}} \,, \\ Z_2 &= -ic_1 \frac{4}{3}\frac{b_2}{M} + c_2 \frac{4}{3}\frac{b_4}{M} - c_3 \frac{2}{\sqrt{6}} \,, \\ Z_3 &= -ir_1 \frac{4}{3}\frac{b_2}{M} + r_2 \frac{4}{3}\frac{b_4}{M} - r_3 \frac{2}{\sqrt{6}} \,. \end{aligned} \tag{VII.30}$$

Then the previous system (VII.26)–(VII.28) reads

$$\begin{aligned} &\lambda_1 i\gamma^5 \hat{D}\Phi_1 + M\Phi_1 + Z_1 \, e\Sigma \, [\, f b_2^* \, \bar{\Phi}_1 - i g b_4^* \, \bar{\Phi}_2 \,] = 0 \,, \\ &\lambda_2 i\gamma^5 \hat{D}\Phi_2 + M\Phi_2 + Z_1 \, e\Sigma \, [\, f b_2^* \, \bar{\Phi}_1 - i g b_4^* \, \bar{\Phi}_2 \,] = 0 \,, \\ &\lambda_3 i\gamma^5 \hat{D}\Phi_3 + M\Phi_3 + Z_3 \, e\Sigma \, [\, f b_2^* \, \bar{\Phi}_1 - i g b_4^* \, \bar{\Phi}_2 \,] = 0 \,; \end{aligned} \tag{VII.31}$$

recall that

$$\hat{D} = \gamma_\mu D_\mu = \gamma_\mu (\partial_\mu - ieA_\mu), \quad \Sigma = \frac{1}{4}(\gamma_\lambda \gamma_\rho - \gamma_\rho \gamma_\lambda) \, F_{\lambda\rho} \,. \tag{VII.32}$$

VIII. MIXING THE COMPONENTS

Let us start with three equations

$$\Phi_1 = a_1 \bar{\Phi}_1 + a_2 \bar{\Phi}_2 + a_3 \bar{\Phi}_3 \,, \quad \Phi_2 = c_1 \bar{\Phi}_1 + c_2 \bar{\Phi}_2 + c_3 \bar{\Phi}_3 \,, \quad \Phi_3 = r_1 \bar{\Phi}_1 + r_2 \bar{\Phi}_2 + r_3 \bar{\Phi}_3 \,.$$

From the third equation one can express $\bar{\Phi}_3$:

$$\bar{\Phi}_3 = \frac{1}{r_3}\Phi_3 - \frac{r_1}{r_3}\bar{\Phi}_1 - \frac{r_2}{r_3}\bar{\Phi}_2 \,;$$

then two first take the form

$$\Phi_1 - \frac{a_3}{r_3}\Phi_3 = (a_1 - \frac{a_3 r_1}{r_3})\bar{\Phi}_1 + (a_2 - \frac{a_3 r_2}{r_3})\bar{\Phi}_2 \,, \quad \Phi_2 - \frac{c_3}{r_3}\Phi_3 = (c_1 - \frac{c_3 r_1}{r_3})\bar{\Phi}_1 + (c_2 - \frac{c_3 r_2}{r_3})\bar{\Phi}_2 \,,$$

or

$$r_3 \Phi_1 - a_3 \Phi_3 = (r_3 a_1 - a_3 r_1)\bar{\Phi}_1 + (r_3 a_2 - a_3 r_2)\bar{\Phi}_2 \,,$$

$$r_3 \Phi_2 - c_3 \Phi_3 = (r_3 c_1 - c_3 r_1)\bar{\Phi}_1 + (r_3 c_2 - c_3 r_2)\bar{\Phi}_2 \,.$$

Thus, we have linear system with respect to the variables $\bar{\Phi}_1, \bar{\Phi}_2$:

$$\begin{cases} (r_3 a_1 - a_3 r_1)\bar{\Phi}_1 + (r_3 a_2 - a_3 r_2)\bar{\Phi}_2 = r_3 \Phi_1 - a_3 \Phi_3 \,, \\ (r_3 c_1 - c_3 r_1)\bar{\Phi}_1 + (r_3 c_2 - c_3 r_2)\bar{\Phi}_2 = r_3 \Phi_2 - c_3 \Phi_3 \,. \end{cases} \tag{VIII.1}$$

