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Кафедра теоретических основ электротехники

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КРАТКОЕ ИЗЛОЖЕНИЕ ДИСЦИПЛИНЫ «ТЕОРИЯ ЭЛЕКТРИЧЕСКИХ ЦЕПЕЙ». ПРИМЕРЫ РЕШЕНИЯ ЗАДАЧ

В двух частях
Часть 1

АНАЛИЗ ЦЕПЕЙ ПОСТОЯННОГО ТОКА

BRIEF ELECTRICAL CIRCUIT THEORY AND PRACTICAL PROBLEMS

In two parts
Part 1

DC ANALYSIS

Рекомендовано УМО по образованию в области информатики и радиоэлектроники в качестве пособия для специальностей 1-45 01 01 «Инфокоммуникационные технологии» (по направлениям), 1-98 01 02 «Защита информации в телекоммуникациях»

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ББК 31.211я73
Б28

Р е ц е н з е н т ы:

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Содержит основные теоретические положения по расчёту линейных электрических цепей постоянного тока, примеры расчёта цепей данного типа, а также задачи для самостоятельного решения.

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Content

PREFACE

The technical manual had been composed in accordance with electric circuit theory syllabus. The part 1 consists of the DC current basic concepts and theoretical information. The Ohm's law, Kirchhoff's current law and Kirchhoff's voltage law are considered. The Mesh (loop) current method and node voltage analysis are reviewed. As well as superposition and Thevenin's theorems are given attention.

For every topic the solutions of the tasks were included that should help clarify the approach and gain a better understanding. The part 1 includes further problems with answers at the end for independent solution.

The technical manual is intended for use by international students getting an education in English.

1. FUNDAMENTALS OF ELECTRIC CIRCUITS

1.1. An electrical circuit elements

A basic electric circuit includes an electrical energy source, a switch, a load and wires, that connect these elements (Fig. 1.1).

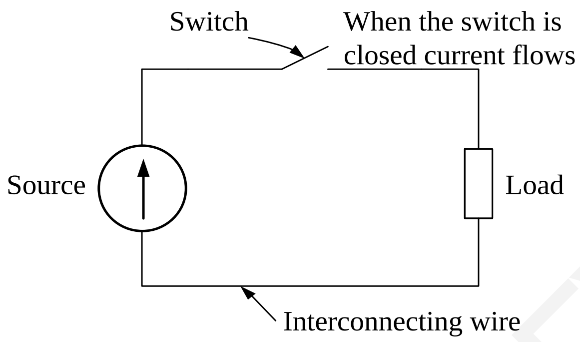


Fig. 1.1. A basic electric circuit

Circuit elements are distinguished to be either active or passive.

Active elements: a voltage or current source of energy. Active elements produce electric energy and supply with it the other elements in an electric circuit.

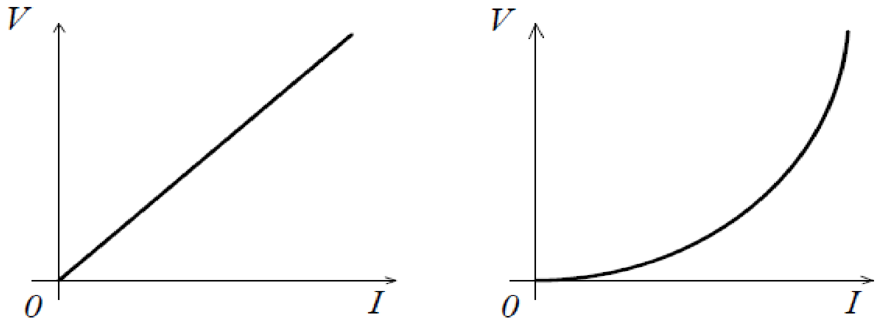
Passive elements: resistors, inductors and capacitors. Passive elements only transform electric energy to another type (light, heat, mechanical work and so on). Passive elements are categorized as linear or nonlinear:

– in linear elements current and voltage are connected by the linear equation;

– in nonlinear elements current and voltage are connected by the nonlinear equation.

The linear element has voltage – current characteristic which looks like a straight line (Fig. 1.2, a). The straight line indicates that current is directly proportional to the voltage across resistor. A resistor possessing such kind of the voltage – current characteristic is said to be a linear device. A linear resistor has constant value.

If the resistor R has the graph shown in Fig. 1.2, b, resistance R is a non-linear device. The non-linear device resistance depends on the current.

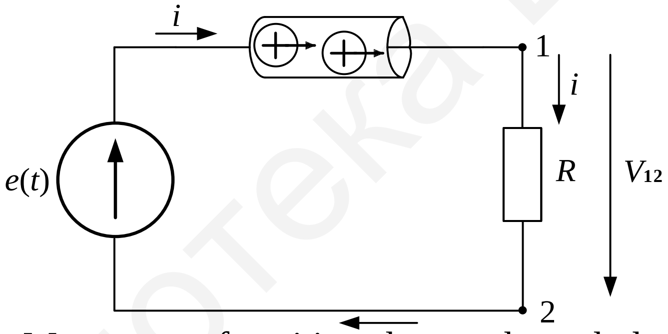


a b

Fig. 1.2. *VI*-characteristics:
a – linear *VI*-characteristic; b – non-linear *VI*-characteristic

1.2. Current and voltage

Movement of electrical charges is called an electrical current and symbolized by *I*. Fig. 1.3 illustrates the flow of positive charges through the wire.



Movement of positive charges through the wire

Fig. 1.3. The flow of positive charges

Charge is measured in coulombs. In the *SI* system one coulomb per second is defined as one ampere. One ampere is the current in an electrical circuit when one coulomb of charge passes a conductor section in one second:

$$I = \frac{Q}{t}, \tag{1.1}$$

where I – the current in ampere; Q – the charge in coulomb; t – the time interval in seconds.

Since one of the battery’s terminals is always positive and the other is always negative, current always directs in the same direction.

Such an unidirectional current is called dc or direct current, and the battery is called dc source.

Fig. 1.4 illustrates a passive part of an electrical circuit. Let arbitrarily indicated the positive direction of an electrical current using the arrow. The arrow shows the conventional positive direction.

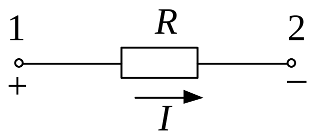


Fig. 1.4. The arrow shows the positive current direction

The positive current direction is the movement of positive charges that moved through the brunch from the positive terminal to the negative terminal, from the point of higher potential to point of less potential. As the positive current direction of the Fig. 1.4 was chosen from the point 1 to the point 2 therefore the potential of terminal 1 is greater than the potential of terminal 2.

Voltage is the potential difference. If the positive current direction (Fig. 1.5) is from terminal 1 to terminal 2 hence

$$V_{12} = \varphi_1 - \varphi_2 > 0. \tag{1.2}$$

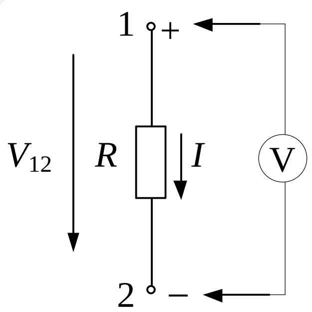


Fig. 1.5. The arrow shows the positive voltage direction

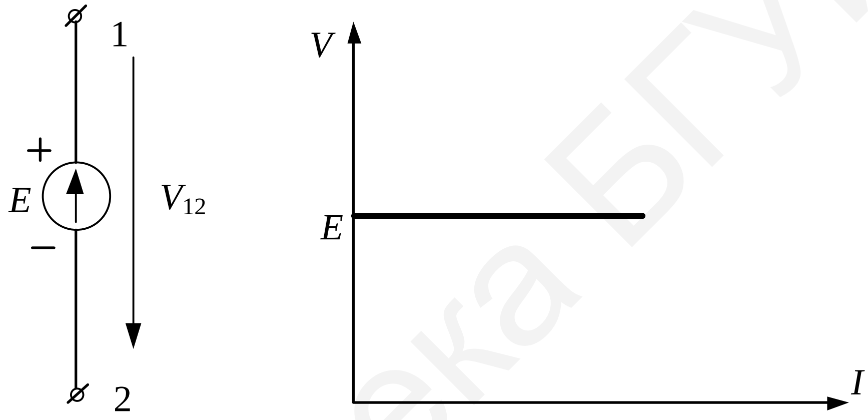
Note if the result of solution gives us a positive value for current and voltage, this means that their actual directions are the same as the reference arrows; if answers are negative therefore their actual directions are opposite to the reference arrows.

1.3. Active elements of electric circuit. Voltage source and current source

Sources of electrical energy produce current and electric voltage in electric circuit.

Voltage source. Often the concept of ideal elements is used in the theory of electric circuits. *An ideal voltage source* (the source of electromotive force – EMF) is an active element that maintains a constant voltage at its terminals regardless of the amount of current, which flows through its terminals.

An ideal voltage source *has no internal resistance*. The diagram for an ideal voltage source is shown in Fig. 1.6, a, and its *VI*-characteristic is in Fig. 1.6, b.



a b

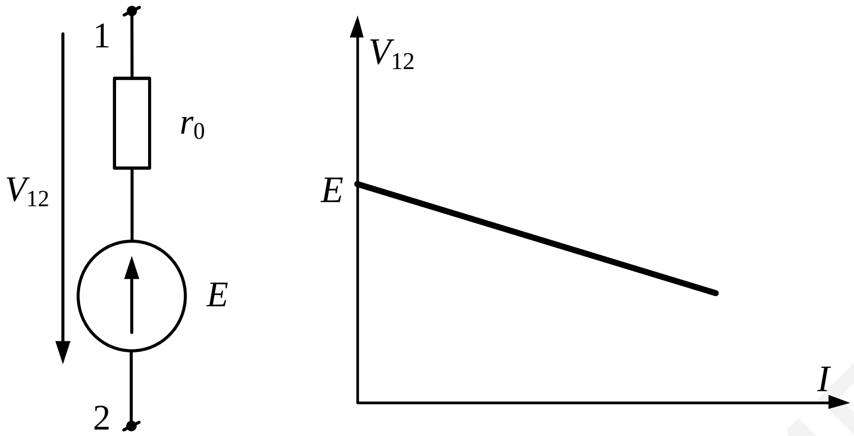
Fig. 1.6. Symbol for *DC* an ideal voltage source and its *VI*-characteristic

Voltage across the ideal source terminals is independent on the current through it.

A practical voltage source has internal resistance. It means that the voltage across its terminals varies as the current changes through the source.

The equivalent circuit of Fig. 1.7 is used for representation a practical voltage source.

The Fig. 1.7 shows a circuit representation of a practical voltage source (Fig. 1.7, a) and its *VI*-characteristic (Fig. 1.7, b).



a b

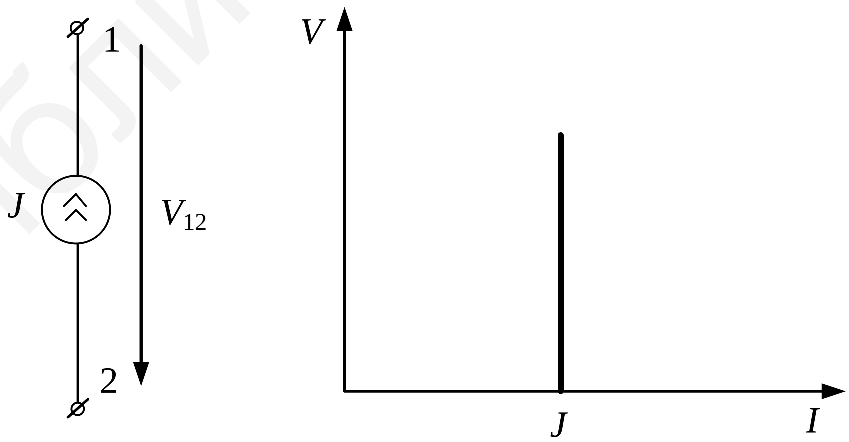
Fig. 1.7. A practical voltage source and its *VI*-characteristic

Current source. *An ideal current source* is an active element that maintains a constant current at its terminals regardless of the voltage across its terminals.

An ideal current source *doesn't possess internal conductivity*, or in other words, its *internal shunt resistance* goes to infinity.

Fig. 1.8, a illustrates the symbol used for representation of an ideal current sources. The arrow inside the circle shows the direction of positive charge movement. The positive voltage exists in opposite direction.

The *VI*-characteristic for an ideal current source is shown in Fig. 1.8, b.



a b

Fig. 1.8. Symbol for *DC* an ideal current source and its *VI*-characteristic

A practical current source has internal conductivity. The equivalent circuit for a practical current source is represented on the Fig. 1.9, a. Fig. 1.9, b shows the *VI*-characteristic for a practical current source. In a practical current source electric current also depends on external load.

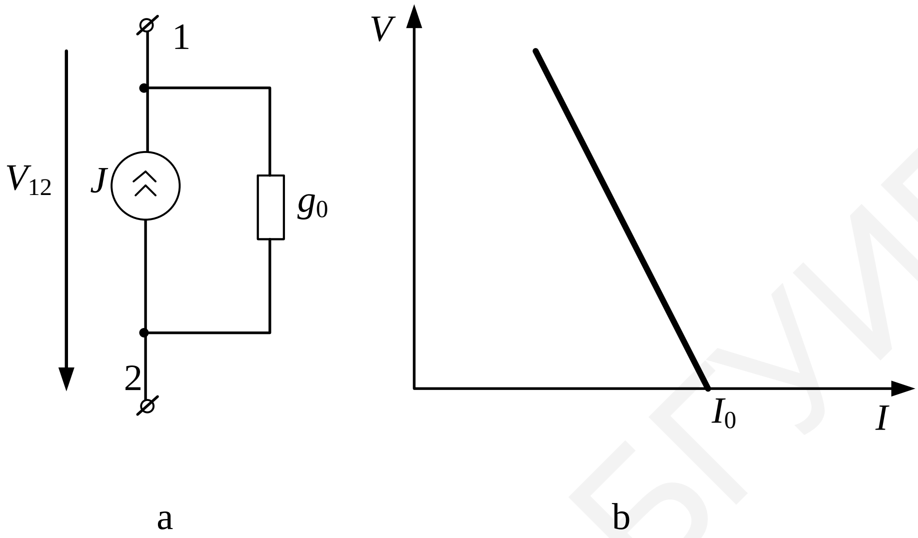


Fig. 1.9. The equivalent circuit for illustration of a practical current source and its *VI*-characteristic

Source conversions

Fig. 1.10 shows conversions of equivalent voltage and current sources. A practical source, whether it is a voltage source or a current source, is easily may be *converted to* the other type. The current source of Fig. 1.10 is equivalent to the voltage source if

$$J = \frac{E}{r_0}, \tag{1.3}$$

where r_0 – the internal resistance of a practical voltage source; g_0 – the internal conductance of a practical current source.

