ONE VERSION OF THE GROUP RESOLUTION PRINCIPLE FOR DISCRETE OPTIMIZATION

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One modification of the group resolution principle (grp) to find a minimum-weighted covering set of 0,1-matrix is presented. The modification makes it possible to use grp in the same way both for non-weighted case of a minimum-size covering problem and for the weighted case. This enables one to use the single algorithm for the generalized covering problem. It is also shown how to modify the existing approach to avoid matrix sizes growing when adding new group resolvents at the iterations of the algorithm. The suggested technique is characterized by quite good computational complexity estimation showing the polynomial complexity of the method. Summarizing all this up, one can conclude that the method may find use for different optimization problems widely presented in data and text mining and the other areas.

Introduction

One of the interesting applied problems is finding a minimum-weighted covering set of the 0,1-matrix. This problem is widely presented in the different applied areas using data mining, text mining and the other optimization techniques. It may be used to solve logical inference problems, inclusing that one known as satisfiability of the given system of disjuncts. Moreover, grp usage can be extended to the first order logic of predicates basing on the same ideas as outlined below. In [1,2], were described the theoretical backgrounds of the group resolution principle and its use for solving a 0,1-matrix covering problem with minimum-size (minimum-weighted) set of rows. Comparing the techniques, used for these problems, one can conclude that they are similar, though not identical. The difference lies in the way of resolvents making. A resolvent represents a new and unique 0,1-column generated in a special way and added to the current 0,1-matrix. It is proved that sooner or later the totally zero resolvent-column would be generated what points to the finishing of the searching procedure. The best covering set found till this moment defines the solution to the initial problem. In this paper, we show two main results. The first result consists in applying grp formulation to the weighted case of the 0,1-matrix covering problem, while the second results gives a possibility to limit the number of the column-resolvents added at the iterations of the algorithm. The necessary complexity estimations show the polynomial efficiency in average. The rest of this short report contains the details.We avoid considering illustrations for the interests of brevity and paper sizes restrictions. Instead, the narration was made as clear as possible.

I. Explanation of the grp

Let us remind the basic principles of grp [1]. The grp-based method uses a 0,1-matrix B with n >0 nonempty rows and m > 0 nonempty columns. One says that row i covers column j if i contains in j a unit. It is required to find a minimum-size set CV of rows such that each column of the initial matrix B is covered by at least one row from CV. The grp-based algorithm performs a finite number of iterations. At each iteration, the algorithm uses some heuristic to find a current covering set P. We use a kind of a greedy algorithm to find P. This greedy algorithm seeks the undeleted column c with minimum number of units and then selects an undeleted row r which covers c and contains maximum number of units among all the rows covering c. The column c is called a syndromic for the covering set P. This syndromic columns corresponds to the row r and the row r is included in P. Then all the columns of the current matrix B, covered by the row r, are (temporarily) being deleted. Also, all the rows covering the syndromic column c (including r) are being deleted as well. The iteration then continues to complete building the covering set P. Evidently, this process stops when there would be no undeleted columns in matrix B. By this, the current covering set P, consisting of the rows $r_1, r_2, \dots, r_z$, is found. Then the group resolvent is being formed accordingy to grp. To form group resolvent, one should build a new matrix on the syndromic columns corresponding to the rows $r_1, r_2, \dots, r_z$. The matrix built on syndromic columns is called a syndromic matrix. Let us designate it by Sdr_P. It is used to generate a new unique column called a group resolvent res. The rule to generate res is the following one: res contains unit in the row i if and only if row i has two or more units in syndromic matrix Sdr_P. Otherwise, res contains zero in row i. Then, all temporarily deleted rows and columns of the matrix B are restored and res is added to B. Provided, that res contains at least one unit, a new iteration starts. The total finiteness of the computational process directly follows from the uniqueness of res what is proved in [1]. As for weighted case of the covering problem, the grp formulation is somewhat different. To form a syndromic matrix at the current iteration, one seeks for a current covering set P as explained before. However, the group resolvent is being formed in a different way.For the weighted case of the covering problem, each row is assigned an integer non-negative weight w_i. The final solution should deliver a minimum value to $\sum w_i$ where index i defines the row number from the optimal covering set. Let as previously, Sdr_P stand for the syndromic matrix found at the curent iteration for the covering set P. Let w(P) stand for the weight of the covering set P . Divide all the rows into two subsets: S_1 and S_2. Subset S_1 consists of the rows which contain in Sdr_P no more than one unit.Subset S_2 consists of the other rows (with more than one unit in each of them). For each column c_j in Sdr_P define a value of v_j corresponding to the minimal weight of the row from S_1 which covers c_j. Define the current low boundary Low_P of P the value $Low_P = \sum v_j$. Then, if Low_P < w(P) one needs to move some row(s) from S_1 to S_2 to provide $Low_P \geqslant$w(P) and form a group resolvent with units in the rows from S_2. Otherwise, provided S_2 is empty, the process terminates with the answer, corresponding to the best covering set found. The details may be found in [2]. Now we show how to unify the Grp formulations for the weighted and non-weighted cases of the covering problem.