Let us write down expressions for parameters (VI.31)–(VI.33)):

$$a_2 = -a_1 \, \frac{ig\lambda_1 b_2 b_4^* (b_1 + b_2/\sqrt{6})}{\lambda_1(\lambda_1 - b_3) b_3 (b_1 + b_2/\sqrt{6}) + (\lambda_1 - b_3)[g b_1 |b_4|^2 + f b_3 |b_2|^2] - \lambda_1 g (b_1 + b_2/\sqrt{6}) |b_4|^2},$$

$$a_3 = -a_1 \frac{4}{\sqrt{6}} \frac{1}{M} \, \frac{i b_2 (\lambda_1 - b_3)[g b_1 |b_4|^2 + f b_3 |b_2|^2]}{\lambda_1(\lambda_1 - b_3) b_3 (b_1 + b_2/\sqrt{6}) + (\lambda_1 - b_3)[g b_1 |b_4|^2 + f b_3 |b_2|^2] - \lambda_1 g (b_1 + b_2/\sqrt{6}) |b_4|^2};$$

$$c_2 = -c_1 \, \frac{ig\lambda_2 b_2 b_4^*(b_1 + b_2/\sqrt{6})}{\lambda_2(\lambda_2 - b_3)b_3(b_1 + b_2/\sqrt{6}) + (\lambda_2 - b_3)[gb_1|b_4|^2 + fb_3|b_2|^2] - \lambda_2 g(b_1 + b_2/\sqrt{6})|b_4|^2},$$

$$c_3 = -c_1 \frac{4}{\sqrt{6}} \frac{1}{M} \, \frac{ib_2(\lambda_2 - b_3)[gb_1|b_4|^2 + fb_3|b_2|^2]}{\lambda_2(\lambda_2 - b_3)b_3(b_1 + b_2/\sqrt{6}) + (\lambda_2 - b_3)[gb_1|b_4|^2 + fb_3|b_2|^2] - \lambda_2 g(b_1 + b_2/\sqrt{6})|b_4|^2};$$

$$r_2 = -r_1 \, \frac{ig\lambda_3 b_2 b_4^*(b_1 + b_2/\sqrt{6})}{\lambda_3(\lambda_3 - b_3)b_3(b_1 + b_2/\sqrt{6}) + (\lambda_3 - b_3)[gb_1|b_4|^2 + fb_3|b_2|^2] - \lambda_3 g(b_1 + b_2/\sqrt{6})|b_4|^2},$$

$$r_3 = -r_1 \frac{4}{\sqrt{6}} \frac{1}{M} \, \frac{ib_2(\lambda_3 - b_3)[gb_1|b_4|^2 + fb_3|b_2|^2]}{\lambda_3(\lambda_3 - b_3)b_3(b_1 + b_2/\sqrt{6}) + (\lambda_3 - b_3)[gb_1|b_4|^2 + fb_3|b_2|^2] - \lambda_3 g(b_1 + b_2/\sqrt{6})|b_4|^2}.$$

Because a_1, c_1, r_1 are still arbitrary, we may fix them to get parameters most simple. These are (they are collected in three sets, symmetrical with respect the roots $\lambda_1, \lambda_2, \lambda_3$))

$$\begin{aligned} a_1 &= \lambda_1(\lambda_1 - b_3)\; b_3(b_1 + b_2/\sqrt{6}) + (\lambda_1 - b_3)\; [gb_1|b_4|^2 + fb_3|b_2|^2] - \lambda_1\; g(b_1 + b_2/\sqrt{6})|b_4|^2 \,, \\ c_1 &= \lambda_2(\lambda_2 - b_3)\; b_3(b_1 + b_2/\sqrt{6}) + (\lambda_2 - b_3)\; [gb_1|b_4|^2 + fb_3|b_2|^2] - \lambda_2\; g(b_1 + b_2/\sqrt{6})|b_4|^2 \,, \\ r_1 &= \lambda_3(\lambda_3 - b_3)\; b_3(b_1 + b_2/\sqrt{6}) + (\lambda_3 - b_3)\; [gb_1|b_4|^2 + fb_3|b_2|^2] - \lambda_3\; g(b_1 + b_2/\sqrt{6})|b_4|^2 \,, \end{aligned} \tag{VIII.2}$$

$$a_2 = -igb_2b_4^*\; (b_1 + b_2/\sqrt{6})\; \lambda_1 \,, \quad c_2 = -igb_2b_4^*\; (b_1 + b_2/\sqrt{6})\; \lambda_2 \,, \quad r_2 = -igb_2b_4^*\; (b_1 + b_2/\sqrt{6})\; \lambda_3 \,, \tag{VIII.3}$$

$$\begin{aligned} a_3 &= -\frac{4}{\sqrt{6}} \frac{1}{M}\; ib_2[gb_1|b_4|^2 + fb_3|b_2|^2]\; (\lambda_1 - b_3) \,, \\ c_3 &= -\frac{4}{\sqrt{6}} \frac{1}{M}\; ib_2[gb_1|b_4|^2 + fb_3|b_2|^2]\; (\lambda_2 - b_3) \,, \\ r_3 &= -\frac{4}{\sqrt{6}} \frac{1}{M}\; ib_2[gb_1|b_4|^2 + fb_3|b_2|^2]\; (\lambda_3 - b_3) \,. \end{aligned} \tag{VIII.4}$$

