# ENERGY LEVELS OF ELECTRON IN CIRCULAR QUANTUM DOT IN THE PRESENCE OF SPIN-ORBIT INTERACTIONS

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## I. INTRODUCTION

The motion of an electron in an inner layer of a double semiconductor heterostructure is usually treated as two-dimensional. In addition, the planar motion is also restricted if an electron is placed in quantum dot localized in a middle layer of heterostructure. The Rashba [1] and Dresselhaus [2] spin-orbit interactions are presented by the formulas $V_R = \alpha_R(\sigma_x p_y - \sigma_y p_x)/\hbar$ and $V_D = \alpha_D(\sigma_x p_x - \sigma_y p_y)/\hbar$, where $\sigma_x$ and $\sigma_y$ are the standard Pauli spin-matrices. The Rashba interaction strength can be controlled by an external electric field, and the Dresselhaus interaction strength can be varied by changing the width of quantum well along the $z$ axis. In the general case the whole spin-orbit interaction has the form $V_R + V_D$. At the same time, the

considerable attention is paid to the special case, when the spin-orbit interactions of Rashba and Dresselhaus have equal strengths $\alpha_R = \alpha_D$.

As a rule, the circular quantum dot is simulated with the help of axially symmetric confinement potentials $V(\rho)$, where $\rho = \sqrt{x^2 + y^2}$. In [3, 4], the simple but sufficiently adequate rectangular potential of finite depth was proposed. This model with discontinuous potential describes the main properties of circular quantum dots but without taking into account the individual characteristics. In [5], the smooth confinement potential of new type was applied in the case of $\alpha_R = \alpha_D$. Now we use this potential in order to calculate the energy levels of electron for unequal but close strengths $\alpha_R \neq \alpha_D$.

## II. METHODS AND RESULTS

The circular quantum dot of radius $\rho_o$ is described by means of the confinement potential $V(\rho) = V_0 v(r)$, where $V_0$ is the depth of potential well. The function $v(r)$ depends on ratio $r = \rho / \rho_0$ in the following way: $v(r) = 0$ for $0 < r < g$, $v(r) = v_1(r)$ for $g < r < s$, $v(r) = v_2(r)$ for $s < r < 1$, and $v(r) = 1$ for $r > 1$. The functions $v_1(r)$ and $v_2(r)$ are of the form

$$v_1(r) = \frac{1}{2}\frac{(1+s^2)}{(1-g^2)(s^2-g^2)}\left(r - \frac{g^2}{r}\right)^2, \qquad v_2(r) = 1 - \frac{1}{2}\frac{(g^2+s^2)}{(1-g^2)(1-s^2)}\left(r - \frac{1}{r}\right)^2. \tag{1}$$

The parameters $g$ and $s$ change within ranges $0 < g < 1$ and $g < s < 1$.

The total Hamiltonian can be written as a sum $H = H_0 + H_1$, where $H_0 = (p_x^2 + p_y^2)/2\mu + V(\rho) + (\alpha_R + \alpha_D)(\sigma_x - \sigma_y)(p_x + p_y)/2\hbar$ $\mu$ is the effective electron mass and $H_1 = \gamma(\alpha_R + \alpha_D)(\sigma_x + \sigma_y)(p_y - p_x)/2\hbar$, $\gamma = (\alpha_R - \alpha_D)/(\alpha_R + \alpha_D)$. It is easy to show that the solutions of the unperturbed Schrödinger equation $H_0\Psi_0 = E_0\Psi_0$ admit a factorization

$$\Psi_0^{\pm}(x,y) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{-i\pi/4} \end{pmatrix} \exp\left[\mp \frac{(\alpha_R - \alpha_D)\mu}{\sqrt{2}\hbar}(x+y)\right] e^{im\phi}u(\rho), \quad m = 0, \pm 1, \pm 2, \ldots \tag{2}$$

Here $m$ is the angular momentum quantum number. Introducing dimensionless quantities $e_0 = 2\mu\rho_o^2 E_0/\hbar$, $v_0 = 2\mu\rho_o^2 V_0/\hbar$ and $a = \mu\rho_o(\alpha_R + \alpha_D)/\hbar$ we get the radial equation

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{m^2u}{r^2} + \left(e_0 + a^2 - v_0 v(r)\right)u = 0. \tag{3}$$

In the regions $0 < r < g$ and $r > 1$, the exact solutions of this equation are expressed via the Bessel functions and in the region $g < r < s$, we get the solutions in terms of the confluent hypergeometric functions.

We introduce the dimensionless perturbation $h_1 = 2\mu\rho_o^2 H_1/\hbar$. Since each energy level of the unperturbed system is doubly degenerate with two eigenfunctions $\Psi_0^{\pm}$ we consider the contribution of $h_1$ with the help of the perturbation theory in the degenerate case for the small value of $\gamma$. We have the following equalities $\left\langle \Psi_0^+ \middle| h_1 \middle| \Psi_0^+ \right\rangle = \left\langle \Psi_0^- \middle| h_1 \middle| \Psi_0^- \right\rangle = 0$, $\left\langle \Psi_0^+ \middle| h_1 \middle| \Psi_0^- \right\rangle = \left\langle \Psi_0^- \middle| h_1 \middle| \Psi_0^+ \right\rangle = \gamma\delta$, where

$$\delta = -2ma\int_0^\infty J_1(2ar)u^2(r)\,dr \Big/ \int_0^\infty u^2(r)\,r\,dr. \tag{4}$$

Then we obtain splitting $e^{\pm} = e_0 \pm \gamma\delta$ for energy levels. Further, we calculate the dependence of energy on potential parameters.

Now we present some numerical results in the case $m = 1$, $a = 1$ for the lower energy levels. If $v_0 = 100$ then $e_0 = 78.2272$, $\delta = 1.87550$ for $g = 0.1$, $s = 0.325$ and $e_0 = 12.1710$, $\delta = 1.65410$ for $g = 0.9$, $s = 0.975$. If $v_0 = 400$ then $e_0 = 154.175$, $\delta = 1.95605$ for $g = 0.1$, $s = 0.325$ and $e_0 = 13.6850$, $\delta = 1.68944$ for $g = 0.9$, $s = 0.975$.

## III. CONCLUSIONS

The confinement model potential for quantum dot considered in the present paper is smooth, has finite depth and width and permits the exact solutions of the separated unperturbed Schrödinger equation for electron states in the presence of spin-orbit interaction of Rashba and Dresselhaus. The contribution of perturbation is really small in comparison with the unperturbed energy $e_0$ if the strength $\alpha_R$ is sufficiently close to the strength $\alpha_D$ ($\gamma \ll$ ).

## REFERENCES


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