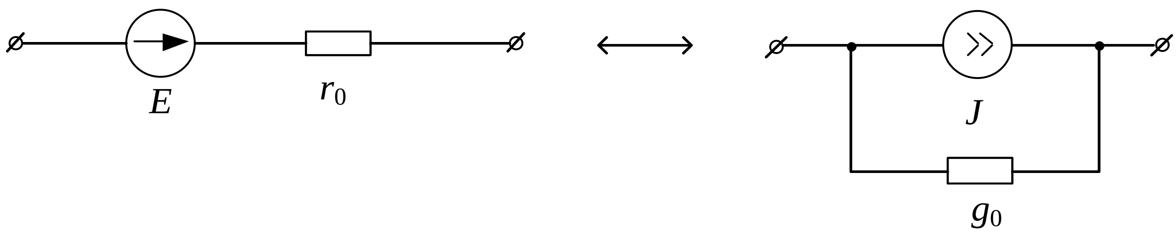


Fig. 1.10. Equivalent voltage and current sources

The internal conductance of a practical current source may be defined using expression:

$$g_0 = \frac{1}{r_0}. \qquad (1.4)$$

The unit of the conductance is siemens. Similarly, a current source may be converted to an equivalent voltage source, using expressions:

$$E = \frac{J}{g_0}; \qquad (1.5)$$

$$r_0 = \frac{1}{g_0}. \qquad (1.6)$$

The sources are only equivalent with respect to elements connected external to the terminal.

Example 1.1

Given: E = 220 V; r_0 = 65 Ω.

Convert the voltage source to an equivalent current source and show its equivalent circuit.

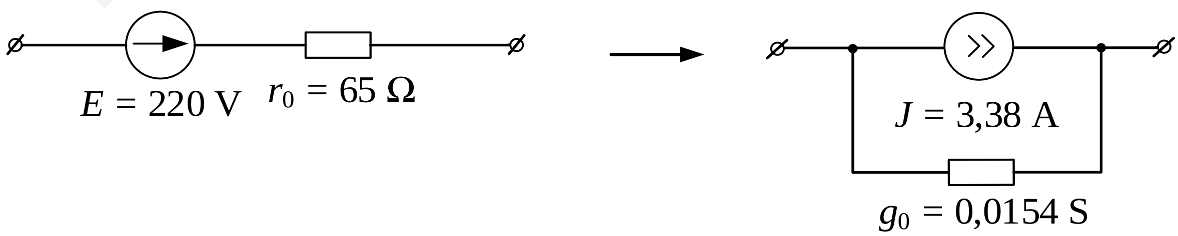
***Solution*:**

Use expressions (1.3), (1.4) and calculate:

$$J = \frac{E}{r_0} = \frac{220}{65} = 3{,}38 \ \text{A};$$

$$g_0 = \frac{1}{r_0} = \frac{1}{65} = 0{,}0154 \ \text{S}.$$

***The equivalent representation*:**



Example 1.2

Given: J = 7,5 mA and g_0 = 0,002 S.

Convert the current source to an equivalent voltage source and show its equivalent circuit.

***Solution*:**

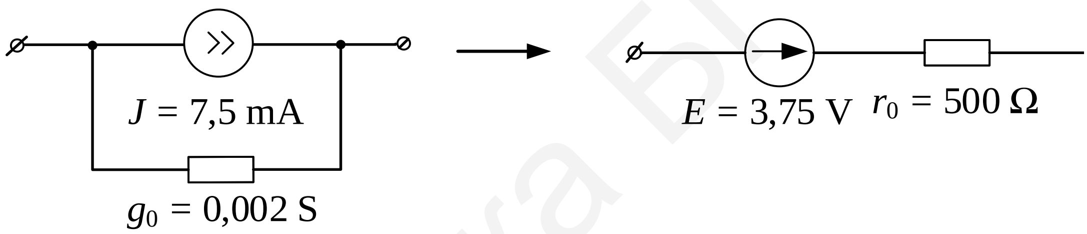
J = 7,5 mA = 7,5·0,001 = 0,0075 A.

Use expressions (1.5), (1.6) and calculate:

$$E = \frac{J}{g_0} = \frac{0,0075}{0,002} = 3,75 \text{ V};$$

$$r_0 = \frac{1}{g_0} = \frac{1}{0,002} = 500 \ \Omega..$$

***The equivalent representation*:**



SELF-ASSESSMENT TEST:

1. Write the formula for converting a voltage source to a current source.
2. Write the formula for converting a current source to a voltage source.
3. Determine J and g_0 of a current source equivalent to a voltage source with E = 60,5 V and r_0 = 12,1Ω.
4. Determine E and r_0 of a voltage source equivalent to a current source with J = 348 mA and g_0 = 0,004 S.

1.4. Passive elements of an electric circuit: resistor, inductor and capacitor

Resistor and resistance. Conductance

When electric current flows through circuit elements, it encounters a certain amount of *resistance*, the magnitude of which depends on the electrical properties of the material [1].

Resistance is opposition to movement of charges. *Resistor* is a component of electric circuit is specifically designed to have a certain amount of resistance.

The Fig. 1.11 represents a circuit symbol and a graphic abbreviation for resistance.

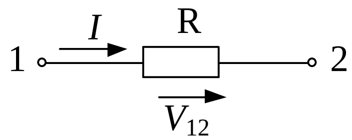


Fig. 1.11. Resistance symbol and notation

The unit of measurement of resistance is the ohm, for which the symbol is Ω. Ohm is defined as the resistance of a conductor when a constant voltage of 1 volt applied between two points produces a current of 1ampere in that conductor.

Thus resistance, in ohms

$$R = \frac{V}{I}, \ \ \Omega, \tag{1.7}$$

where V – the voltage across the two points in volts; I – the current flowing between the two points in amperes.

The principle applications of resistor are:

- to limit current in a circuit;
- to convert electrical energy into another form of energy such as heat;
- to divide voltage.

The reciprocal of resistance is called *conductance*. It is measured in siemens (S). Thus, conductance in siemens

$$G = \frac{1}{R}, \tag{1.8}$$

where R – the resistance in ohms.

Example 1.3

Find the conductance of a conductor of resistance:

a) R_1 = 0,5 Ω;

b) R_2 = 5 kΩ;

c) R_3 = 500 Ω.

***Solution*:**

Use expression (1.8) and calculate:

a) $G_1 = \dfrac{1}{R_1} = \dfrac{1}{0,5} = 2 \quad \text{S};$

b) $G_2 = \dfrac{1}{R_2} = \dfrac{1}{5 \cdot 10^3} = 2 \cdot 10^{-3} \quad \text{S};$

c) $G_3 = \dfrac{1}{R_3} = \dfrac{1}{500} = 0,002 \quad \text{S}.$

Inductor and inductance

Inductance is the name given to the property of a circuit whereby there is an electromotive force (EMF) induced into the circuit by the change of flux linkages produced by a current change.

Inductance is the effect of circuit operation to oppose any change in current. *Inductor* is a component of electric circuit, which designed to possess an inductance.

The Fig. 1.12 represents a circuit symbol and a graphic abbreviation for inductance.

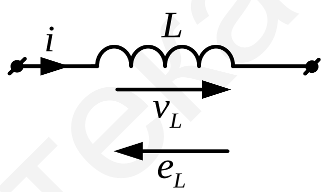


Fig. 1.12. Inductance symbol and notation

The basic form of an inductor is simply coil of wire. The unit of measurement of inductance is the henry, for which the symbol is H. A circuit has an inductance of 1 henry when an EMF of 1 volt is induced in it by the current changing at the rate of 1 ampere per second.

The inductance in henry determine by

$$L = \frac{d\psi}{di}, \tag{1.9}$$

where $d\psi$ – a flux changing; di – a current changing.

The basic equation, connecting voltage across inductor and current through it is:

$$v_L = L\frac{di}{dt}. \tag{1.10}$$

Note:

– when the EMF is induced in the same circuit as that in which the current is changing, the property is called *self inductance, L*;

– when the EMF is induced in a circuit by a change of flux due to current changing in an adjacent circuit, the property is called *mutual inductance*, *M*.

Energy stored

An inductor possesses an ability to store energy in magnetic field around it. Energy in magnetic field of an inductor is determined by the formula

$$W_M = \frac{Li^2}{2}. \tag{1.11}$$

Capacitor and capacitance

Capacitor is a component of electric circuit, which possesses capacitance. *Capacitance* is the property of electrical conductors to store charge corresponding to a given voltage. The unit of measurement of capacitance is farad, F (or more usually μF = 10^{-6} F or pF = 10^{-12} F).

Capacitance in farad determine by

$$C = \frac{Q}{V}, \tag{1.12}$$

where Q – charge of conductor; V – voltage across conductor.

The Fig. 1.13 represents a circuit symbol and a graphic abbreviation for capacitance.

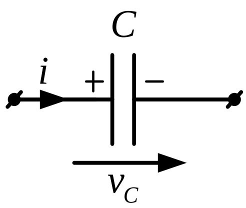


Fig. 1.13. Capacitance symbol and notation

Capacitor has the ability to store a quantity of static electricity.

Every system of electrical conductors possesses capacitance. For example, there is capacitance between the conductors of transmission lines and also between the wires of a telephone cable.

The basic equation, connecting voltage across capacitor and current through it is:

$$i = C\frac{du}{dt}. \tag{1.13}$$

Energy stored

A capacitor possesses an ability to store energy in electric field between plates. The letter for indication of this kind of energy is W_C. Energy in electric field of acapacitor is is determined by the formula:

$$W_C = \frac{Cv^2}{2}. \tag{1.14}$$

1.5. Open and short circuit. Ground

Open circuit

A circuit element whose resistance approaches infinity is called *an open circuit* (Fig. 1.14). Or in other words there is no conducting path through an open circuit. Hence the current through an open circuit is zero $I = 0$. But an open circuit can have a voltage across it.

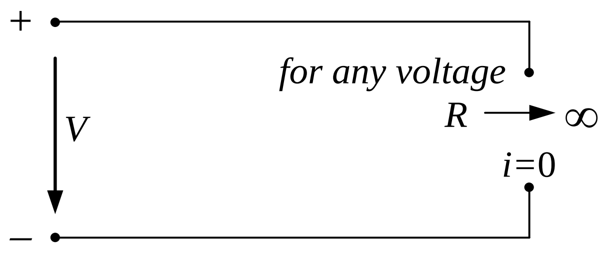


Fig. 1.14. An open circuit

Short circuit

A circuit element with resistance approaching zero is called *a short circuit*.

A short circuit has no voltage across it, so V = 0. However, a short circuit is a *perfectly conducting path.* For example, a short wire of large diameter approximates to the behavior of a short circuit.

Formally, a short circuit is defined as a circuit element across which the *voltage is zero*, regardless of the current flowing through it.

We indicate a short circuit by drawing a straight line in a circuit diagram (Fig. 1.15).

Fig. 1.15. A short circuit

Ground

Ground is the reference point in electric circuit (Fig. 1.16). Voltage is always specified with respect to another point. Reference ground has *potential of zero volts* (0 V) with respect to all other points in the circuit.

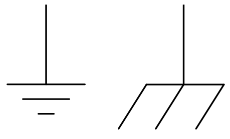
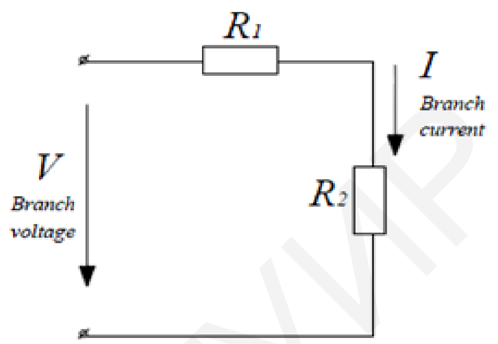
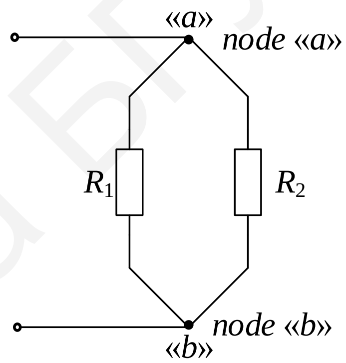
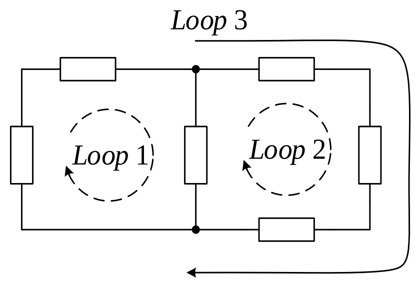


Fig. 1.16. Commonly used ground symbols

1.6. Branches, nodes and loops of electric circuit

Table 1.1

Branches, nodes and loops. The concepts and definitions

The Concepts and definitions	Images for illustration
1. *A branch of a circuit* is any part of the circuit that includes one or more elements in series	
2. *A node* is a connection point between too or more branches	
3. *A loop* is a closed path in a circuit. *A mesh* is a loop that does not contain other loops. The circuit with loops 1, 2 and 3 consists of two meshes: loops 1 and 2 are meshes, but loop 3 isn't a mesh, because it includes both loops 1 and 2	

2. ELECTRICAL CIRCUIT LAWS

There are three basic laws of electric circuits for circuit analysis: *Ohm's law*, *Kirchhoff's current law* and *Kirchhoff's voltage law*.

2.1. Ohm's law

Relationship between current, voltage and resistance is called *Ohm's law*.

Ohm's law states: **current in a resistive circuit is directly proportional to its applied voltage and inversely proportional to its resistance** (Fig. 2.1).

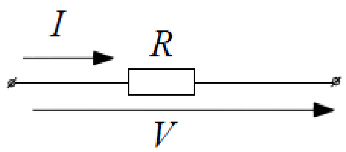


Fig. 2.1. Current through and voltage across a resistor

In equation form, Ohm's law states:

$$I = \frac{V}{R}, \qquad (2.1)$$

where I – current in amperes; V – voltage in volts; R – resistance in ohms.

Ohm's law can be restated in terms of conductance as

$$I = V \cdot G, \qquad (2.2)$$

where I – current in amperes; V – voltage in volts; G – conductance in siemence.

Example 2.1

Given: E = 28 V; R = 7 Ω.

Find: current in the circuit of Fig. 2.2.

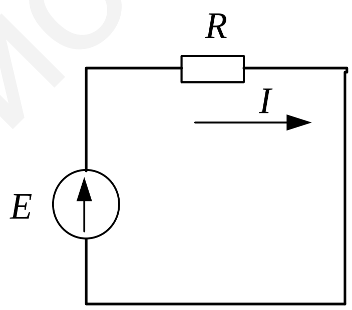


Fig. 2.2. One-loop circuit

***Solution*:**

Use the formula (2.1), and substitute 28 V for voltage and 7 Ω for resistance:

$$I = \frac{V}{R} = \frac{28\ \text{V}}{7\ \Omega} = 4\ \text{A}.$$

***Related Problem*:** If R is changed to 14 Ω in Fig. 2.2, what is the current?

Example 2.2

Given: R = 93 Ω and I = 3,75 A.

How much voltage is required to cause this current in the circuit of Fig. 2.2?

***Solution*:**

Substitute 3,75 A for I and 93 Ω for R into the formula $V = IR$:

$$V = IR = 3{,}75 \cdot 93 = 348{,}75 \text{ V}.$$

Thus, 348,75 V are required to produce 3,75 A of current through a 93 Ω resistor.