With notations

$$K = gb_1b_4b_4^* + fb_3b_2b_2^*, \qquad L = (b_1 + b_2/\sqrt{6}), \tag{VIII.5}$$

the above formulas become shorter:

$$\begin{aligned} a_1 &= \lambda_1(\lambda_1 - b_3)\; b_3L + (\lambda_1 - b_3)\; K - \lambda_1\; L\; gb_4b_4^* \,, \\ c_1 &= \lambda_2(\lambda_2 - b_3)\; b_3L + (\lambda_2 - b_3)\; K - \lambda_2\; L\; gb_4b_4^* \,, \\ r_1 &= \lambda_3(\lambda_3 - b_3)\; b_3L + (\lambda_3 - b_3)\; K - \lambda_3\; L\; gb_4b_4^* \,, \end{aligned} \tag{VIII.6}$$

$$a_2 = -igb_2b_4^*\; L\; \lambda_1, \quad c_2 = -igb_2b_4^*\; L\; \lambda_2 \,, \quad r_2 = -igb_2b_4^*\; L\; \lambda_3 \,, \tag{VIII.7}$$

$$a_3 = -i\frac{4}{\sqrt{6}} \frac{1}{M}\; b_2K(\lambda_1 - b_3), \quad c_3 = -i\frac{4}{\sqrt{6}} \frac{1}{M}\; b_2K(\lambda_2 - b_3), \quad r_3 = -i\frac{4}{\sqrt{6}} \frac{1}{M}\; b_2K(\lambda_3 - b_3). \tag{VIII.8}$$

To get solutions of the system

$$\begin{cases} (r_3a_1 - a_3r_1)\bar{\Phi}_1 + (r_3a_2 - a_3r_2)\bar{\Phi}_2 = r_3\Phi_1 - a_3\Phi_3 \,, \\ (r_3c_1 - c_3r_1)\bar{\Phi}_1 + (r_3c_2 - c_3r_2)\bar{\Phi}_2 = r_3\Phi_2 - c_3\Phi_3 \,, \end{cases} \tag{VIII.9}$$

let us find three determinants:

$$\Delta = (r_3a_1 - a_3r_1)(r_3c_2 - c_3r_2) - (r_3a_2 - a_3r_2)(r_3c_1 - c_3r_1) =$$

$$= r_3r_3a_1c_2 - r_3r_2a_1c_3 - r_1r_3a_3c_2 + r_1r_2a_3c_3-$$

$$-r_3r_3a_2c_1 + r_3r_1a_2c_3 + r_2r_3a_3c_1 - r_2r_1a_3c_3 \quad \Longrightarrow$$

$$\Delta = r_3r_3(a_1c_2 - a_2c_1) + r_3r_2(a_3c_1 - a_1c_3) + r_1r_3(a_2c_3 - a_3c_2)\,, \tag{VIII.10}$$

$$\Delta_1 = (r_3\Phi_1 - a_3\Phi_3)(r_3c_2 - c_3r_2) - (r_3\Phi_2 - c_3\Phi_3)(r_3a_2 - a_3r_2) \quad \Longrightarrow$$

$$\Delta_1 = r_3(c_2r_3 - r_2c_3)\cdot\Phi_1 + r_3(a_3r_2 - a_2r_3)\cdot\Phi_2 + r_3(c_3a_2 - c_2a_3)\cdot\Phi_3\,, \tag{VIII.11}$$

$$\Delta_2 = (r_3a_1 - a_3r_1)(r_3\Phi_2 - c_3\Phi_3) - (r_3c_1 - c_3r_1)(r_3\Phi_1 - a_3\Phi_3) \quad \Longrightarrow$$

$$\Delta_2 = r_3(c_3r_1 - c_1r_3)\cdot\Phi_1 + r_3(a_1r_3 - a_3r_1)\cdot\Phi_2 + r_3(a_3c_1 - a_1c_3)\cdot\Phi_3\,. \tag{VIII.12}$$

These yield the formulas

$$\begin{aligned}\bar\Phi_1 &= \frac{(c_2r_3 - r_2c_3)\cdot\Phi_1 + (a_3r_2 - a_2r_3)\cdot\Phi_2 + (c_3a_2 - c_2a_3)\cdot\Phi_3}{r_1(a_2c_3 - a_3c_2) + r_2(a_3c_1 - a_1c_3) + r_3(a_1c_2 - a_2c_1)}\\ \bar\Phi_1 &= \frac{(c_3r_1 - c_1r_3)\cdot\Phi_1 + (a_1r_3 - a_3r_1)\cdot\Phi_2 + (a_3c_1 - a_1c_3)\cdot\Phi_3}{r_1(a_2c_3 - a_3c_2) + r_2(a_3c_1 - a_1c_3) + r_3(a_1c_2 - a_2c_1)}\end{aligned} \tag{VIII.13}$$

