FLAT PROBLEM OF ALLOWANCE DISTRIBUTION AS DUAL CONTROL PROBLEM

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The problem of the allowance distribution is described. This problem is formulated as a dual control problem.

Introduction

The allowance is the layer of the material that is removed from the workpiece (billet) in the machining or finishing. The example of the flat workpiece (solid line) and the template (dotted line) is showed in Fig. 1. The difference between these two curves forms the allowance that should be cut from this workpiece. The allowance is shaded in Fig. 1. The allowance distribution is the process of the placing of the template on the workpiece in the best possible way [1]. If the allowance is too small, then it is possible to spoil the template in the machining. If the allowance is too big, then the big part of material will go in chips, and the cost of processing will increase . Therefore, the production is always trying to make the allowance as uniform as possible.The allowance distribution problem is a complex problem, the various aspects of which are considered in works [2–5].

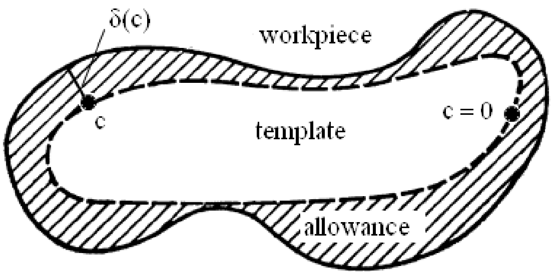

Рис. 1 – The workpiece and the template in the problem of the allowance distribution

I. Mathematical model of the controlled object

In this report, we consider the flat problem of the allowance distribution. The value of the allowance is measured by the normal to the curve characterizing the shape of the template. Then the allowance is some function of the position: $\delta = \delta(c)$, where c is the curvilinear coordinate directed along the contour of the template, $0 \leq c \leq c_0$, c_0 is the perimeter of the contour (Fig. 1). The function δ depends on the position of the template's contour on the workpiece. The position of the template's contour is characterized in the flat case by three coordinates. They may be, for instant, the abscissa x, ordinate y of the mass center and the rotation angle ϕ relative the axis x (or basis direction): $\delta = \delta(c, x, y, \phi)$.

The non-uniformity of the allowance could be defined, for instance, as follows [1]:

$$Q = Q(x, y, \phi) = \max_c \delta(c, x, y, \phi) - \min_c \delta(c, x, y, \phi), \quad (1)$$

i.e. as the difference between the maximum and minimum values of the allowance (Fig. 2).

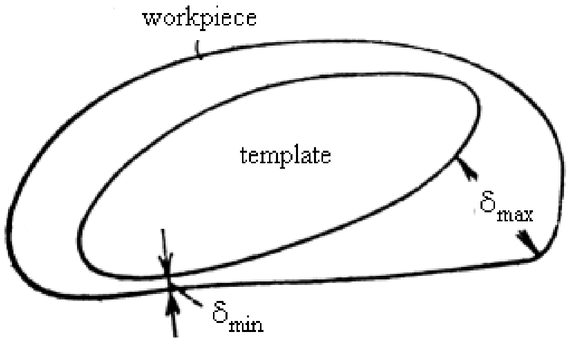

Рис. 2 – The relation between the workpiece and the template in the problem of the allowance distribution, $\delta_{max} = \max_c \delta(c, x, y, \phi)$, $\delta_{min} = \min_c \delta(c, x, y, \phi)$

The problem of the optimal allowance distribution can be formulated as follow: it is needed to minimize the function $Q(x, y, \phi)$ (1) on the variables x, y, ϕ:

$$Q(x, y, \phi) \to \min_{x,y,\phi} .$$

The function (1) is unknown beforehand and could be estimated on the base of measurements. Measurements could be organized as follows. The values of the allowance $\delta = \delta(c, x, y, \phi)$ are determined practically in the m_t points $p_1, p_2, ..., p_{m_t}$ on the contour of the template by the sensors. As a results, they receive the values $\delta_i = \delta(p_i, x, y, \phi)$, $i = 1, 2, ..., m_t$, when x, y, ϕ are fixed.

Thus, the allowance distribution object is described practically by the following function:

$$Q = Q(x, y, \phi) = max(\delta_1, ..., \delta_{m_t}) - min(\delta_1, ..., \delta_{m_t}). \quad (2)$$

The measurements $\delta_1, ..., \delta_{m_t}$ give one value of the output variable Q.

The contour of the workpiece is defined practically also by its points $r_1, ..., r_{m_w}$. The measurements Q under fixed x, y, ϕ contain errors due to the point-defined outlines of the workpiece and the template and are random. This means that the object with description (2) is a regression object and is unknown beforehand.