***Related Problem*:** In Fig. 2.2, how much voltage is required to produce 7,5 A of current?

SUMMARY:

– voltage and current are linearly proportional;
– Ohm's law gives the relationship of voltage, current, and resistance;
– current is inversely proportional to resistance.

SELF-ASSESSMENT TEST:

1. Ohm's law states that…
 a) current equals voltage times resistance;
 b) voltage equals current times resistance;
 c) resistance equals current divided by voltage;
 d) voltage equals current squared times resistance.
2. When the voltage across a resistor is doubled, the current will…
 a) triple;
 b) halve;
 c) double;
 d) not change.
3. When 15 V are applied across a 30 Ω resistor, the current is:
 a) 10 A;
 b) 0,5 A;
 c) 0,2 A;
 d) 5 A.
4. When current of 0,25 mA flows through 1,0 kΩ resistor, the voltage across the resistor is:
 a) 25 V;

b) 250 V;
c) 2,5 kV;
d) 0,25 V.

5. If current of 6,5 mA due to voltage of 50 V flows through resistor, the resistance is:
a) 769 Ω;
b) 0,13 kΩ;
c) 769 kΩ;
d) 7,69 kΩ.

6. A resistance of 1,25 MΩ is connected across a 2,5 kV source. The resulting current is approximately:
a) 2,2 mA;
b) 2 mA;
c) 0,2 μA;
d) 2 A.

2.2. Kirchhoff's current law

Kirchhoff's current law (KCL) states the following: **the algebraic sum of all currents at a node must equal to zero**.

Formally:

$$\sum I_i = 0. \qquad (2.3)$$

Fig. 2.3 illustrates KCL: KCL applies to node. Here we see that the node has three currents entering I_1, I_3, J_6 and three currents leaving I_2, I_4, J_5.

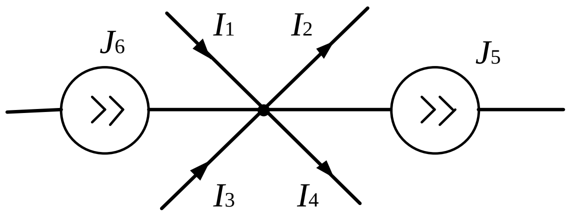


Fig. 2.3. A generalized circuit node

Note: we choose to apply the following convention:
– sign «+» for currents entering a node;

– sign «–» for currents leaving a node.

Thus, the resulting expression of KCL for the node is:

$$I_1 + I_3 + J_6 - I_2 - I_4 - J_5 = 0.$$

Example 2.3

Verify that KCL applies at the node, shown in Fig. 2.4.

Given: $I_1 = 3$ A; $I_2 = 2$ A; $J_5 = 1$ A; $I_4 = 6$ A.

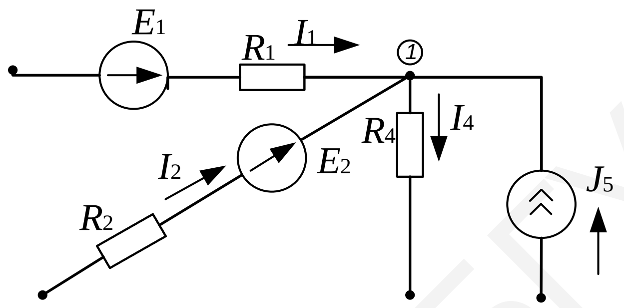


Fig. 2.4. The one node of an electrical circuit

Solution: $I_1 + I_2 + J_5 - I_4 = 3 + 2 + 1 - 6 = 0.$

Example 2.4

Determine the magnitude and correct direction of the currents I_3 and I_5 for the network of Fig. 2.5.

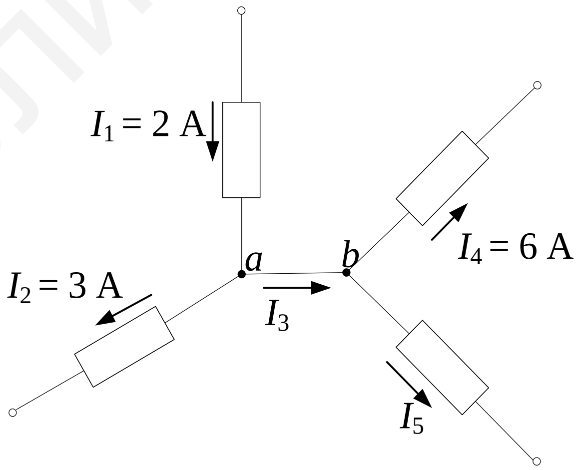


Fig. 2.5. A part of electrical circuit with two nodes

Although points «*a*» and «*b*» are in fact the same node, we treat the points as two separate nodes with 0 Ω resistance between them.

***Solution*:** KCL for point «*a*»:

$$I_1 - I_2 - I_3 = 0,$$

hence

$$I_3 = I_1 - I_2 = 2 - 1 = -1, \text{ A}.$$

Calculation shows that the current has a negative sign. The negative sign simply indicates that the current is in fact *opposite to the direction* selected as the reference. Reference direction of current I_3 was taken to be from «*a*» to «*b*», while the negative sign indicates that the current is in fact from «*b*» to «*a*». Similarly, using KCL at point «*b*» gives

$$I_3 - I_4 - I_5 = 0,$$

which gives current I_5 as

$$I_5 = I_3 - I_4 = -1 - 6 = -7, \text{ A}.$$

The negative sign indicates that the current I_5 is actually towards node «*b*».

SUMMARY:

– Kirchhoff's current law: the sum of the currents into a junction (total current in) equals the sum of the currents out of the junction (total current out);

– The algebraic sum of all the currents entering and leaving a junction is equal to zero.

SELF-ASSESSMENT TEST:

1. State Kirchhoff's current law in two ways.

2. There is a total current of 15 mA into a node and the current out of the node divides into three parallel branches. What is the sum of all three branch currents?

3. Two branch currents enter a node, and two branch currents leave the same node. One of the currents entering the node is 1 mA, and one of the currents leaving

the node is 3 mA. The total current entering and leaving the node is 8 mA. Determine the value of the unknown current entering the node and the value of the unknown current leaving the node.

2.3. Kirchhoff’s voltage law

Kirchhoff’s voltage law (KVL) states the following: **the algebraic sum of voltage drops around a closed loop is equal to the resulting EMF acting in that loop**.

Formally:

$$\sum V_i = \sum E_i \quad \text{or} \quad \sum I_i \cdot R_i = \sum E_i \,. \tag{2.4}$$

Kirchhoff’s voltage law applies to loops.

We can consider a loop by moving clockwise or anticlockwise direction around the loop. To avoid mistakes you should do the following procedure:

1. Label positive current directions in the original circuit diagram.
2. Choose the positive moving direction around the closed loop (clockwise or anticlockwise) at will.
3. If the voltage drop direction is the same as the movement direction around the loop, we put «+» before it; if the voltage drop direction is the opposite to the movement direction, we put «–» before one.

Application of Kirchhoff’s voltage law around the closed loop of the Fig. 2.6 gives the following mathematical statement:

$$I_1R_1 - I_2R_2 + I_3R_3 = E_1 - E_3 + E_4 \,.$$

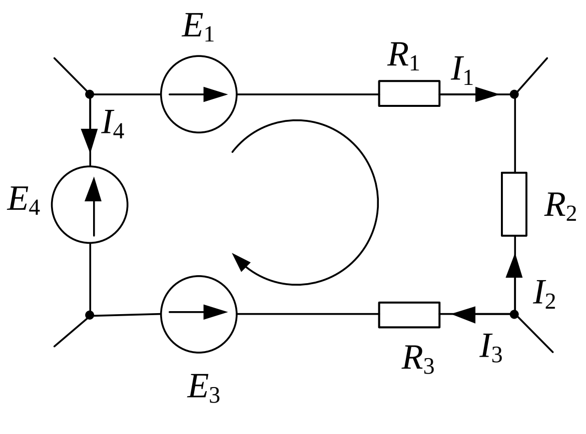


Fig. 2.6. An illustration of KVL application

Note, that the source polarity is minus to plus and each voltage drop is plus to minus.

Example 2.5

Given: $V_1 = 65$ V; $V_2 = 50$ V.

Determine the source voltage E in Fig. 2.7. There is no voltage drop across the fuse.

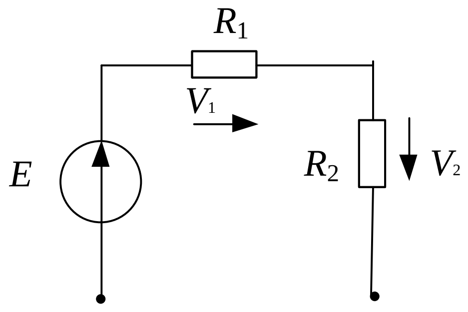


Fig. 2.7. Determining the source voltage by KVL

***Solution*:**

By Kirchhoff's voltage law, the source voltage (applied voltage) must be equal the sum of the voltage drops. Adding the voltage drops gives the value of the source voltage:

$$E = V_1 + V_2 = 65 + 50 = 115 \quad \text{V}.$$

Example 2.6

Given: $V_1 = 12$ V; $V_2 = 6$ V.

Determine the unknown voltage drop, V_3, in Fig. 2.8, if $E_1 = 50$ V and $E_2 = 15$ V.

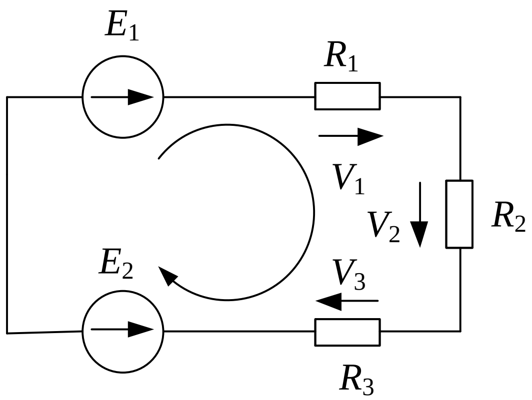


Fig. 2.8. Determining the unknown voltage drop by KVL

***Solution*:**

By Kirchhoff's voltage law, the algebraic sum of voltage drops around a closed loop is equal to the resulting *EMF* acting in that loop:

$$V_1 + V_2 + V_3 = E_1 - E_2.$$

The value of each voltage drop except V_3 is known. Substitute these values into the equation:

$$12 + 6 + V_3 = 50 - 15;$$
$$18 + V_3 = 35.$$

Solving equation, we find $V_3 = 17$ V. Its polarity is as shown in Fig. 2.8.

***Related Problem*:**

Determine V_3 if the polarity of E_2 is reversed in Fig. 2.8.

Solution of the set of equations including KCL and KVL allows us to calculate the current in each branch of circuit. Method that includes KCL and KVL equations is calls the brunch-currents method.

Example 2.7

Given: for the circuit of the Fig. 2.9 E_1 = 12 V; E_2 = 20 V; R_1 = 4 Ω; R_2 = 6 Ω; R_3 = 4 Ω.

Find the currents I_1, I_2, I_3.

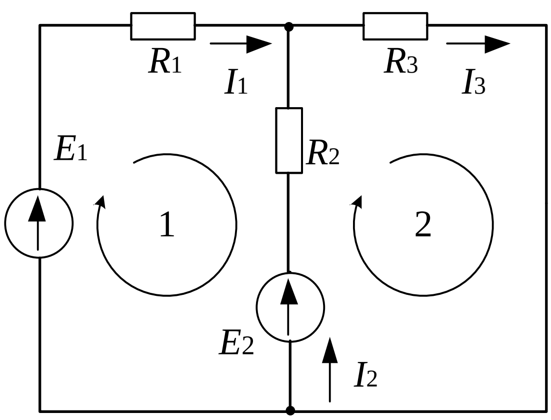


Fig. 2.9. Determining the unknown currents by KCL and KVL

***Solution*:**

Step 1. Assign currents I_1, I_2, I_3.

Step 2. Indicate the positive direction of moving around the loop (clockwise).

Step 3. Write the KCL and KVL equations:

$$\begin{cases} I_1 + I_2 - I_3 = 0; \\ I_1 R_1 - I_2 R_2 = E_1 - E_2; \\ I_2 R_2 + I_3 R_3 = E_2; \end{cases} \quad (2.5)$$

$$\begin{cases} I_1 + I_2 - I_3 = 0; \\ I_1 \cdot 4 - I_2 \cdot 6 = -8; \\ I_2 \cdot 6 + I_3 \cdot 4 = 20. \end{cases} \quad (2.6)$$

Step 4. Solution gives the currents:

$$I_1 = 0{,}625 \text{ A}; \; I_2 = 1{,}75 \text{ A}; \; I_3 = 2{,}375 \text{ A}.$$

SUMMARY:

– Kirchhoff's voltage law: the sum of all the voltage drops around a single closed path in a circuit is equal to the total source voltage in that loop;

– Kirchhoff's voltage law: the algebraic sum of all the voltages (both source and drops) around a single closed path is zero;

– the voltage drops in a circuit are always opposite in polarity to the total source voltage.

SELF-ASSESSMENT TEST:

1. State Kirchhoff's voltage law.

2. A 67,5 V source is connected to a series resistive circuit. What is the sum of the voltage drops in this circuit?

3. Two equal-value resistors are connected in series across a 26,4 V battery. What is the voltage drop across each resistor?

4. In a series circuit with a 37,3 V source, there are three resistors. One voltage drop is 5 V, and the other is 10,3 V. What is the value of the third voltage drop?

5. The individual voltage drops in a series string are as follows: 1; 3; 1,5; 5,4 and 7 V. What is the total voltage applied across the series string?

3. SERIES AND PARALLEL CIRCUITS. SERIES-PARALLEL CIRCUITS

3.1. Series circuits

When resistors are connected «*end-to-end*» so that the *same current* flows through them *all are connected in series*.

Such a circuit is shown in Fig. 3.1.

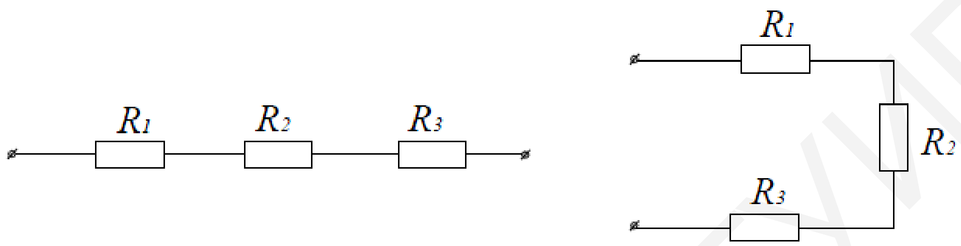


Fig. 3.1. Resistors are in series

When resistors are connected in series the total resistance is found simply by adding together the resistor values.

$$R_t = \sum R_n \ \Omega. \tag{3.1}$$

For the circuit of Fig. 3.2

$$R_t = R_1 + R_2 + R_3 \ \Omega.$$

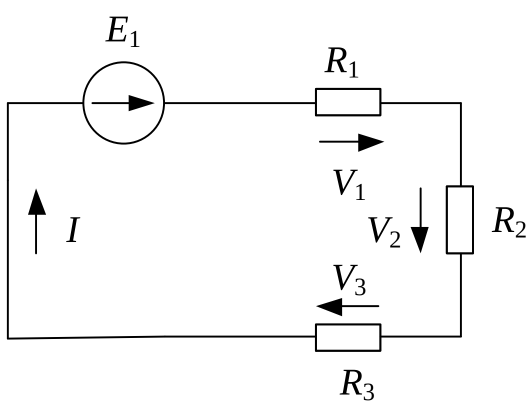


Fig. 3.2. Resistors are in series

In a case more than one resistor of the same value in series circuits the total resistance is found by simple multiplication the resistance value by the number of equal-value resistors that are in series. In general, the formula is expressed as

$$R_t = n \cdot R_n \ \Omega\,, \tag{3.2}$$

where n – the number of equal-value resistors; R_n – the resistance value.