Taking into account formulas (VIII.6)–(VIII.8) in (VIII.13), we obtain

$$\begin{aligned}\bar\Phi_1 &= \frac{(-\Phi_2 + \Phi_3)\,\lambda_1 + (\Phi_1 - \Phi_3)\,\lambda_2 - \lambda_3\,(\Phi_1 - \Phi_2)}{b_3\,L\,(\lambda_2 - \lambda_3)\,(-\lambda_3 + \lambda_1)\,(\lambda_1 - \lambda_2)}\,,\\ \bar\Phi_2 &= \frac{1}{b_2\,b_3\,L\,g\,b_4^*\,(\lambda_2 - \lambda_3)\,(-\lambda_3 + \lambda_1)\,(\lambda_1 - \lambda_2)}\times\\ &\times\Big[i\Phi_1\,(\lambda_2 - \lambda_3)\big((\lambda_3 - b_3)\,\lambda_2 + gb_4\,b_4^* + {b_3}^2 - \lambda_3\,b_3\big) -\\ &-i\Phi_2\,(-\lambda_3 + \lambda_1)\big((\lambda_3 - b_3)\,\lambda_1 + gb_4\,b_4^* + {b_3}^2 - \lambda_3\,b_3\big) +\\ &+i\Phi_3\,(\lambda_1 - \lambda_2)\big((\lambda_2 - b_3)\,\lambda_1 + gb_4\,b_4^* + {b_3}^2 - \lambda_2\,b_3\big)\Big]\,.\end{aligned} \tag{VIII.14}$$

Allowing for notations (VIII.5) in (VIII.14), we find

$$\begin{aligned}\bar\Phi_1 &= \frac{(-\Phi_2 + \Phi_3)\,\lambda_1 + (\Phi_1 - \Phi_3)\,\lambda_2 - \lambda_3\,(\Phi_1 - \Phi_2)}{b_3\,\big(b_1 + b_2/\sqrt{6}\big)\,(\lambda_2 - \lambda_3)\,(-\lambda_3 + \lambda_1)\,(\lambda_1 - \lambda_2)} =\\ &= -\frac{\Phi_1(\lambda_2 - \lambda_3) + \Phi_2(\lambda_3 - \lambda_1) + \Phi_3(\lambda_1 - \lambda_2)}{b_3\,\big(b_1 + b_2/\sqrt{6}\big)\,(\lambda_2 - \lambda_3)\,(\lambda_3 - \lambda_1)\,(\lambda_1 - \lambda_2)}\,,\\ \bar\Phi_2 &= -\frac{1}{b_3\,\big(b_1 + b_2/\sqrt{6}\big)\,(\lambda_2 - \lambda_3)\,(\lambda_3 - \lambda_1)\,(\lambda_1 - \lambda_2)}\,\frac{i}{b_2\,g\,b_4^*}\times\\ &\times\Big\{\Phi_1\,(\lambda_2 - \lambda_3)\,\big[(\lambda_3 - b_3)\,\lambda_2 + gb_4\,b_4^* + {b_3}^2 - \lambda_3\,b_3\big] +\\ &+\Phi_2\,(\lambda_3 - \lambda_1)\,\big[(\lambda_3 - b_3)\,\lambda_1 + gb_4\,b_4^* + {b_3}^2 - \lambda_3\,b_3\big] +\\ &+\Phi_3\,(\lambda_1 - \lambda_2)\,\big[(\lambda_2 - b_3)\,\lambda_1 + gb_4\,b_4^* + {b_3}^2 - \lambda_2\,b_3\big]\Big\}\,.\end{aligned} \tag{VIII.15}$$

Now we calculate the term from (VII.31):

$$\Phi \equiv fb_2^*\,\bar\Phi_1 - igb_4^*\,\bar\Phi_2 =$$

$$= -\frac{1}{\big(b_1 + b_2/\sqrt{6}\big)\,b_2b_3\,(\lambda_2 - \lambda_3)\,(\lambda_3 - \lambda_1)\,(\lambda_1 - \lambda_2)}\times$$

$$\times \left[\Phi_1 \; (\lambda_2 - \lambda_3) \left(f b_2^* \, b_2 + \lambda_2 \, \lambda_3 - (\lambda_2 + \lambda_3) \, b_3 + {b_3}^2 + g b_4 \, b_4^* \right) + \right.$$
$$+ \Phi_2 \; (\lambda_3 - \lambda_1) \left(f b_2^* \, b_2 + \lambda_3 \lambda_1 - (\lambda_3 + \lambda_1) \; b_3 + {b_3}^2 + g b_4 \, b_4^* \right) +$$
$$\left. + \Phi_3 \; (\lambda_1 - \lambda_2) \left(f b_2^* \, b_2 + \lambda_1 \, \lambda_2 - (\lambda_1 + \lambda_2) \, b_3 + {b_3}^2 + g b_4 \, b_4^* \right) \right] \, , \qquad \text{(VIII.16)}$$

and coefficients Z_1, Z_2, Z_3 from (VII.30):

$$Z_1 = i \frac{4}{3} \frac{b_2 b_3 \, L}{M} \; \lambda_1 \, (b_3 - \lambda_1) \; , \quad Z_2 = i \frac{4}{3} \frac{b_2 b_3 \, L}{M} \; \lambda_2 \, (b_3 - \lambda_2) \; , \quad Z_3 = i \frac{4}{3} \frac{b_2 b_3 \, L}{M} \; \lambda_3 \, (b_3 - \lambda_3) \; . \qquad \text{(VIII.17)}$$