II. GrP unification

The unified version of grp is obtained as follows. At each iteration, one seeks for the column with minimum number of units (this is a syndromic column c as earlier) and selects a row which covers it and has minimal weight among all the rows covering c. To be applicable to the non-weighted case of the problem, the conditional weights are introduced computed as $1/n_i$, where n_i stands for the number of units in the row i. So, the more units contains the row, the less is its weight. Now, let syndromic matrix Sdr_P be defined corresponding to the current covering set P . Evidently, there is no need to define the sets S_1 and S_2. One can directly use the grp, formulated for the non-weighted case of the problem: the rule to generate *res* is the following one – *res* contains unit in the row i if and only if row i has two or more units in syndromic matrix Sdr_P . Otherwise, *res* contains zero in row i. We need to prove correctness of this rule. It should be clear that each column j in Sdr_P has value of v_j corresponding to the row with minimal weight from those covering column j due to heuristic used to form curennt covering set P. By this, $Low_P \geq$w(P) and it is impossible to improve te solution, provided there are no rows with two or more units in Sdr_P . This formulation, evidently, does not require to move some rows from S_1 to S_2 and saves time to make the algorithm faster than in [2]. This, moreover, is not the single possible enhancement. The second one is connected to restriction on the sizes of the 0,1-matrix B which are permanently gtowing in [1,2]. Consider, how one can do this. The idea is to use the previously added column-resolvents in order to overwrite them with new resolvents [3]. The common rule is as follows: if at the current iteration some previous group resolvent was not included into Sdr_P then it may be overwritten by a new generated resolvent without loss of solution. Provide the following reasoning. Let a cover P_i was found at iteration i by sequantial including rows $r_1, r_2, ..., r_z$. Supose that new iteration i + 1 entirely repeats the previous iteration i. This means that the same syndromic columns and the same rows $r_1, r_2, ..., r_z$ are selected in the same order including some additional new row(s). At the moment of including row r_i into P_{i+1} (r_z is the last one in P_i) matrix B cannot be entirely destroyed, otherwise one gets $P_i = P_{i+1}$, which is impossible according to grp theoretical properties [1,2]. This means that at least one column c should remain undeleted and c is not covered by any one of rows $r_1, r_2, ..., r_z$. But column c must be at this moment totally zero as all the rows having «1sc in c will be deleted (because the same syndromic columns are selected for rows $r_1, r_2, ..., r_z$ at the iterations i and $i + 1$). Evidently, this is impossible and enables one to come to one of the next conclusions:either P_{i+1} has less than z rows or at one of the steps 1,2, . . . , z when forming cover P_{i+1} in the selected syndromic column there would be smaller amount of units in comparison to the syndromic column selected at the same step while forming cover P_i. The last remark concerns computation complexity of the method. Clearly, as the suggested technique is based on grp for non-weighted case of the minimum-size covering problem, the estimations should preserve the same order. According to [1], grp converges to a solution approximately for a number of steps estimated as $O(nmp/(1 \ \overline{p}))$ where p denotes density of units in matrix B, n stands for the number of rows, and m stands for the number of columns. The expressions points to polynomial complexity for the value of p not close to 1 or 0. In conclusion, let us note that the unified version of grp is more natural for programming aims, clear for understanding and may be successfully used in the university courses dealing with discrete optimization.

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