Example 3.1

Given: for the circuit of Fig. 3.2 R_1 = 0,75 kΩ; R_2 = 240 Ω; R_3 = 655 Ω; E = 57 V.

Calculate:

1. The total circuit resistance.
2. The circuit current.
3. The voltage across each element.

***Solution*:**

1. The total circuit resistance

$$R_t = R_1 + R_2 + R_3 = 750 + 240 + 655 = 1645\ \Omega = 1{,}645\ \text{k}\Omega.$$

2. Using Ohm's law calculate the circuit current

$$I = \frac{E}{R_t} = \frac{57}{1645} = 0{,}0346\ \ \text{A}.$$

3. Voltages across each resistor

$$V_1 = IR_1 = 0{,}0346 \cdot 0{,}75 \cdot 10^3 = 25{,}95\,\text{V};$$
$$V_2 = IR_2 = 0{,}0346 \cdot 240 = 8{,}304\,\text{V};$$
$$V_3 = IR_3 = 0{,}0346 \cdot 655 = 22{,}663\,\text{V}.$$

Let test results using KVL

$$E = \sum U_i = V_1 + V_2 + V_3 = 25{,}95 + 8{,}304 + 22{,}663 = 56{,}653\,\text{V}.$$

Note, that the sum of voltages is 56,653 V instead of 57 V due to the rounding errors in the calculation.

Example 3.2

Find the R_t of eight 22 Ω resistors in series.

Solution:

$R_t = 22 \cdot 8 = 176\ \Omega$.

Example 3.3

Given: for the circuit of Fig. 3.3 $E_1 = E_2 = E_3 = 15$ V.

Define the total voltage source for the circuits of the Fig. 3.3, a and b.

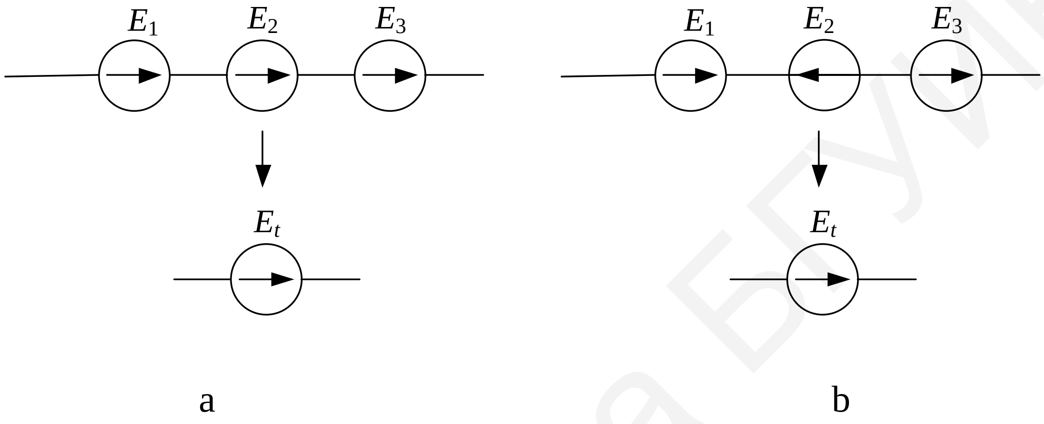


Fig. 3.3. Voltage source in series add algebraically

***Solution*:**

When two or more voltage source *are in series*, the total voltage is equal to *the algebraic sum* of the individual source voltages, in other words the polarities of the sources must be taken into consideration.

1. For the circuit (Fig. 3.3, a) as the voltage sources are all in the same direction in terms of their polarities all off the voltage have the same sign when added:

$$E_t = E_1 + E_2 + E_3 = 15 + 15 + 15 = 45 \quad \text{V}.$$

2. For the circuit (Fig. 3.3, b) the middle voltage source is opposite to the other two; so, its EMF has an opposite sign when added to the others:

$$E_t = E_1 - E_2 + E_3 = 15 - 15 + 15 = 30 \quad \text{V}.$$

SUMMARY:

– the current is the same at all points in a series circuit;

– the total series resistance is the sum of all resistors in the series circuit;

– if all of the resistors in a series circuit are of equal value, the total resistance is the number of resistors multiplied by the resistance value of one resistor;

– the order of series components may be change without affecting the operation of the circuit;

– voltage sources in series add algebraically.

SELF-ASSESSMENT TEST:

1. What is the total resistance for the following series resistors: 1,0; 2,2; 3,3 and 5,6 kΩ?
2. Find R_t for three 1,7 kΩ resistors and two 947 Ω resistors in series.
3. A voltage source of 25 V supplies with current a series connection of 15; 7,5; 2,8 and 13,2 Ω. The current flowing through the connection is equal to:

a) 0,32 A;

b) 0,649 A;

c) 64,9 mA;

d) 3,2 mA.

4. Three voltage sources are connected in series in the same direction in terms of their polarities: E_1 = 3 V, E_2 = 5 V, E_3 =10 V. What is the total EMF?
5. Four 1,5 V flashlight batteries are connected in series plus to minus. What is the total voltage of all four cells?
6. How many 11 V batteries must be connected in series to produce 44 V? Draw a schematic that shows the battery connections.

3.2. Parallel circuits

When resistors are joined «side-by-side» so that their corresponding ends are connected together are said to be connected in parallel. A parallel circuit provides more than one path for current (Fig. 3.4).

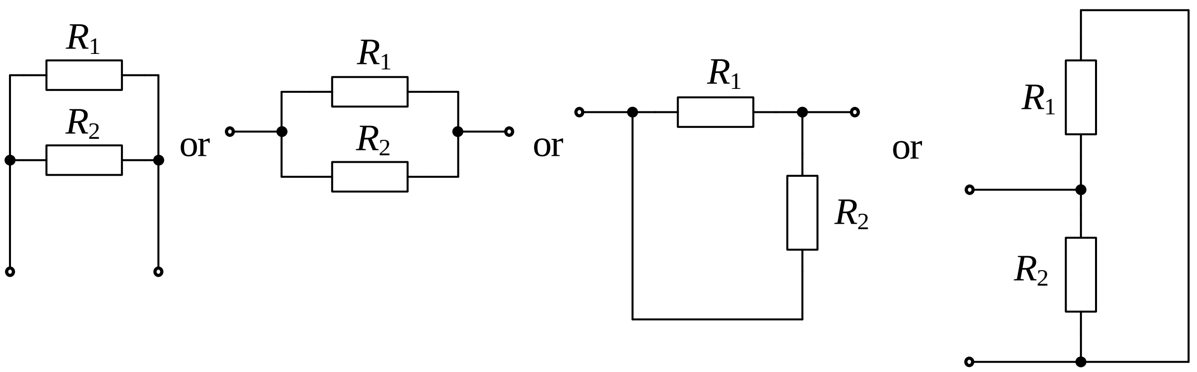


Fig. 3.4. Resistors are in parallel

Three parallel resistors are shown in Fig. 3.5.

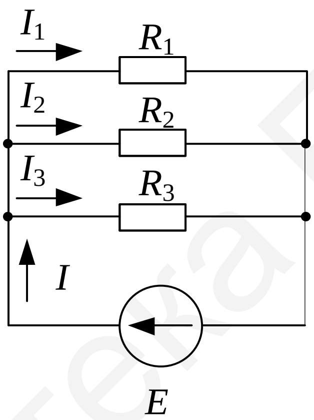


Fig. 3.5. The circuit includes three parallel resistors

For *n* resistors in parallel

$$G_t = G_1 + G_2 + ... + G_n.$$

Thus, *conductance is the reciprocal of resistance*, so, to obtain the circuit resistance you must then take the reciprocal of the answer obtained from the equation

$$R_t = \frac{1}{G_t},$$

or

$$R_t = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}}. \qquad (3.3)$$

This is the general formula for calculation the total resistance for any number of resistors in parallel.

The combination of two resistors in parallel occurs commonly in practice. The formula for the total resistance in parallel is

$$R_t = \frac{R_1 R_2}{R_1 + R_2}. \qquad (3.4)$$

Another special case of parallel circuit is the parallel connection of several resistors each having the same resistance value: $R_1 = R_2 = R_3 = \ldots = R_n = R$.

The total resistance for this case is

$$R_t = \frac{R}{n}.$$

Note: when resistors are connected in parallel, the total resistance of the circuit decreases. The total resistance of a parallel circuit is always less than the value of the smallest resistor.

Example 3.4

For the circuit of Fig. 3.5 calculate:

1. The total resistance of the circuit.
2. The tree brunch currents.
3. The total current from the battery.

Given: E = 24 V; R_1 = 330 Ω; R_2 = 1500 Ω; R_3 = 470 Ω.

***Solution*:**

1. The total circuit resistance is defined in such way:

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{330} + \frac{1}{1500} + \frac{1}{470} =$$

$$= 0,00303 + 0,000667 + 0,00213 = 0,005827 \text{ S.}$$

so

$$R_t = \frac{1}{0,005845} = 171,614 \ \Omega.$$

2. Currents through each brunch:

$$I_1 = \frac{E}{R_1} = \frac{24}{330} = 72,73 \text{ mA}; \ I_2 = \frac{E}{R_2} = \frac{24}{1500} = 16 \text{ mA};$$

$$I_3 = \frac{E}{R_3} = \frac{24}{470} = 51,06 \text{ mA}.$$

3. The total current according KCL:

$$I_t = I_1 + I_2 + I_3 = 72,73 + 16 + 51,06 = 139,79 \text{ mA}.$$

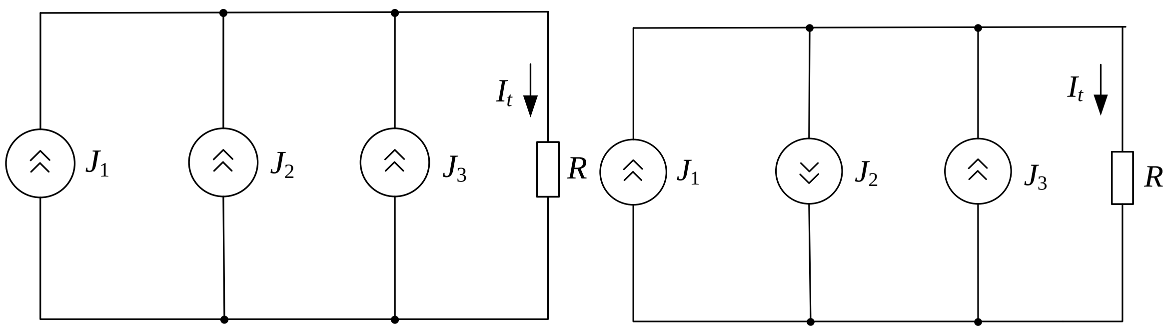
By Ohm's law:

$$I_t = \frac{E}{R_t} = \frac{24}{171,614} = 139,85 \text{ mA}.$$

Example 3.5

Given (Fig. 3.6): J_1 = 15 mA; J_2 = 7 mA; J_3 = 10,5 mA.

Define current through resistor R.



a b

Fig. 3.6. Current source in parallel add algebraically

***Solution*:**

1. For example, in Fig. 3.6, a the three current sources in parallel provide current in the same direction.

So

$$J_t = J_1 + J_2 + J_3 = 15 + 7 + 10{,}5 = 32{,}5 \ \text{ mA.}$$

2. In Fig. 3.6, b the middle source provides current in a direction opposite to the other:

$$J_t = J_1 - J_2 + J_3 = 15 - 7 + 10{,}5 = 18{,}5 \ \text{ mA.}$$

SUMMARY:

– resistors in parallel are connected between two points (nodes);
– a parallel combination has more than one path for current;
– the total parallel resistance is less than the lowest-value resistor;
– the voltages across all branches of a parallel circuit are the same;
– current sources in parallel add algebraically.

SELF-ASSESSMENT TEST:

1. How are the resistors connected in a parallel circuit?
2. How do you identify a parallel circuit?
3. Does the total resistance increase or decrease as more resistors are connected in parallel?
4. The total parallel resistance is always less than what value?

5. Write the general formula for R_t with any number of resistors in parallel.

6. Write the special formula for two resistors in parallel.

7. Write the special formula for any number of equal-value resistors in parallel.

8. If four identical lamps are connected in parallel and the combined resistance is 100 Ω, find the resistance of one lamp.

9. A circuit has the following resistors in parallel with a voltage source: 220, 100, 47 and 22 kΩ. Which resistor has the most current through it? The least current?

10. How are voltages across each branch of a parallel circuit related?

11. Four 0,5 A current sources are connected in parallel in the same direction. What current will be produced through a load resistor?

3.3. Series-parallel circuits

Series-parallel networks are networks *that contain both series and parallel configuration.*

Fig. 3.7 shows an example of a simple series-parallel combination of resistors R_1, R_2 and R_3.

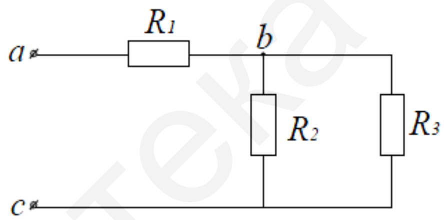


Fig. 3.7. A series-parallel combination of three resistors

Notice that the resistance from point «*a*» to point «*b*» is R_1. The resistance from point «*b*» to point «*c*» is R_2 and R_3 in parallel. The total resistance from point «*a*» to point «*c*» is R_1 in series with series with the parallel combination of R_2 and R_3 (Fig. 3.8).

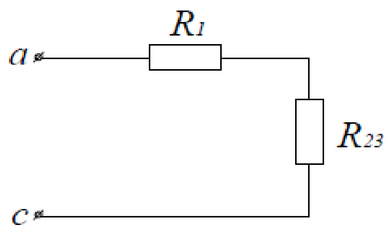


Fig. 3.8. Equivalent circuit with R_1 and R_{23} in series

Example 3.6

Given: R_1 = 16,6 Ω; $R_2 = R_3 = R$ = 64,8 Ω.

Determine the total resistance for the circuit of Fig. 3.7.