There formulas should be taken into account in equations (VII.31) :

$$\lambda_1 i \gamma^5 \hat{D} \Phi_1 + M \Phi_1 + Z_1 \; e \Sigma \left[\; f b_2^* \; \bar{\Phi}_1 - i g b_4^* \; \bar{\Phi}_2 \; \right] = 0 \, ,$$
$$\lambda_2 i \gamma^5 \hat{D} \Phi_2 + M \Phi_2 + Z_1 \; e \Sigma \; \left[\; f b_2^* \; \bar{\Phi}_1 - i g b_4^* \; \bar{\Phi}_2 \; \right] = 0 \, , \qquad \text{(VIII.18)}$$
$$\lambda_3 i \gamma^5 \hat{D} \Phi_3 + M \Phi_3 + Z_3 \; e \Sigma \; \left[\; f b_2^* \; \bar{\Phi}_1 - i g b_4^* \; \bar{\Phi}_2 \; \right] = 0 \, .$$

We readily can see that both in Z_i and Φ one multiplier vanishes due to identity $b_2 b_3 L = b_2 b_3 (b_1 + b_2/\sqrt{6})$. Therefore, the main system may be re-written as follows

$$i \gamma^5 \hat{D} \Phi_1 + \frac{M}{\lambda_1} \Phi_1 - \frac{4i}{3M} \; \frac{(b_3 - \lambda_1)}{(\lambda_2 - \lambda_3)(\lambda_3 - \lambda_1)(\lambda_1 - \lambda_2)} \; e \Sigma \; F = 0 \, ,$$
$$i \gamma^5 \hat{D} \Phi_2 + \frac{M}{\lambda_2} \Phi_2 - \frac{4i}{3M} \; \frac{(b_3 - \lambda_2)}{(\lambda_2 - \lambda_3)(\lambda_3 - \lambda_1)(\lambda_1 - \lambda_2)} \; e \Sigma \; F = 0 \, , \qquad \text{(VIII.19)}$$
$$i \gamma^5 \hat{D} \Phi_3 + \frac{M}{\lambda_3} \Phi_3 - \frac{4i}{3M} \; \frac{(b_3 - \lambda_3)}{(\lambda_2 - \lambda_3)(\lambda_3 - \lambda_1)(\lambda_1 - \lambda_2)} \; e \Sigma \; F = 0 \, ,$$

where

$$F(x) = \Phi_1 L_1 + \Phi_2 L_2 + \Phi_3 L_3 =$$
$$= \Phi_1 \, (\lambda_2 - \lambda_3) \left[\, f b_2^* \, b_2 + \lambda_2 \, \lambda_3 - (\lambda_2 + \lambda_3) b_3 + {b_3}^2 + g b_4 \, b_4^* \, \right] +$$
$$+ \Phi_2 \, (\lambda_3 - \lambda_1) \left[\, f b_2^* \, b_2 + \lambda_3 \lambda_1 - (\lambda_3 + \lambda_1) \, b_3 + {b_3}^2 + g b_4 \, b_4^* \, \right] +$$
$$+ \Phi_3 \, (\lambda_1 - \lambda_2) \left[\, f b_2^* \, b_2 + \lambda_1 \, \lambda_2 - (\lambda_1 + \lambda_2) b_3 + {b_3}^2 + g b_4 \, b_4^* \, \right] . \qquad \text{(VIII.20)}$$

Recall that $\hat{D} = \gamma_\mu D_\mu = \gamma_\mu (\partial_\mu - i e A_\mu), \; \Sigma = \frac{1}{4} (\gamma_\lambda \gamma_\rho - \gamma_\rho \gamma_\lambda) \, F_{\lambda \rho}$.

