DEDUCTIVE MACHINE FOR MODAL LOGIC IN A MODULE OF A VIRTUAL TEACHER

Juan Zhou, O.V. German

Belarusian State University of Informatics and Radioelectronics, Minsk, Belarus,
zhoujuan_minck@yahoo.com

Abstract. The paper presents modal logic, deductive machine, application of modal logic in decision support and distance learning.

Interesting problems for the creation of interactive distance learning is the creation of a virtual teacher. Such a teacher should respond to signals from the student, taking into account the fact that the teacher does not know exactly beforehand what condition the student is on in fact. Therefore, when developing the model of “student teachers” we use formulas manipulating uncertainty. In this sense, modal logic allows us to describe estimated situations in terms of possibilities and necessities. Here is an example.

Rule 1: If a student sequentially looks through the pages of an electronic textbook, it is possible that he studies.

Rule 2: If a student chaotically looks through the page, it must be that he does not study.

Rule 3: If a student does not study a text, he should be given some leading questions.

Thus, when constructing a virtual teacher, we need to create a model of knowledge about the behavior of student and teacher based on modal logic and make a decision, deduced from the knowledge base using the deductive machine [3].

Modal Logic.

Narrowly constructed, modal logic studies reasoning that involves the use of the expressions ‘necessarily’ and ‘possibly’. However, the term ‘modal logic’ is used more broadly to cover a family of logics with similar rules and a variety of different symbols.

The most familiar logics in the modal family are constructed from a weak logic called K (after Saul Kripke). Under the narrow reading, modal logic concerns necessity and possibility. A variety of different systems may be developed for such logics using **K** as a foundation.

The symbols of **K** include ‘~’ for ‘not’, ‘→’ for ‘if…then’, and ‘□’ for the modal operator ‘it is necessary that’. (The connectives ‘&’, ‘∨’, and ‘↔’ may be defined from ‘~’ and ‘→’ as is done in propositional logic.) **K** results from adding the following to the principles of propositional logic.

Necessitation Rule: If A is a theorem of **K**, then so is $\Box A$.

Distribution Axiom: $\Box(A\to B) \to (\Box A\to\Box B)$.

(In these principles we use ‘A’ and ‘B’ as metavariables ranging over formulas of the language.) According to the Necessitation Rule, any theorem of logic is necessary. The Distribution Axiom says that if it is necessary that if A then B, then if necessarily A then necessarily B.

The operator ◊ (for ‘possibly’) can be defined from □ by letting $\Diamond A = \sim\Box\sim A$. In **K**, the operators □ and ◊ behave very much like the quantifiers ∀ (all) and ∃ (some). For example, the definition of ◊ from □ mirrors the equivalence of $\forall xA$ with $\sim\exists x\sim A$ in predicate logic. Furthermore, $\Box(A\&B)$ entails $\Box A\&\Box B$ and vice versa; while $\Box A\vee\Box B$ entails $\Box(A\vee B)$, but not vice versa. Similar parallels between ◊ and ∃ can be drawn. [1].

Deductive (Logical) Inference.

Inferences are made when a person (or machine) goes beyond available evidence to form a conclusion. With a deductive inference, this conclusion always follows the stated premises. In other words, if the premises are true, then the conclusion is valid. Studies of human efficiency in deductive inference involves conditional reasoning problems which follow the "if A, then B" format.

The task of making deductions consists of three stages. First, a person must understand the meaning of the premises. Next they must be able to formulate a valid conclusion. Thirdly, a person should evaluate their conclusion to tests its validity. Although deductive inference is easy to test or model, the results of this type of inference never increase the semantic information above what is already stated in the premises [2].

Application of modal logic in decision support.

Modal logic can be used in decision support. It is convenient to interpret the values of modal formulas, with the position of several experts in expert systems. For example, there are two experts in the system of Resher and more than two experts in the system of Kripke, then □φ means that φ is true in each expert's knowledge system, and ◊ φ means that φ is true at least in an expert's knowledge system. It should be kept in mind that the formulas with modalities (□, ◊) represent meta-knowledge, i.e. knowledge about knowledge, so we can say that $□_S φ$ is true, if φ is true, and it is only truth in the system S.

Suppose that

$$S = \begin{cases} x_1 \vee x_2 \vee \sim x_3 \\ \sim x_1 \vee x_2 \\ \sim x_2 \vee \sim x_3 \end{cases}, \quad (1)$$

then $\square\sim x_3$ is a true formula, since every interpretation for S contains $\sim x_3$ and does not contain x_3. On the other hand, a meta-formula $\lozenge x_1$ is true, since there is an interpretation for S with x_1 and an interpretation for S with $\sim x_1$. Thus, it should be kept in mind that the system itself as well as the reasoning system with respect to S, which is constructed using the modalities of necessities and possibilities.

Basing on this we can associate meta-formulas with formulas of the system S as follows:

$$\begin{aligned} & x_1 \vee x_2 \vee \sim x_3 \equiv f_1 \\ & \sim x_1 \vee x_2 \equiv f_2 \\ & \sim x_2 \vee \sim x_3 \equiv f_3 \end{aligned}, \quad (2)$$

then, for example,

$$\square x_1 \equiv \begin{cases} f_1 \vee f_2 \vee f_3 \to x_1 \\ x_1 \to f_1 \vee f_2 \vee f_3 \end{cases}.$$

To write $\square x$, we use the formula of Shannon:

$$F(x_1, x_2, \cdots x_n) = x_1 \cdot F(1, x_2, \cdots x_n) \vee \sim x_1 \cdot F(0, x_2, \cdots x_n), \quad (3)$$

then, for example,

$$\Diamond x_1 \equiv x_1 \cdot F(1, x_2, \cdots, x_n) \equiv x_1 \cdot f_1(1, x_2, x_3) \cdot f_2(1, x_2) \cdot f_3(1, x_2, x_3) \equiv$$
$$\equiv x_1 \cdot (1 \vee x_2 \vee \sim x_3) \cdot x_2 \cdot (\sim x_2 \vee \sim x_3) \equiv x_1 \cdot x_2 \cdot \sim x_3 . \quad (4)$$

We can build inference machine for modal logic based on the usual Boolean Machine, using the principle of Robinson's resolution. This machine allows us to check the deductibility of meta-formulas with respect to a given system S, or check the deductibility of meta-formulas with respect to S.

Return to the example and verify the validity of $\Diamond x_1$ for S (1). According to (4)

$$\Diamond x_1 \equiv x_1 \cdot x_2 \cdot \sim x_3$$

$$S = \begin{cases} x_1 \vee x_2 \vee \sim x_3 \\ \sim x_1 \vee x_2 \\ \sim x_2 \vee \sim x_3 \end{cases}, \quad (5)$$

It is easy to see that $\Diamond x_1$ occurs, i.e. $\Diamond x_1$ - is true.

Conclusion.

If we interpret modal logic as a meta-system, then it is possible to realize the inference machine on basis of binary logic, as we can construct corresponding binary equivalents for the formulas ◊ φ and □ φ.

References

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