***Solution*:**

Step 1. Calculate the equivalent parallel resistance of R_2 and R_3. For the case of two equal value resistors (R_2 and R_3) in parallel R_{23} is equal to the resistance R divided by two:

$$R_{23} = \frac{R}{2} = \frac{64,8}{2} = 32,4 \ \ \Omega.$$

Step 2. Total resistance:

$$R_t = R_1 + R_{23} = 16,6 + 32,4 = 49 \ \ \Omega.$$

SUMMARY:

– a series-parallel circuit is a combination of both series and parallel current paths;

– to determine total resistance in a series-parallel circuit, identify the series and parallel relationships, and then apply the formulas for series resistance and parallel resistance;

– to find the total current, apply Ohm’s law and divide the total voltage by the total resistance;

– to determine branch currents, using Kirchhoff’s current law or Ohm’s law. Consider each circuit problem individually to determine the most appropriate method;

– to determine voltage drops across any portion of a series-parallel circuit, using Kirchhoff’s voltage law or Ohm’s law. Consider each circuit problem individually to determine the most appropriate method;

– to find total resistance of a ladder network, start at the point farthest from the source and reduce the resistance in steps.

SELF-ASSESSMENT TEST:

1. Which of the following statements are true concerning the Fig. 3.9:

a) R_1 and R_2 are in series with R_3, R_4, and R_5;

b) R_1 and R_2 are in series;

c) R_3, R_4, and R_5 are in parallel;

d) the series combination of R_1 and R_2 is in parallel with the series combination of R_3, R_4 and this parallel combination is in series with R_5.

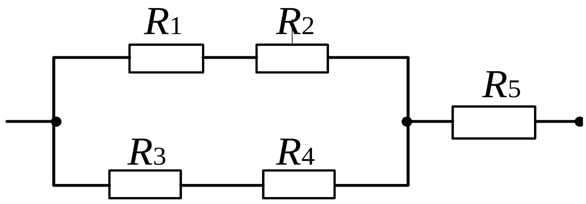


Fig. 3.9. Series-parallel combination

2. Two 3,5 kΩ resistors are in series and this series combination is in parallel with a 6 *k*Ω resistor. The voltage across one of the 3,5 kΩ resistors is 4,25 V. The voltage across the 6 kΩ resistor is:

a) 6 V;

b) 4,25 V;

c) 8,5 V.

4. METHODS OF DC CIRCUIT ANALYSIS

The network analysis includes *determining* each of *unknown currents in branches of electrical circuit* and *voltage in nodes.* There are complex circuits, that configuration cannot be solved by reduction according to series-parallel rules. Such circuits are *complex circuits*. The Fig. 4.1 illustrates complex circuits.

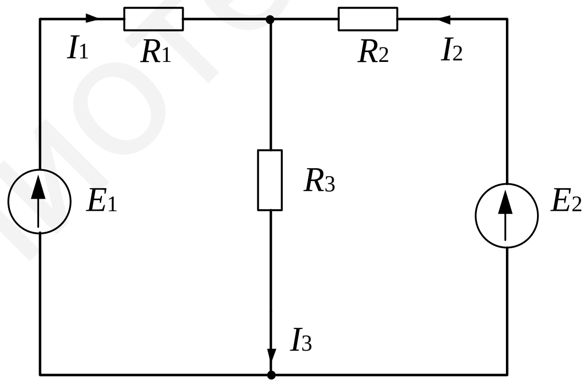


Fig. 4.1. Complex circuit

Resistors R_1 and R_3 aren't in parallel with each other because E_1 has been inserted into R_1's branch. There are no two resistors in this circuit directly in series or parallel with each other.

Note, that complex circuits have more than one source.

4.1. Mesh (loop) current method

The loop or mesh current method allows us to simplify a problem solution. It consists of determining loop currents. The loop or mesh current method is based on KVL.

Loop (or mesh) currents aren't real physical currents. They are *mathematical tools that help us to define* branch currents.

The number of the loop equations equals the number of the meshes:

$$N_{eq} = N_{mesh}.$$

Format approach for mesh analysis. There is simple procedure to form a linear equations set. If a network has n independent loops the system of equations is the following:

$$\begin{cases} I_{11}R_{11} + I_{22}R_{12} + \ldots + I_{nn}R_{1n} = E_{11}; \\ I_{11}R_{21} + I_{22}R_{22} + \ldots + I_{nn}R_{2n} = E_{22}; \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ I_{11}R_{n1} + I_{22}R_{n2} + \ldots + I_{nn}R_{nn} = E_{nn}. \end{cases} \qquad (4.1)$$

The coefficients R_{11}, R_{22}, R_{33}, ..., R_{nn} represent *the total resistance* in each loop and are found by simply adding all the resistances in a particular loop. The terms containing R_{11}, R_{22}, R_{33}, ..., R_{nn} *are positive*.

The remaining resistance coefficients are called *the mutual resistance*. These resistances represent resistance which is common for two loops.

The mutual resistance terms *are positive*, if loop currents through it are in the same direction. The mutual resistance terms *may be negative* if mesh currents through it are in the opposite direction. If the linear equations are correctly written, the coefficients along the principal diagonal (R_{11}, R_{22}, R_{33}, ..., R_{nn}) will be positive.

Also, if the equations are correctly written, the terms will be symmetrical about the principal diagonal, e. g., $R_{12} = R_{21}$.

The terms E_{11}, E_{22}, E_{33}, ..., E_{nn} *are the algebraic sum of all EMF* in this loop.

Example 4.1

Given: $E_1 = 40$ V; $E_2 = 5$ V; $E_3 = 25$ V; $R_1 = 5\ \Omega$; $R_2 = R_3 = 10\ \Omega$.

Find: the currents in each branch for the circuit in Fig. 4.2.

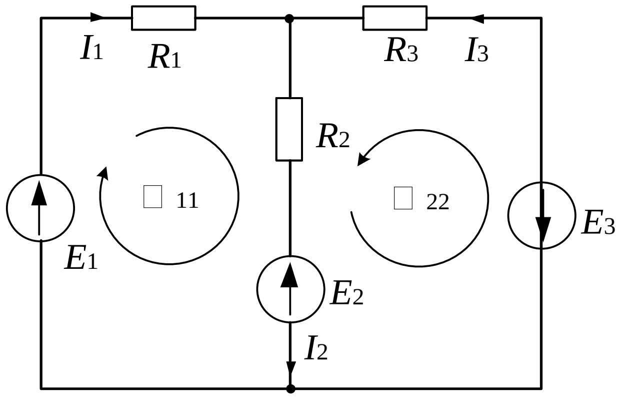


Fig. 4.2. Determining the unknown currents using mesh analysis

***Solution*:**

Step 1. Arbitrarily assign a clockwise mesh current to each interior closed loop in that network. These currents are designated I_{11} and I_{22}.

Step 2. The loop equations are written by applying KVL in each of the loops. The equations are as follows:

$$\text{Loop 1: } I_{11}(R_1 + R_2) + I_{22}R_2 = E_1 - E_2;$$
$$\text{Loop 2: } I_{22}(R_2 + R_3) + I_{11}R_2 = -E_2 - E_3,$$

where R_2 is resistor common to both loops.

Substitution gives us

$$\begin{cases} I_{11} \cdot 15 + I_{22} \cdot 10 = 35; \\ I_{11} \cdot 10 + I_{22} \cdot 20 = -30. \end{cases}$$

Solution of the set gives mesh currents: I_{11} = 5 A; I_{22} = –4 A.

The currents through resistors R_1 and R_2 equal to I_{11} and I_{22} respectively. So, the branch currents $I_1 = I_{11}$ = 5 A and $I_3 = I_{22}$ = –4 A.

The current I_2 is found by algebraic sum the loop currents

$$I_2 = I_{11} + I_{22} = 5 + (-4) = 1 \text{ A}.$$

Whereas branch-current analysis required three equations, this method requires the solution of only two simultaneous linear equations.

Note: if the circuit being analyzed contains current sources, the procedure is simply. In this case the current source will provide one of the loop currents.

Example 4.2

Given: $E_1 = 60$ V; $E_2 = 20$ V; $J_3 = 2$ A; $R_1 = R_2 = R_3 = 10$ Ω.

Find the currents in each branch of the circuit above, using mesh current method in Fig. 4.3.

***Solution*:**

Step 1. Assign mesh currents I_{11} and I_{22}.

The loop current $I_{22} = J_3 = 2$ A.

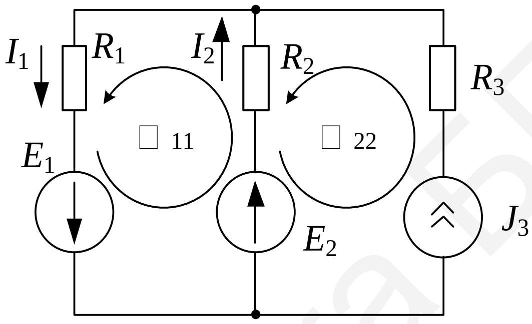


Fig. 4.3. Determining the unknown currents using mesh analysis in case the circuit includes a current source

Step 2. The mesh equation for the loop is

$$I_{11}(R_1 + R_2) - I_{22}R_2 = E_1 + E_2 .$$

Substitution gives us

$$I_{11}\,(10 + 10) - 2 \cdot 10 = 60 + 20 ;$$
$$I_{11} \cdot 20 - 20 = 80 .$$

Solution of the set gives us

$$I_{11} = 5 \text{ A}.$$

The branch currents are:

$$I_1 = I_{11} = 5\ \text{A};$$
$$I_2 = I_{11} - I_{22} = 5 - 2 = 3\ \text{A};$$
$$I_3 = J_3 = 2\ \text{A}.$$

SUMMARY:

– the loop current method is based on Kirchhoff's voltage law;

– a loop current is not necessarily the actual current in a branch.

SELF-ASSESSMENT TEST:

1. Do the loop currents necessarily represent the actual currents in the branches ?
2. When you solve for a current using the loop method and get a negative value, what does it mean?
3. What circuit law is used in the loop current method?
4. Write down the mesh current system equations for three-loop circuit.

4.2. Nodal analysis (or node voltage method)

The *node voltage method* is based on defining the voltage in each node as an independent variable. One of the nodes is selected as *a reference node* and each of the other node voltages is referenced to this node. When each node voltage is defined, Ohm's law may be applied in order to determine the current flowing in each branch.

The node voltage method is based on Kirchhoff's current law and Ohm's law.

The number of the nodal equations equals the number of the nodes minus one:

$$N_{eq} = N_{node} - 1.$$

Format approach for nodal analysis. There is a simple method to write the nodal equations for any network having (n + 1) nodes. When one of these nodes is assign as the reference node, there will be n simultaneous linear equations which will appear as follows:

$$
\begin{cases}
G_{11} \cdot V_1 - G_{12} \cdot V_2 - G_{13} \cdot V_3 - ... - G_{1n} \cdot V_n = I_{node1}; \\
-G_{21} \cdot V_1 + G_{22} \cdot V_2 - G_{23} \cdot V_3 - ... - G_{2n} \cdot V_n = I_{node2}; \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
-G_{n1} \cdot V_1 - G_{n2} \cdot V_2 - G_{n3} \cdot V_3 - ... + G_{nn} \cdot V_n = I_{node\ n}.
\end{cases}
\tag{4.2}
$$

The coefficients G_{11}, G_{22}, G_{33}, ..., G_{nn} represent *the summation of the brunch conductances* attached to the particular node. These terms are always *positive terms*.

The remaining coefficients are called the *mutual conductance terms*. If there is no conductance that is common to two nodes, then this term would be zero. The mutual conductance terms are always *negative*.

If the equations are written correctly, then the terms will be symmetrical about the principal diagonal, e. g., $G_{23} = G_{32}$.

The terms V_1, V_2, ..., V_n are the unknown node voltages.

The terms I_{node1}, I_{node2}, ..., $I_{node\ n}$ represent the algebraic sum of currents of sources entering the node and terms E_n/R_n. If a current source has a current such that it is leaving the node, then the term is included as negative. If a particular current source is shared between two nodes, then this current must be included in both nodal equations.

***The steps used in solving a circuit using nodal analysis are as follows*:**

Step 1. Arbitrarily assign a reference node within the circuit and indicate this node as ground.

Step 2. Arbitrarily assign voltages (V_1, V_2, ..., V_n) to the remaining nodes in the circuit. Remember that you have already assigned a reference node, so these voltages will all be with respect to the chosen reference.

Step 3. Arbitrarily assign a current direction to each branch in which there is no current source.

Step 4. Write the linear equation for each node using the format outlined. If a circuit has a total of (n + 1) nodes (including the reference node), there will be n simultaneous linear equations.

Step 5. Solve the resulting simultaneous linear equations for V_1, V_2, …, V_n.

Step 6. Define each brunch current by Ohm's law, using known node voltages and brunch resistance.

Example 4.3

Given: E_1 = 8 V; E_5 = 12 V; R_1 = 1 Ω; R_2 = 2 Ω; R_3 = 1 Ω; R_4 = 2 Ω; R_5 = 3 Ω.

Find: The nodal voltage using nodal analysis and branch currents in the circuit of the Fig. 4.4.

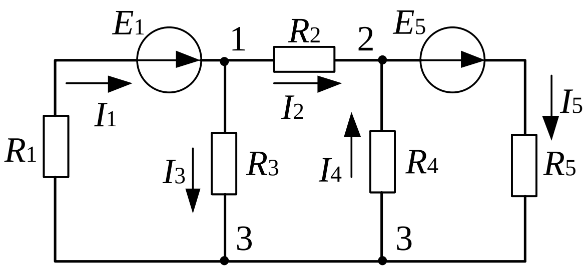


Fig. 4.4. Determining the unknown currents using nodal analysis

***Solution*:**

Step 1. Arbitrarily assign currents at each branch as shown in Fig. 4.4.

Step 2. Label the nodes as shown in Fig. 4.4.

Step 3. Select node «3» as a reference.

Step 4. Write the nodal equations (4.3):

$$\begin{cases} G_{11}\cdot V_1 - G_{12}\cdot V_2 = I_{node1}; \\ -G_{21}\cdot V_1 + G_{22}\cdot V_2 = I_{node2}. \end{cases} \tag{4.3}$$

In the set of equations (4.3):

$$G_{11} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{1} = 2{,}5 \quad \text{S};$$

$$G_{22} = \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{2} + \frac{1}{2} + \frac{1}{3} = 1{,}33 \quad \text{S};$$

$$G_{12} = G_{21} = \frac{1}{R_2} = \frac{1}{2} = 0{,}5 \quad \text{S};$$

$$I_{node1} = \frac{E_1}{R_1} = \frac{8}{1} = 8 \quad \text{A};$$

$$I_{node2} = -\frac{E_5}{R_5} = -\frac{12}{3} = -4 \quad \text{A}.$$

Step 5. The nodal equations (4.3) may be simplified as

$$\begin{cases} 2{,}5V_1 - 0{,}5V_2 = 8; \\ -0{,}5V_1 + 1{,}33V_2 = -4. \end{cases}$$

The solutions are as the following: $V_1 = 2{,}811$ V; $V_2 = -1{,}95$ V.