IX. EXTENSION TO GENERAL RELATIVITY

To generalize the model to requirements of General relativity, we should turn back to initial system (VII.1)–(VII.3):

$$b_1 \hat{D} (\gamma_\mu \Psi_\mu) - \frac{4 b_2}{\sqrt{6}} \left[(D_\mu \Psi_\mu) - \frac{1}{4} \hat{D} (\gamma_\mu \Psi_\mu) \right] - i \gamma_5 M (\gamma_\mu \Psi_\mu) = 0 \, , \qquad \text{(IX.1)}$$
$$b_3 \hat{D} \Psi_0 - i \frac{4 b_4}{\sqrt{6}} \left[(D_\mu \Psi_\mu) - \frac{1}{4} \hat{D} (\gamma_\mu \Psi_\mu) \right] \; - i \gamma_5 M \Psi_0 = 0 \, , \qquad \text{(IX.2)}$$
$$\frac{2}{\sqrt{6}} \left\{ b_5 \left[D_\lambda (\gamma_\mu \Psi_\mu) - \frac{1}{4} \gamma_\lambda \hat{D} (\gamma_\mu \Psi_\mu) \right] - \; - i b_6 \left[D_\lambda \Psi_0 - \frac{1}{4} \gamma_\lambda \hat{D} \Psi_0) \right] \right\} - i \gamma_5 M \left[\Psi_\lambda - \frac{1}{4} \gamma_\lambda (\gamma_\mu \Psi_\mu) \right] = 0 \, . \qquad \text{(IX.3)}$$

and make several changes..

1. Instead of *ict*-metric in Minkowski space now we are to apply the local metric tensor $g_{\alpha\beta}(x)$ (assuming the signature $(+, -, -, -)$); also we should make the formal change

$$M \longrightarrow i M \, . \qquad \text{(IX.4)}$$

2. Now the Dirac matrices in spinor basis are the following

$$\gamma^0 = \begin{vmatrix} 0 & I \\ I & 0 \end{vmatrix} , \quad \gamma^i = \begin{vmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{vmatrix} . \qquad \text{(IX.5)}$$

3. Derivative operators are modified in accordance with the rule

$$D_\alpha(x) = \nabla_\alpha + \Gamma_\alpha(x) - ieA_\alpha(x)\,, \quad \hat{D} = \gamma^\alpha(x) D_\alpha(x)\,, \tag{IX.6}$$

where $\Gamma_\alpha(x)$ stands for bispinor connection, and local Dirac matrices are defined with the use of tetrads: $\gamma^\alpha(x) = \gamma^a e^\alpha_{(a)}(x)$.

4. The following commutation rules are valid

$$\gamma^\rho(x) D_\beta = D_\beta \gamma^\rho(x)\,, \quad \hat{D}\hat{D} = \Box - ieF_{\alpha\beta}\sigma^{\alpha\beta}(x) + \frac{R}{4}\,, \quad \Box = D^\alpha D_\alpha, \tag{IX.7}$$

$R(x)$ is the Rici scalar.

5. Note the properties of covariant matrix $\gamma^5(x)$:

$$\gamma^5(x) = \frac{i}{4!}\epsilon_{\alpha\beta\rho\sigma}(x)\gamma^\alpha(x)\gamma^\beta(x)\gamma^\rho(x)\gamma^\sigma(x)\,, \quad \epsilon^{\alpha\beta\rho\sigma}(x) = \epsilon^{abcd}\, e^\alpha_{(a)}(x)\, e^\beta_{(b)}(x)\, e^\rho_{(c)}(x)\, e^\sigma_{(d)}(x), \quad \epsilon_{0123} = -1\,. \tag{IX.8}$$

Levy-Civita symbol $\epsilon^{\alpha\beta\rho\sigma}(x)$ varies at tetrad transformation according to the rule

$$\epsilon'^{\alpha\beta\rho\sigma}(x) = -\det[L_{ai}(x)]\, \epsilon^{\alpha\beta\rho\sigma}(x)\,. \tag{IX.9}$$

In particular, at tetrad P-reflection it behaves as a pseuvdoscalar: $\epsilon^{(p)\alpha\beta\rho\sigma}(x) = (-1)\,\epsilon^{\alpha\beta\rho\sigma}(x)$. We readily can prove the identity

$$\gamma^5(x) = \frac{i}{4!}\epsilon^{abcd} e_{\alpha(a)} e_{\beta(b)} e_{\rho(c)} e_{\sigma(d)}\; e^\alpha_{(m)}\gamma^m e^\beta_{(n)}\gamma^n e^\rho_{(k)}\gamma^k e^\sigma_{(l)}\gamma^l) = \frac{i}{4!}\epsilon_{mnkl}\gamma^m\gamma^n\gamma^k\gamma^l = \gamma^5\,. \tag{IX.10}$$