II. Dual control of the allowance distribution

The allowance distribution problem can be considered as the problem of dual control of the regression object. The dual control is recommended in automatic search systems and in particular automatic optimization systems [6]. The dual control allows to store the information about the object in the control process. The control actions must have a reciprocal, "dual" character: to a certain extent they must be of a probing nature, but also controlling to a known degree.

Fig. 3 shows the block diagram of the proposed closed-loop automatic control system for the optimal (dual) allowance distribution. We will suppose that the system depicted in Fig. 3 is discrete-continuous. It means that all the variables in the diagram are considered only at the discrete time instants $s = 0, 1, 2, ..., n$. Any of the variables at the time instant s is provided with a subscript s, for example, Q_s, $u_s = (x_s, y_s, \phi_s)$. We have the measurements $Q_s = Q(x_s, y_s, \phi_s)$, $s = 1, 2, ..., n$,, where n is the amount of the time instants (the volume of the sample) , Q_s, $u_s = (x_s, y_s, \phi_s)$ are the output and input variables of the controlled object respectively.

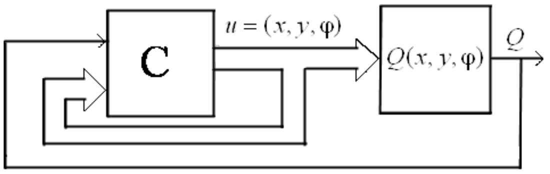

Рис. 3 – The block diagram of the closed-loop automatic control system for the optimal (dual) allowance distribution

Let $f_{Q_s}(q_s/x_s, y_s, \phi_s, \Theta) = f_{Q_s}(q_s/u_s, \Theta)$ be the known conditional probability density function of the random variable Q_s, where Θ is the set of the unknown parameters of the probability density function. We will suppose that the parameters Θ are random and their probability density function $f_\Theta(\theta)$ is known. The control vector u_s is restricted by the condition $u_s \in U$, where U is a certain admissible region, for example, $U = \{u_s : u_s \geq 0, \psi_j(u_s) \leq 0, s = 0, 1, ..., n, j = 1, 2, ..., l\}$, and $\psi_j(u_s)$ are some scalar functions of the vector u_s. It is assumed, that the controller C utilizes past information in the form of the measurements $\vec{u}_{s-1} = (u_0, u_1, ..., u_{s-1})$, $\vec{Q}_{s-1} = (Q_0, Q_1, ..., Q_{s-1})$ to form the control action u_s at the time instant s. It is assumed, that the controller C has a random behavior and is described at the time instant s by the conditional probability density function $\Gamma_s = f(u_s/\vec{u}_{s-1}, \vec{Q}_{s-1})$ called the strategy of the controller. The quality of the functioning of the system at the time instant s is evaluated by the loss function $W_s(Q_s)$. Mathematical expectation of the loss function $R_s = E\{W_s(Q_s)\}$ is called mean specific risk. The sum of the mean specific risks on the $n + 1$ time instants

$$R = E\{\sum_{s=0}^{n} W_s\} = \sum_{s=0}^{n} E\{W_s\} = \sum_{s=0}^{n} R_s \quad (3)$$

is called the mean total risk. The system will be optimal when the mean total risk R will be minimal. The problem consists of determining the sequence of the strategies of the controller $\Gamma_s = f(u_s/\vec{u}_{s-1}, \vec{Q}_{s-1}), s = 0, 1, 2, ..., n$, minimizing the mean total risk R (3) .

The strategies of the controller $\Gamma_s = f(u_s/\vec{u}_{s-1}, \vec{Q}_{s-1})$ minimizing the mean total risk R (3) are not randomized, i.e. $\Gamma_s = \delta(u_s - u_s^*)$, δ is the unit impulse function, u_s^* is optimal control action [6]. The sequence of the control actions $u_n^*, u_{n-1}^*, ..., u_0^*$ is defined from the functional equations which can be found in the work [7].

We will suppose also that the probability density function $f_{Q_s}(q_s/u_s, \Theta)$ is normal $N(\psi(u_s, \Theta), \sigma^2)$, where $\psi(u_s, \Theta)$ is the regression function and σ^2 is constant. We propose to use the quadratic regression function

$$\psi(u_s, \Theta) = \theta_0 + {}^{0,1}(\theta_1 u_s) + {}^{0,2}(\theta_2 u_s^2),\ \Theta = \{\theta_0, \theta_1, \theta_2\},$$

where θ_0 is constant, θ_k is the k-dimensional matrix of the order 3, ${}^{0,k}(\theta_k u_s^k)$, $k = 1, 2$, is the $(0, k)$-folded product of the multidimensional matrices [8]. The loss function is proposed to be $W_s = Q_s$.

III. References

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