Step 6. Determine the branch currents using Ohm's law:

$$I_1 = \frac{V_3 - V_1 + E_1}{R_1} = \frac{0 - 2{,}811 + 8}{1} = 5{,}189 \text{ A};$$

$$I_2 = \frac{V_1 - V_2}{R_2} = \frac{2{,}811 + 1{,}95}{2} = 2{,}378 \text{ A};$$

$$I_3 = \frac{V_1 - V_3}{R_3} = \frac{2{,}811 - 0}{1} = 2{,}811 \text{ A};$$

$$I_4 = \frac{V_3 - V_2}{R_4} = \frac{0 - (-1{,}95)}{2} = 0{,}973 \text{ A};$$

$$I_5 = \frac{V_2 - V_3 + E_5}{R_5} = \frac{-1{,}95 - 0 + 12}{3} = 3{,}351 \text{ A}.$$

SUMMARY:

– a node is the junction of two or more components;

– the node voltage method is based on Kirchhoff's current law and Ohm's law;

– in a circuit containing (n + 1) nodes, we can write at most n independent equations.

SELF-ASSESSMENT TEST:

1. What circuit law is the basis for the node voltage method?
2. What is the reference node?
3. Which coefficients in the set of nodal analysis equations are always positive and which coefficients are negative?
4. Which law is used for determine brunch currents in nodal analysis?
5. Write down the system of nodal equations for circuit, that has four nodes.

5. NETWORK THEOREMS

5.1. The superposition theorem

Some circuits include more than one voltage or current source. Such multiple sources circuits may be analyzed with a method, based on the superposition theorem.

A general statement of the superposition theorem is as follows: **the total current through or voltage across a resistor or branch may be determined *as algebraic sum* of the currents or voltages produced independently by each source**.

The following are *the general steps used in applying the superposition method:*

Step 1. Leave one voltage (or current) source at a time in the circuit and replace each of the other voltage (or current) sources with its internal resistance. For ideal sources a short represents zero internal resistance and an open represents infinite internal resistance.

Step 2. Determine the particular current (or voltage) just as if there were only one source in the circuit.

Step 3. Take the next source in the circuit and repeat steps 1 and 2. Do this for each source.

Step 4. To find the actual current in a given branch, algebraically sum the currents due to each individual source. If the particular currents are in the same direction, they are added. If the particular currents are in opposite directions, they are subtracted with the direction of the resulting current the same as the larger of the original quantities. Once you find the current, you can determine the voltage using Ohm's law.

The approach to superposition is demonstrated in Fig. 5.1 for a complex circuit with voltage source and current source. Study the steps in this figure.

Example 5.1

Given: $J = 2$ A; $E = 20$ V; $R_1 = 24$ Ω; $R_2 = 16$ Ω.

Find: the currents in the resistors R_1 and R_2 in the circuit of Fig. 5.1.

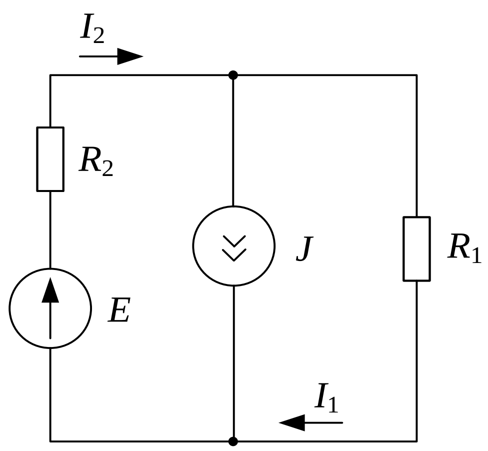
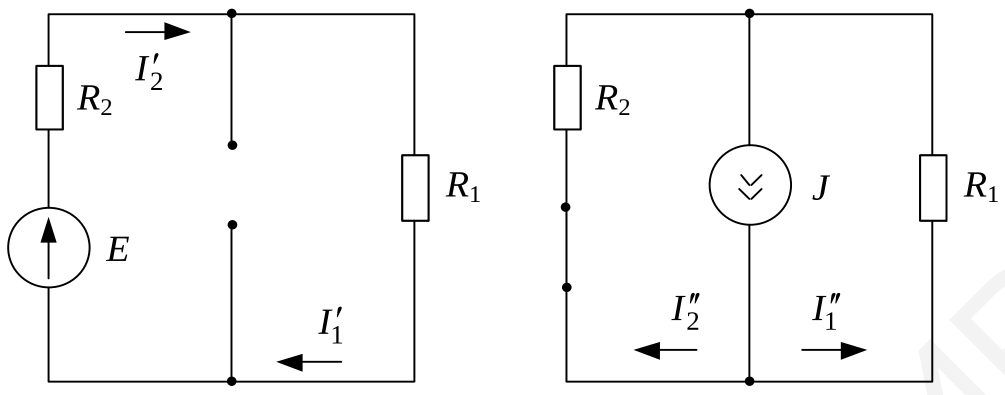


Fig. 5.1. Determining the unknown currents using the method of superposition

***Solution*:**

Step 1. The current source replaced with an open circuit (Fig. 5.2, a).



a b

Fig. 5.2. Leaving voltage source (a) and current one (b) at a time in the circuit and replacing with their internal resistances

Note, that the resistor R_1 is in series with resistor R_2, so the individual current $I_1' = I_2'$ due to the voltage source is determined by Ohm's law:

$$I_1' = \frac{E}{R_1 + R_2} = \frac{20}{16+24} = 0,5 \text{ A.}$$

Step 2. Determine the currents through R_1 and R_2 due to the current source by removing the voltage source and replacing it with a short circuit (*zero* volts) as shown in Fig. 5.2, b.

The currents through R_1 and R_2 are found using the current divider rule [1]:

$$I_1'' = J\frac{R_2}{R_1 + R_2} = 2 \cdot \frac{16}{24+16} = 0,8 \text{ A;}$$

$$I_2'' = J\frac{R_1}{R_1 + R_2} = 2 \cdot \frac{24}{24+16} = 1,2 \text{ A.}$$

Step 3. The resulting currents through R_1 and R_2 are found by applying the superposition theorem:

$$I_1 = I_1' - I_1'' = 0,5 - 0,8 = -0,3 \text{ A;}$$

$$I_2 = I_2' + I_2'' = 0,5 + 1,2 = 1,7 \text{ A.}$$

Note: the superposition theorem *does not apply to power*, since power *is not linear quantity*, but rather is found as the square of either current or voltage:

$$P \neq P' + P''.$$

Verify in example 1 that the superposition theorem does not apply to power.

If we assume (incorrectly) that the superposition theorem applies for power, we would have the power due the first source given as

$$P_2' = (I_1')^2 R_2 = (0{,}5)^2 \cdot 16 = 4 \quad \text{W},$$

and the power due to the second source as

$$P_2'' = (I_2'')^2 R_2 = (1{,}2)^2 \cdot 16 = 23{,}04 \quad \text{W}.$$

The total power, if superposition applies, would be:

$$P_{t2} = P_2' + P_2'' = 4 + 23{,}04 = 27{,}04 \quad \text{W}.$$

Clearly, this result is wrong, since the actual power dissipated by the resistor R_2 is correctly given as

$$P_{t2} = I_2{}^2 \cdot R_2 = (0{,}7)^2 \cdot 16 = 7{,}84 \quad \text{W}.$$

SUMMARY:

– the principle of superposition can easily be applied to circuits containing multiple sources and is sometimes an effective solution technique;

– the superposition theorem is applied to voltage, so voltage is linear function of current;

– the superposition theorem does not apply to power.

SELF-ASSESSMENT TEST:

1. State the superposition theorem.

2. Why is an ideal voltage source shorted when the superposition theorem is applied ?

3. Why is an ideal current source opened when the superposition theorem is applied?

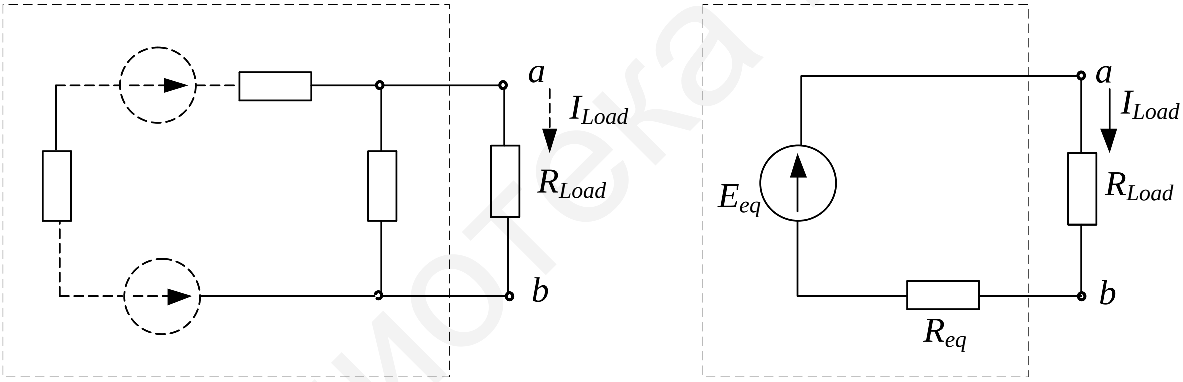
5.2. Thevenin's theorem

Thevenin's theorem is one of the most important theorems of electrical circuits.

Thevenin's Theorem states the following: **any two-terminal complex linear DC network can be replaced by an equivalent circuit consisting of a single voltage source and one series resistor. The voltage source has EMF, that equals to $E_{eq} = E_{Th}$ and the series resistor equals $R_{eq} = R_{Th}$.**

Thevenin's theorem allows even the most complicated circuit to reduce to a single voltage source and a single resistance.

Thevenin's theorem conversion. The Fig. 5.3 illustrates Thevenin's conversation for simplification to a single voltage source and a single resistance.



a b

Fig. 5.3. The effect of applying Thevenin's theorem

Ohm's law may be applied for the circuit of Fig. 5.3, b:

$$I_L = \frac{E_{eq}}{R_{eq} + R_L}, \qquad (5.1)$$

where E_{eq} is called *Thevenin voltage* ($E_{eq} = E_{Th}$), R_{eq} is called *Thevenin resistor* ($R_{eq} = R_{Th}$).

Thevenin voltage E_{eq} is *potential difference* between the terminals «*a*» and «*b*» with the element R_L removed from the circuit (Fig. 5.4, a).

Thevenin resistor R_{eq} is the total resistance between the terminals «*a*» and «*b*» with the element removed and with all sources replaced by their internal resistances (Fig. 5.4, b).

Recall, that ideal voltage source is replaced by short circuit and ideal current source is replaced by open circuit.

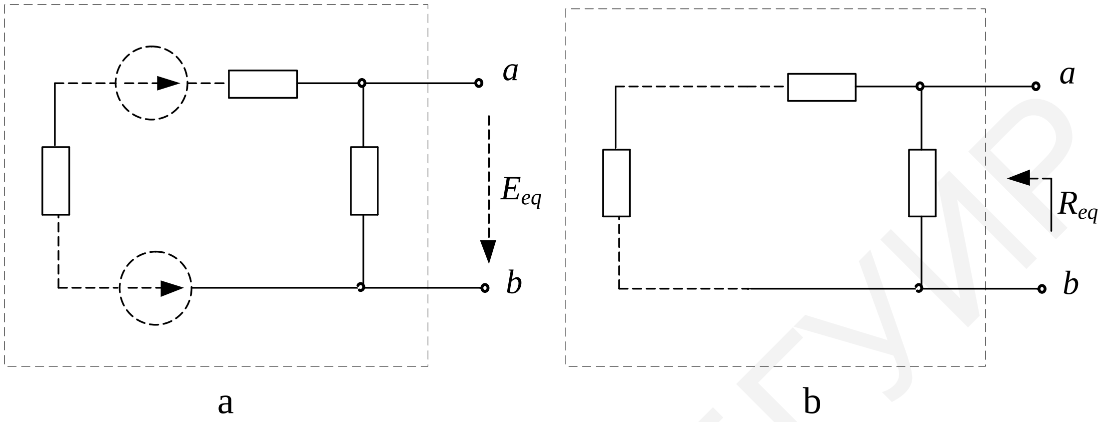


Fig. 5.4. The effect of applying Thevenin's theorem

The basic procedure for solving a circuit using Thevenin's theorem is as follows:

Step 1. Remove the load resistor R_{Load}.

Step 2. Find $E_{eq} = E_{Th}$ using KVL. Currents for KVL are defined by the usual circuit analysis methods (see Fig. 5.3, a).

Step 3. Find $R_{eq} = R_{Th}$ by shorting all voltage sources or by open circuiting all the current sources (see Fig. 5.3, b).

Step 4. Find the current flowing through the load resistor R_L according the formula (5.1).

Example 5.2

Given: E = 20 V; R_1 = 24 Ω; R_L = 16 Ω; J = 2 *A*.

Determine the Thevenin equivalent circuit external to the resistor R_L for the circuit of Fig. 5.5. Use the Thevenin equivalent circuit to calculate the current through R_L .

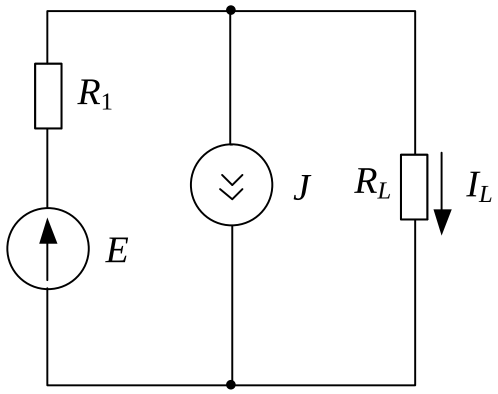


Fig. 5.5. Determining the current through the load resistor applying Thevenin’s theorem

***Solution*:**

Step 1. Removing the load resistor from the circuit, we obtain the circuit shown in Fig. 5.6.

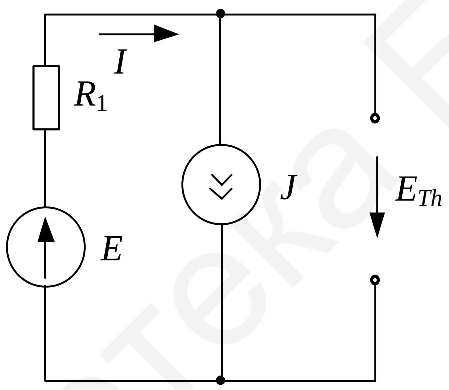


Fig. 5.6. Removing the load resistor R_L.

Step 2. From Fig. 5.6 the Thevenin voltage E_{Th} is found as

$$E_{Th} = E - JR_1 = 20 - 24 \cdot 2 = -28 \text{ V}.$$

Step 3. Setting the source to zero, we have the circuit shown in Fig. 5.7.

The Thevenin resistance is $R_{Th} = R_1 = 24\ \Omega$. The resulting Thevenin equivalent circuit is shown in Fig. 5.8.