The analysis performed above is valid in this more general case only with small formal modifications. Taking in mind the formal changes:

$$M \Rightarrow iM, \quad \bar{\Phi}_1 = \gamma^\mu \Psi_\mu\,, \quad \bar{\Phi}_2 = \Psi_0\,, \quad \bar{\Phi}_3 = \gamma_5 D_\mu \Psi^\mu\,; \tag{IX.11}$$

$$\begin{aligned} Z_1 &= -ia_1 \frac{4}{3}\frac{b_2}{iM} + a_2 \frac{4}{3}\frac{b_4}{iM} - a_3 \frac{2}{\sqrt{6}}\,, \\ Z_2 &= -ic_1 \frac{4}{3}\frac{b_2}{iM} + c_2 \frac{4}{3}\frac{b_4}{iM} - c_3 \frac{2}{\sqrt{6}}\,, \\ Z_3 &= -ir_1 \frac{4}{3}\frac{b_2}{iM} + r_2 \frac{4}{3}\frac{b_4}{iM} - r_3 \frac{2}{\sqrt{6}}\,; \end{aligned} \tag{IX.12}$$

$$e\Sigma = eF_{\mu\nu}\sigma^{\mu\nu} \Longrightarrow (eF_{\mu\nu}\sigma^{\mu\nu} + i\frac{R(x)}{4})\,. \tag{IX.13}$$

instead the system (VII.31) we obtain

$$\begin{aligned} &\lambda_1 i\gamma^5 \hat{D}\Phi_1 + iM\Phi_1 + Z_1\,(eF_{\mu\nu}\sigma^{\mu\nu} + i\frac{R}{4})\,[\, fb_2^*\,\bar{\Phi}_1 - igb_4^*\,\bar{\Phi}_2\,] = 0\,, \\ &\lambda_2 i\gamma^5 \hat{D}\Phi_2 + iM\Phi_2 + Z_1\,(eF_{\mu\nu}\sigma^{\mu\nu} + i\frac{R}{4})\,[\, fb_2^*\,\bar{\Phi}_1 - igb_4^*\,\bar{\Phi}_2\,] = 0\,, \\ &\lambda_3 i\gamma^5 \hat{D}\Phi_3 + iM\Phi_3 + Z_3\,(eF_{\mu\nu}\sigma^{\mu\nu} + i\frac{R}{4})\,[\, fb_2^*\,\bar{\Phi}_1 - igb_4^*\,\bar{\Phi}_2\,] = 0\,; \end{aligned} \tag{IX.14}$$

and further arrive at (compare with (VIII.19))

$$\begin{aligned} &i\gamma^5\hat{D}\Phi_1 + i\frac{M}{\lambda_1}\Phi_1 - \frac{4}{3M}\,\frac{(b_3-\lambda_1)}{(\lambda_2-\lambda_3)(\lambda_3-\lambda_1)(\lambda_1-\lambda_2)}\,(eF_{\mu\nu}\sigma^{\mu\nu} + i\frac{R}{4})\,F = 0\,, \\ &i\gamma^5\hat{D}\Phi_2 + i\frac{M}{\lambda_2}\Phi_2 - \frac{4}{3M}\,\frac{(b_3-\lambda_2)}{(\lambda_2-\lambda_3)(\lambda_3-\lambda_1)(\lambda_1-\lambda_2)}\,(eF_{\mu\nu}\sigma^{\mu\nu} + i\frac{R}{4})\,F = 0\,, \\ &i\gamma^5\hat{D}\Phi_3 + i\frac{M}{\lambda_2}\Phi_3 - \frac{4}{3M}\,\frac{(b_3-\lambda_3)}{(\lambda_2-\lambda_3)(\lambda_3-\lambda_1)(\lambda_1-\lambda_2)}\,(eF_{\mu\nu}\sigma^{\mu\nu} + i\frac{R}{4})\,F = 0\,, \end{aligned} \tag{IX.15}$$

where

$$
\begin{aligned}
F(x) = \Phi_1 L_1 + \Phi_2 L_2 + \Phi_3 L_3 = \\
= \Phi_1 (\lambda_2 - \lambda_3) \left[f b_2^* b_2 + \lambda_2 \lambda_3 - (\lambda_2 + \lambda_3) b_3 + {b_3}^2 + g b_4 b_4^* \right] + \\
+ \Phi_2 (\lambda_3 - \lambda_1) \left[f b_2^* b_2 + \lambda_3 \lambda_1 - (\lambda_3 + \lambda_1) b_3 + {b_3}^2 + g b_4 b_4^* \right] + \\
+ \Phi_3 (\lambda_1 - \lambda_2) \left[f b_2^* b_2 + \lambda_1 \lambda_2 - (\lambda_1 + \lambda_2) b_3 + {b_3}^2 + g b_4 b_4^* \right] .
\end{aligned}
\qquad \text{(IX.16)}
$$

X. CONCLUSIONS

The theory of a P-asymmetric spin 1/2 fermion with three mass parameters is developed. It is based on 20-component representation of the Lorentz group. Starting with the Gel'fand - Yaglom approach, we derive a spin-tensor form of a generalized wave equation. In absence of external electromagnetic fields, the system of equation can be transformed to the form of three separate P-asymmetric Dirac-like equations for bispinor wave functions with different masses. The values of masses values vary within some freedom, determined by parameters of the model. In presence of external electromagnetic fields, three bispinors turn out to be linked into the unified system of equations. If the scalar curvature of the space differs from zero, then between three bispinor components there arises additional terms of geometrical interaction. Extension of the theory to General relativity is performed, at this there arises additional interaction term through Ricci scalar function. The model for *P*-asymmetric fermion fermion with three mass parameters allows for restriction to Majorana case.