Step 4. Using the Thevenin equivalent circuit, we easily find current through R_L as

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{-28}{24+16} = -0{,}7 \quad \text{A}.$$

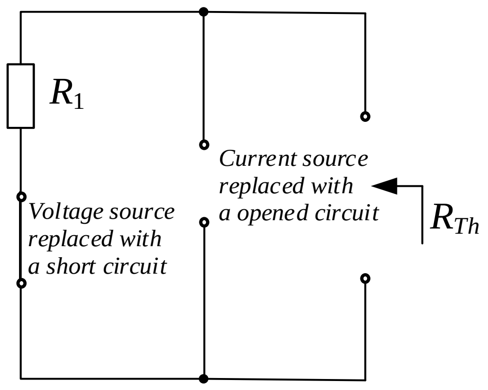


Fig. 5.7. Setting the source to zero

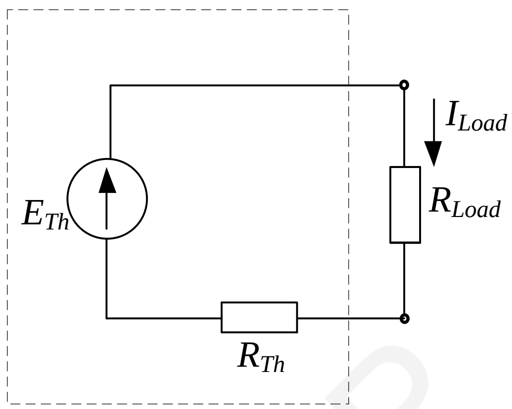


Fig. 5.8. The resulting Thevenin equivalent circuit

SUMMARY:

– Thevenin's theorem provides for the reduction of any two-terminal linear resistive circuit to an equivalent form consisting of an equivalent voltage source in series with an equivalent resistance;

– the term equivalency, as used in Thevenin's, means that when a given load resistance is connected to the equivalent circuit, it will have the same voltage across it and the same current through it as when it was connected to the original circuit.

SELF-ASSESSMENT TEST:

1. State the Thevenin's theorem.
2. What are the two components of a Thevenin equivalent circuit?
3. Thevenin's theorem converts a circuit to an equivalent form consisting of:
   a) a current source and a series resistance;
   b) a voltage source and a parallel resistance;
   c) a voltage source and a series resistance;
   d) a current source and a parallel resistance.
4. The Thevenin equivalent voltage for a given circuit is found by:
   a) shorting the output terminals;
   b) opening the output terminals;
   c) shorting the voltage source;
   d) removing the voltage source and replacing it with a short.
5. Draw the general form of a Thevenin equivalent circuit.
6. List steps for applying Thevenin's theorem.

6. DC CIRCUIT. TASKS

6.1 Ohm's law. Tasks

Task №1

Given: R = 17,5 Ω. How much voltage is needed to produce:

a) 2 A of current?

b) 6,4 A of current?

Answer: a) E = 35 V; b) E = 112 V.

Task №2

Given: I = 55 mA; R = 80 Ω. How much voltage will be measured across the resistor in the circuit of the Fig. 6.1? Indicate the polarity of voltmeter terminals.

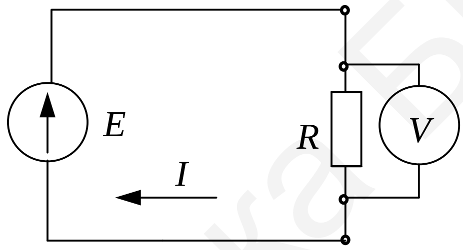


Fig. 6.1. The circuit for the task №2

Answer: V = 4,4 V.

Task №3

Current of 4 mA flows through series resistors 65 kΩ and 30 kΩ. How much voltage is across the resistors?

Answer: V = 380 V.

Task №4

If the ammeter in the circuit of the Fig. 6.2 indicates 7,5 mA and the voltmeter reads 150 V, what is the value of R?

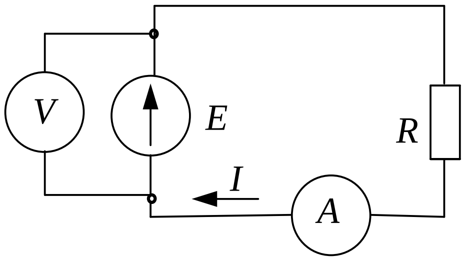


Fig. 6.2. The circuit for the task №4

Answer: R = 20 kΩ.

6.2. Kirchhoff's current law. Tasks

Task №5

Given: I_t = 125 mA; I_1 = 40 mA; I_3 = 50 mA. Determine the current I_2 through R_2 in the circuit of the Fig. 6.3.

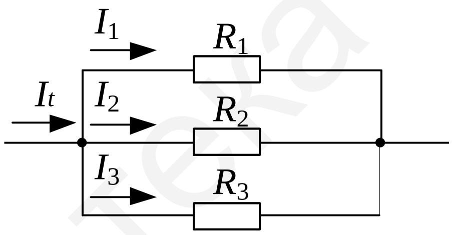


Fig. 6.3. The circuit for the task №4

Answer: I_2 = 35 mA.

Task №6

Ammeters measured the following currents: I_1 = 7,75 mA; I_3 = 3,25 mA and I_5 = 1,1 mA. Use KCL to find the currents I_2 and I_4 in Fig. 6.4.

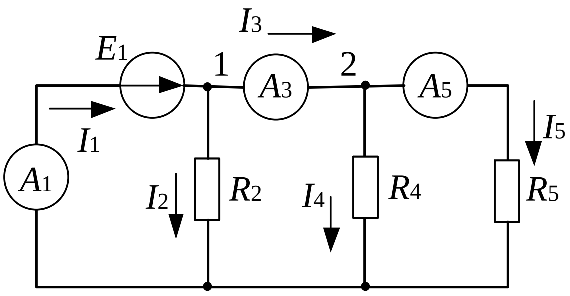


Fig. 6.4. The circuit for the task №6

Answer: I_3 = 3,25 mA; I_5 = 1,1 mA.

6.3. Kirchhoff's voltage law. Tasks

Task №7

Given: I = 0,25 mA; R_1 = 7,2 kΩ; R_2 = 5,6 kΩ; R_3 = 6,4 kΩ. Determine the voltage drop across each resistor and the value of battery voltage in the circuit of Fig. 6.5.

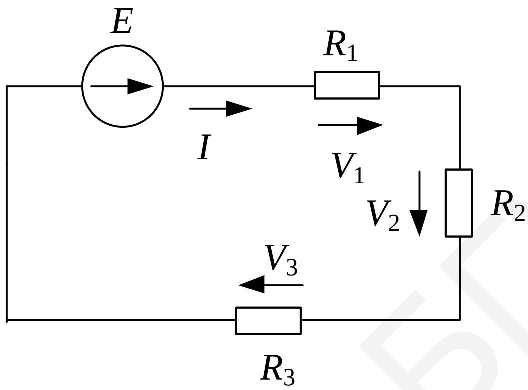


Fig. 6.5. The circuit for the task №7

Answer: V_1 = 1,8 V; V_2 = 1,4 V; V_3 = 1,6 V; E = 4,8 V.

Task №8

Given: in the circuit of the task №7 E = 16 V; V_1 = 20 V; V_2 = 5 V. Find the voltage drop across resistor R_3. What is the real third voltage direction?

Answer: V_3 = – 9 V. To the right.

6.4. Series and parallel circuits. Series-parallel circuits. Tasks

Task №9

Given: E = 25 V; R_1, R_2 and R_3 are in series. R_1 = 1,2 kΩ; R_2 = 6,6 kΩ; R_3 = 1,5 kΩ. What the current in the circuit?

Answer: I = 2,68 mA.

Task №10

Three 470 Ω resistors are connected in series with 48 V source:

a) What is the current in the circuit?

b) What is the voltage across each resistor?

Answer: I = 34 mA; V = 16 V.

Task №11

Given: E_1 = 10 V; E_2 = 50 V; E_3 = 25 V; R = 5 Ω. Determine the voltmeter's and ammeter's readings in the circuit of Fig. 6.6.

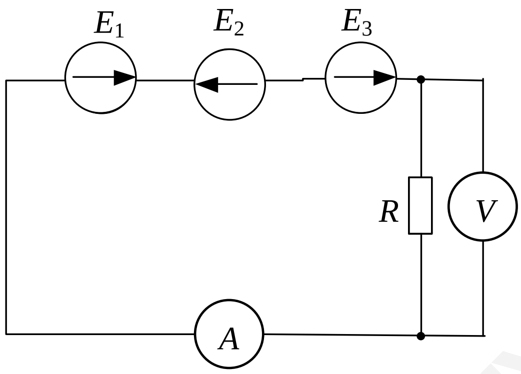


Fig. 6.6. The circuit for the task №11

Answer: I = –3 A; V = –15 V.

Task №12

Given: R_1, R_2 and R_3 are in parallel. R_1 = 78 Ω; R_2 = 84 Ω; R_3 = 84 Ω. Calculate the total parallel resistance between points «a» and «b» in the circuit of Fig. 6.7.

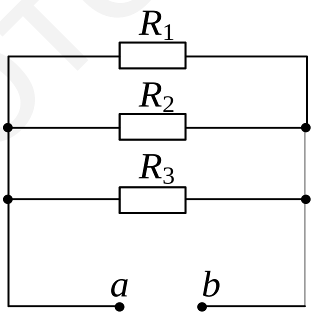


Fig. 6.7. The circuit for the task №12

Answer: R_t = 27,3 Ω.

Task №13

Given: assume that resistors in the task №12 are $R_1 = R_2 = R_3$ = 81 kΩ. Find the total resistance between points «a» and «b».

Answer: R_t = 27 kΩ.

Task №14

Given: in the circuit of Fig. 6.8 E = 120 V; R_1 = 10 kΩ; R_2 = 3,3 kΩ; R_3 = 20 kΩ. Determine:

a) the current through each resistor, using Ohm's law;

b) the total current, using KCL.

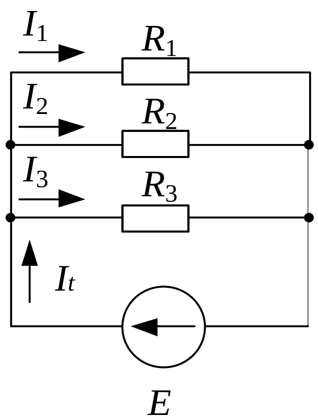


Fig. 6.8. The circuit for the task №14

Answer:

a) I_1 = 12 mA; I_2 = 36,36 mA; I_3 = 6 mA;

b) I_t = 54,36 mA.

Task №15

Given: J_1 = 10 mA; J_2 = 20 mA; R_L = 1,75 kΩ. Determine the current through the resistor R_L in the circuit of Fig. 6.9.

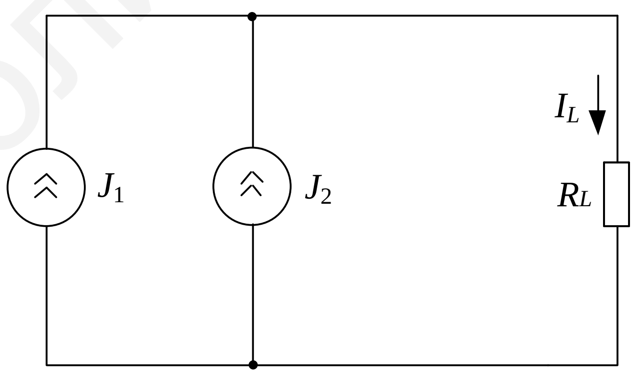


Fig. 6.9. The circuit for the task №15

Answer: I_L = 30 mA.

Task №16

Given: R_1 = 14 Ω is in series with two equal-value resistors $R_2 = R_3$ = 60 Ω in parallel. Find the total resistance of this circuit.

Answer: R_t = 44 Ω.

Task №17

Given: R_1 = 51 Ω; R_6 = 34 Ω; $R_2 = R_3$ = 60 Ω; R_4 = 12 Ω; R_5 = 18 Ω. Find the total resistance looking into source's terminals in the circuit of Fig. 6.10.

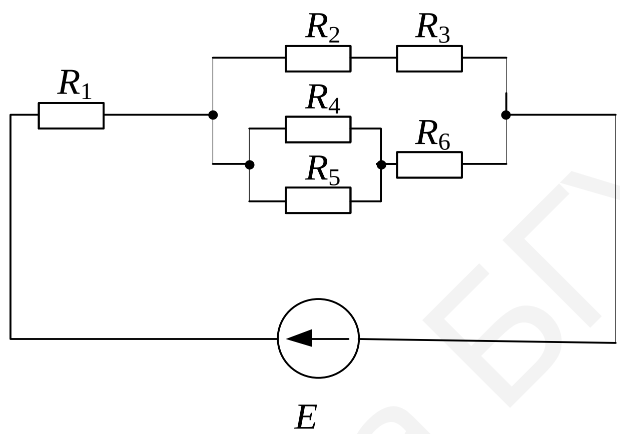


Fig. 6.10. The circuit for the task №17

Answer: R_t = 81,67 Ω.

Task №18

Given: E = 100 V; R_1 = 40 Ω; R_2 = 17,5 Ω; R_3 = 45 Ω; R_4 = 45 Ω. Determine the input current and the voltage across resistor R_3 in the circuit of Fig. 6.11.

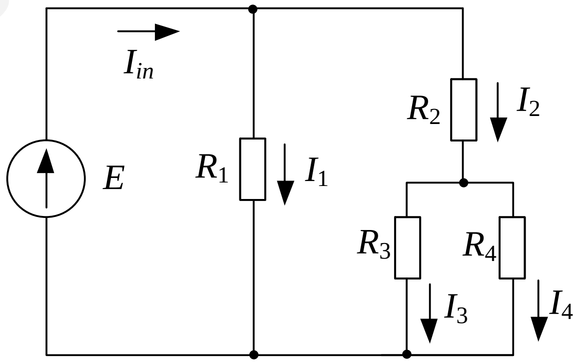


Fig. 6.11. The circuit for the task №18

Answer: I_{in} = 5 A; V_3 = 56,25 V.

Task №19

Given: E = 15 V; R_1 = 10 Ω; R_2 = R_3 = 30 Ω; R_4 = 12 Ω. Find the total resistance and current I_2 in the circuit of Fig. 6.12.

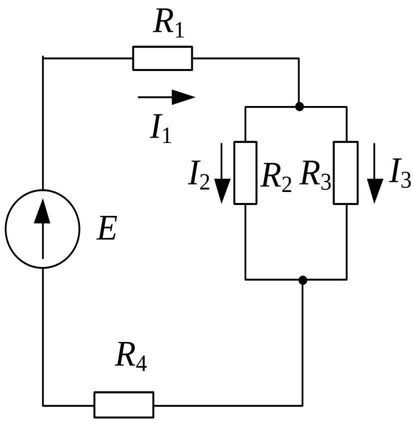


Fig. 6.12. The circuit for the task №19

Answer: R_t = 37 Ω; I_2 = 0,2 A.

6.5. Brunch-current method. Tasks

Task №20

Given: E_1 = 12 V; E_2 = 8 V; E_3 = 4 V; R_1 = 4 Ω; R_2 = 4 Ω; R_3 = 8 Ω. Find all currents in the circuit of Fig. 6.13.

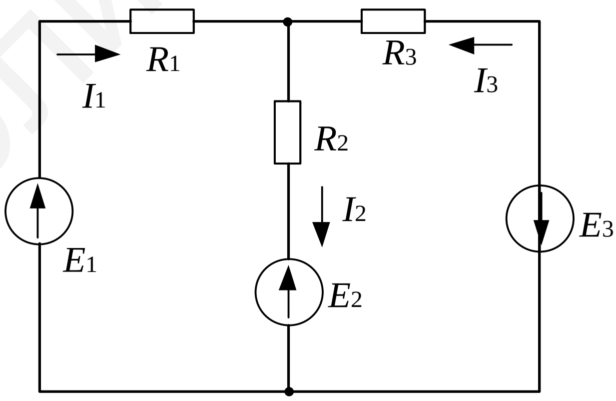


Fig. 6.13. The circuit for the task №20

Answer: I_1 = 1,2 A; I_2 = – 0,2 A; I_3 = –1,4 A.