The next step should be the development of a model accounting for both P-symmetric and P-asymmetric.

---

[1] Gel'fand, I.M. Representations of the rotation group and the Lorentz group, their applications / I.M. Gel'fand, R.A. Minlos, Ya.Ya. Shapito. – Moscow: Fizmatgiz, 1958. – 368 p.

[2] Gel'fand, I.M. General relativistically invariant equations and infinitly dimensional representations of the Lorentz group / I.M. Gel'fand, A.M. Yaglom // JETP. – 1948. – Vol. 18, 8. – P. 703–733.

[3] Fedorov, F.I. Wave equations with repeated representations of the Lorentz group. Half-integer spin / F.I. Fedorov, V.A. Pletyukhov // Vesti AN BSSR. Ser. Phys.-Mat. – 1970. – no 3. – P. 78–83.

[4] Pletyukhov, V.A.Relativistic wave equatiuons and intrincis degrees of freedom / V.A. Pletyukhov, V.M. Red'kov, V.I. Strazhev. – Belaruskaya navuka: Minsk, 2015. – 328 p. (in Russian) , . .

[5] V.V. Kisel, V.A. Pletyukhov, V.V. Gilewsky, E.M. Ovsiyuk, O.V. Veko, V.M. Red'kov. Spin 1/2 particle with two mass states. interaction with external fields // Nonlinear Dynamics and Applications. 2017. Vol. 23. – P. 210–230.

[6] V.V. Kisel, V.A. Pletyukhov, V.V. Gilewsky, E.M. Ovsiyuk, O.V. Veko, V.M. Red'kov. Spin 1/2 particle with two mass states, interaction with external fields. Nonlinear Phenomena in Complex Systems, 2017. Vol. 20, no. 4. P. pp. 404–423.

[7] E.M. Ovsiyuk, O.V. Veko, Ya.A. Voynova, V.V. Kisel, V. Balan, V.M. Red'kov. Spin 1/2 particle with two masses in magnetic field // Applied Sciences. – 2018. – Vol. 20. – P. 148–166.

[8] Elementary Particles with Internal Structure in External Fields. Vol I. General Theory / V.V. Kisel, E.M. Ovsiyuk, O.V. Veko, Y.A. Voynova, V. Balan, V.M. Red'kov. - New York: Nova Science Publishers Inc., 2018. - 404 pages

[9] Elementary Particles with Internal Structure in External Fields. Vol II. Physical Problems / V.V. Kisel, E.M. Ovsiyuk, O.V. Veko, Y.A. Voynova, V. Balan, V.M. Red'kov. - New York: Nova Science Publishers Inc., 2018. - 402 pages.

[10] Fermion with Three Mass Parameters: General Theory, Interaction with External Fields / V.V. Kisel, V.A. Pletjukhov, E.M. Ovsiyuk, Y.A. Voynova, O.V. Veko, V.M. Red'kov // Nonlinesr Dynamics and Applications. – 2018. – Vol. 24. – P. 141–164.

[11] Fermion with three mass parameters: interaction with external fields / V.V. Kisel, V.A. Pletjukhov, E.M. Ovsiyuk, Y.A. Voynova, O.V. Veko, V.M. Red'kov // Doklady NAN Belarusi. – – 2018. – Vol. 62, 6. – P. 661–667.

[12] Fermion with three mass parameters: general theory, interaction with external fields / V.A. Pletjukhov, V.V. Kisel, E.M. Ovsiyuk, Y.A. Voynova, O.V. Veko, V.M. Red'kov // Uchenye Zapiski Brest State University named after A.S. Pushkin. Natural sciences. – 2018. – Vol. 14, Part 2. – P. 16–50.

[13] On P-noninvariant wave equation for a spin 1/2 particle with anomalous magnetic moment / V.V. Kisel, V.A. Pletyukhov, E.M. Ovsiyuk, V.M. Red'kov // NPCS. – 2019. – Vol. 22, no. 1 – P. 18–40

[14] On modeling neutrinos oscillations by geometry methods in the frames of the theory for a fermion with three mass parameters E.M. Ovsiyuk, Ya.A. Voynova, V.V. Kisel, V.A. Pletyukhov, V.V. Gilewsky, V.M. Red'kov // J. Phys.: Conf. Ser. – 2019. – **1416**. – Paper 012040 [9 pages].

[15] V.A. Pletyukhov, I.V. Kapitsa, V.V. Kisel, V.M. Red'kov P-noninvariant relativistic wave equation for a spin 1/2 particle with three mass parameters // Vestnik of the Brest State University. Ser. 4. Physica and Mathematics. – 2019. – no 2. – P. 35–43.