Task №21

Given: V = 20 V; E_2 = 100 V; R_1 = 10 Ω; R_2 = 20 Ω; R_3 = 10 Ω. Find all currents in the circuit of Fig. 6.14.

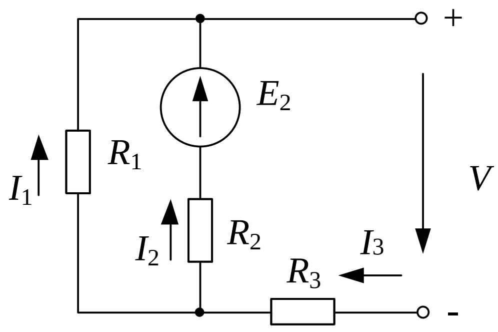


Fig. 6.14. The circuit for the task №21

Answer: I_1 = –2,8 A; I_2 = 3,6 A; I_3 = 0,8 A.

6.6. Mesh-current method. Tasks

Task №22

Given: E_1 = 8 V; E_2 = 4 V; R_1 = 2 Ω; R_2 = 2 Ω; R_3 = 8 Ω. Calculate all branch currents in the circuit of Fig. 6.15.

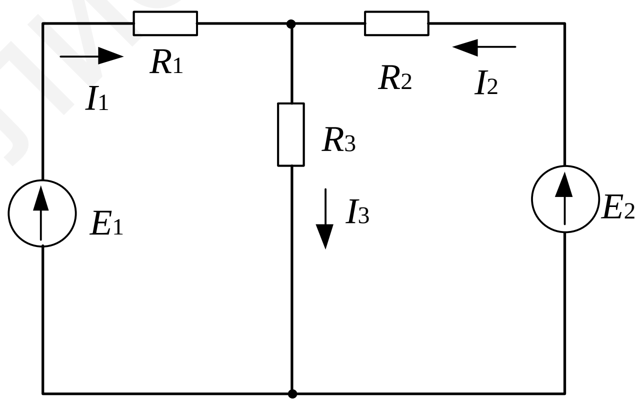


Fig. 6.15. The circuit for the task №22

Answer: I_1 = 1,333 A; I_2 = –0,666 A; I_3 = 0,0667 A.

Task №23

Given: E_1 = 10 V; E_3 = 20 V; R_1 = 2 Ω; R_2 = 5 Ω. Find all currents in the circuit of Fig. 6.16.

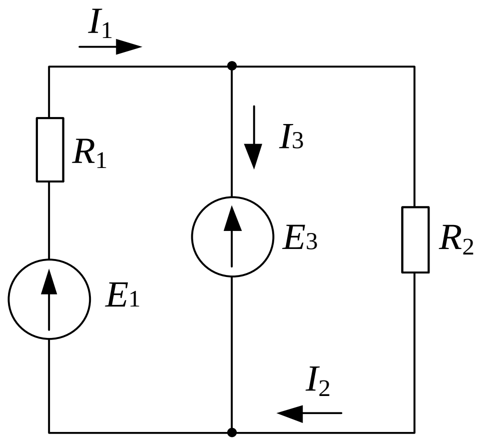


Fig. 6.16. The circuit for the task №23

Answer: I_1 = –5 A; I_2 = 4 A; I_3 = –9 A.

Task №24

Given: E = 20 V; J = 0,5 A; R_1 = 2 Ω; R_2 = 8 Ω. Find all currents in the circuit of Fig. 6.17.

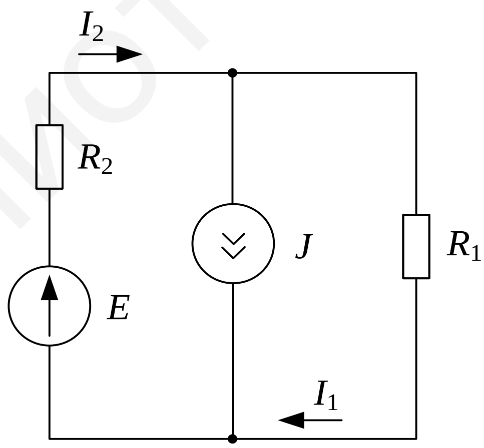


Fig. 6.17. The circuit for the task №24

Answer: I_1 = 1,6 A; I_2 = 2,1 A.

6.7. Nodal analysis. Tasks

Task №25

Given: E_1 = 24 V; E_3 = 12 V; J = 20 A; R_1 = 6 Ω; R_2 = 6 Ω; R_3 = 3 Ω; R_4 = 12 Ω; R_5 = 4 Ω. Find all currents by nodal analysis in the circuit of Fig. 6.18.

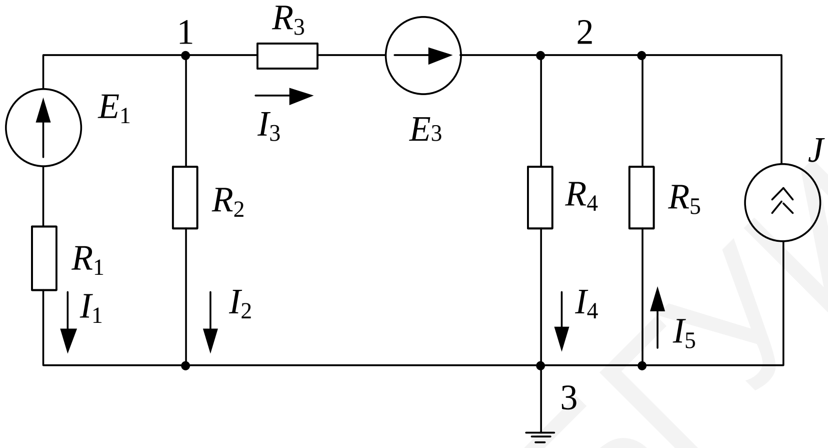


Fig. 6.18. The circuit for the task №25

Answer: I_1 = 0 A; I_2 = 4 A; I_3 = –4 A; I_4 = 4 A; I_2 = –12 A.

Task №26

Given: E_1 = 10 V; J = 2,5 A; R_1 = 5 Ω; R_2 = 6,2 Ω; R_3 = 4,7 Ω; R_4 = 8,2 Ω. Find all currents by nodal analysis in the circuit of Fig. 6.19.

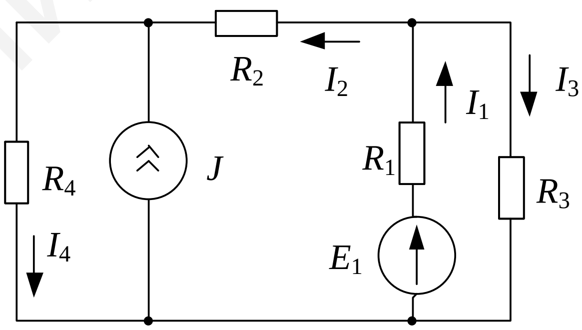


Fig. 6.19. The circuit for the task №26

Answer: I_1 = – 0,577 A; I_2 = 0,931 A; I_3 = –1,5 A; I_4 = 1,569 A.

Task №27

Given: $E_1 = E_2 = 120$ V; $R_1 = 60$ Ω; $R_2 = 15$ Ω; $R_3 = 90$ Ω; $R_4 = 60$ Ω; $R_5 = 12$ Ω. Find all currents by nodal analysis in the circuit of Fig. 6.20.

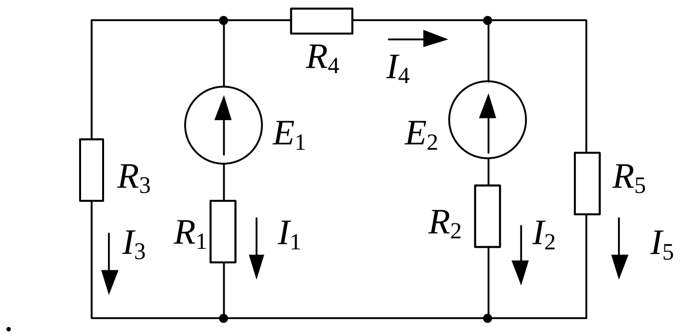


Fig. 6.20. The circuit for the task №27

Answer: $I_1 = 0{,}56$ A; $I_2 = -1{,}92$ A; $I_3 = -0{,}96$ A; $I_4 = 1{,}52$ A; $I_5 = -0{,}4$ A.

6.8. The superposition theorem. Tasks

Task №28

Given: $E_1 = 15$ V; $E_3 = 7{,}5$ V; $R_1 = R_2 = R_3 = 150$ Ω. Find current I_2, using the superposition theorem (Fig. 6.21).

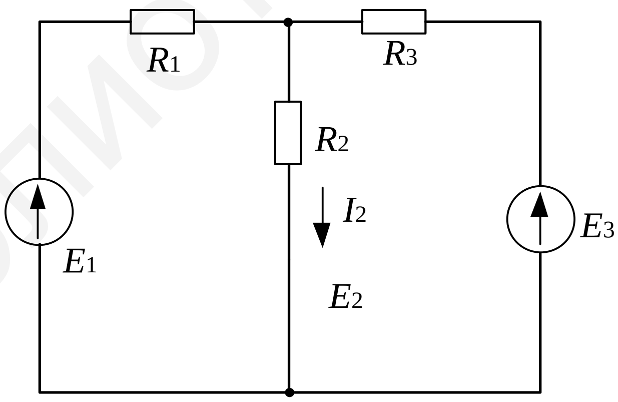


Fig. 6.21. The circuit for the task №28

Answer: $I_2 = 0{,}05$ A.

Task №29

Solve task №24 using the superposition theorem.

Answer: $I_1 = 1{,}6$ A; $I_2 = 2{,}1$ A.

Task №30

Given: J_1 = 10 mA; J_2 = 3 mA; R_1 = 100 Ω; R_2 = 56 Ω. Find the current by I_1 in the circuit of Fig. 6.22.

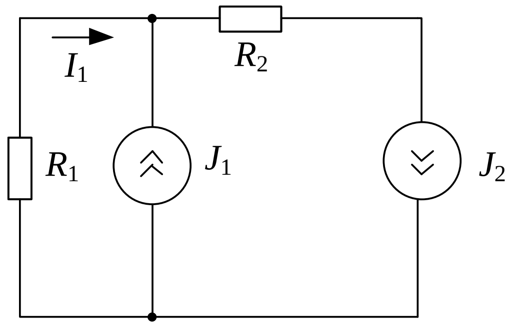


Fig. 6.22. The circuit for the task №30

Answer: I_1 = –7 mA.

Task №31

Given: E_1 = 20 V; J = 2 A; R_1 = 24 Ω; R_2 = 16 Ω. Find the currents I_1 and I_2 in the circuit of Fig. 6.23.

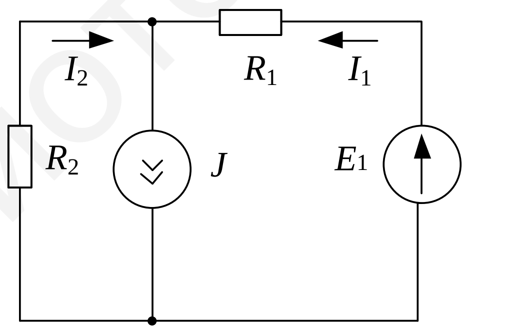


Fig. 6.23. The circuit for the task №31

Answer: I_1 = 1,3 A; I_2 = –0,7 A.

6.9. Thevenin's theorem. Tasks

Task №32

Given: E = 10 V; R_1 = 10 Ω; R_2 = 20 Ω; J_4 = 5 A. Find current I_2 in the circuit of Fig. 6.24.

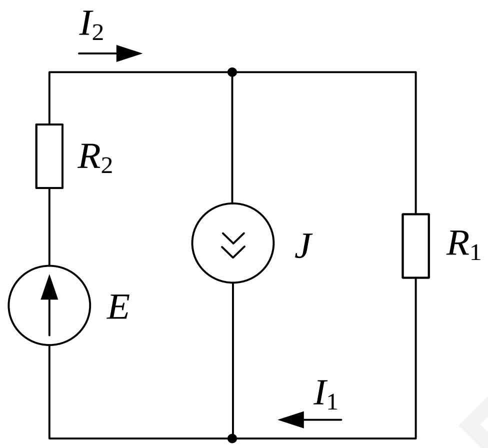


Fig. 6.24. The circuit for the task №32

Answer: E_{Th} = 60 V; R_{Th} = 10 Ω; I_2 = 2 A.

Task №33

Given: E_1 = 22 V; E_1 = 10 V; R_1 = 6 Ω; R_2 = 3 Ω; R_3 = 5 Ω; R_4 = 3,5 Ω; R_5 = 2,5 Ω. Find current I_2 in the circuit of Fig. 6.25.

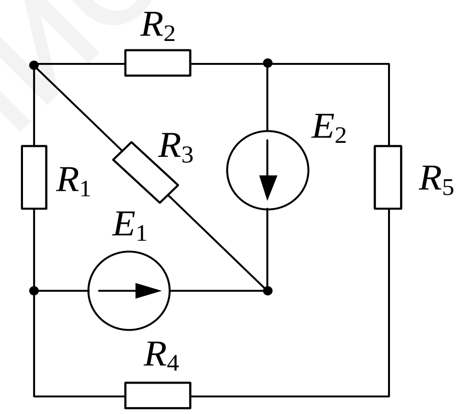


Fig. 6.25. The circuit for the task №33

Answer: E_{Th} = 14 V; R_{Th} = 7 Ω; I_3 = 2 A.

Task №34

Given: E_1 = 10 V; J = 50 mA; R_1 = 300 Ω; R_2 = 200 Ω; R_3 = 80 Ω. Use Thevenin's theorem to determine the current I_3 in the circuit of Fig. 6.26.

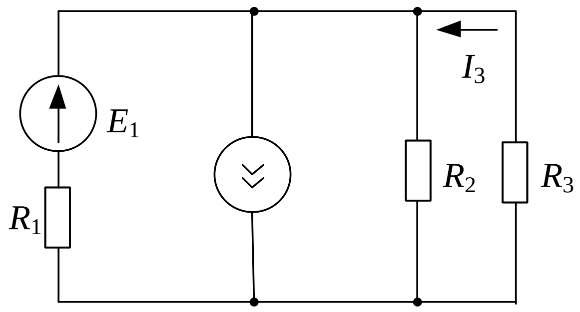


Fig. 6.26. The circuit for the task №34

Answer: I_3 = 10 mA.

List of literature

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В двух частях
Часть 1

АНАЛИЗ ЦЕПЕЙ ПОСТОЯННОГО ТОКА

BRIEF ELECTRICAL CIRCUIT THEORY AND PRACTICAL PROBLEMS

In two parts
Part 1

DC ANALYSIS

ПОСОБИЕ

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