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ON THE ALLOWANCE CALCULATION IN THE FLAT PROBLEM OF THE ALLOWANCE DISTRIBUTION

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Introduction

The allowance is the layer of the material that is removed from the workpiece in the machining or finishing. The example of the flat workpiece (solid line) and the template (dotted line) is showed in Fig. 1. The difference between these two curves forms the allowance that should be cut from this workpiece. The allowance is shaded in Fig. 1. The allowance distribution is the process of the placing of the template on the workpiece in the best possible way [1]. The allowance distribution problem is a complex problem, the various aspects of which are considered in works [2, 3].

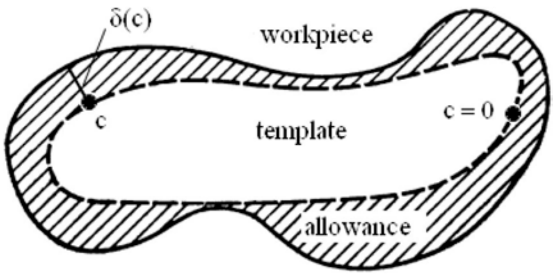

Fig. 1. The workpiece and the template in the problem of the allowance distribution

Mathematical model of the allowance distribution object

The allowance is some function of the position: $\delta = \delta(c)$, where c is the curvilinear coordinate directed along the contour of the template, $0 \le c \le c_0$, c_0 is the perimeter of the contour (Fig. 1). The function δ depends on the position of the template contour on the workpiece. The position of the template contour is characterized in the flat case by three coordinates: the abscissa x, the ordinate y of the mass center and the rotation angle φ relative the axis x (or basis direction), $\delta = \delta(c, x, y, \varphi)$.

The non-uniformity of the allowance can be defined as follows [1] (Fig. 2):

$$Q = Q(x, y, \varphi) = \max_c \delta(c, x, y, \varphi) - \min_c \delta(c, x, y, \varphi) = \delta_{\max} - \delta_{\min}. \quad (1)$$

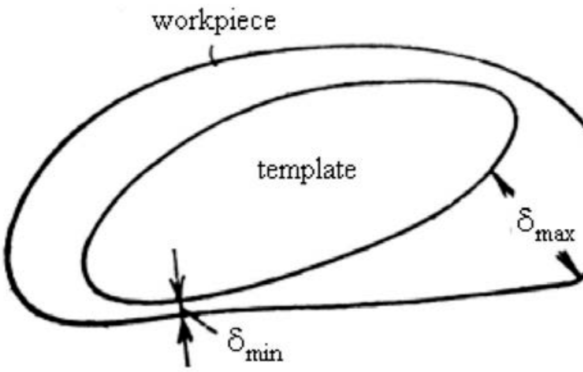

Fig. 2. The relation between the workpiece and the template

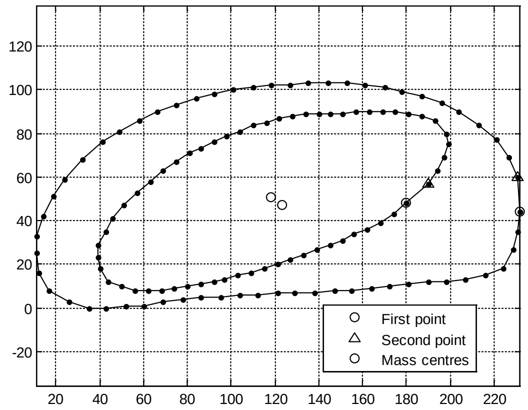

Fig.3. Pointwise defined workpiece and template

The problem of the optimal allowance distribution consists of minimize the function $Q(x, y, \varphi)$ (1) on the variables x, y, φ:

$$Q(x, y, \varphi) \to \min_{x,y,\varphi}.$$

The function (1) is unknown and can be estimated on the base of measurements. The values of the allowance $\delta = \delta(c, x, y, \varphi)$ can be measured practically in the m_t points $p_{1,t}, p_{2,t}, ..., p_{m_t}$ of the template contour (fig. 3). In this case we obtain the values $\delta_i = \delta(p_i, x, y, \varphi)$, $i = 1,2,..., m_t$, provided x, y, φ are fixed. Thus, the allowance distribution object is described practically by the following function:

$$Q = Q(x, y, \varphi) = \max_c(\delta_1, ..., \delta_{m_t}) - \min_c(\delta_1, ..., \delta_{m_t}). \quad (2)$$

The contour of the workpiece is defined also by its points $r_1, r_2, ..., r_{m_w}$ (Fig. 3). The measurements Q under fixed x, y, φ contain errors due to the point-defined outlines of the workpiece and the template and are random. This means that the object with description (2) is stochastic.

One of the important part in the problem of the allowance distribution by computer equipment is the algorithm of the allowance calculation.

Calculation of the allowance values in the template contour points

The value of the allowance is measured by the normal to the template contour in given points $p_{1,t} = (x_{1,t}, y_{1,t})$, $p_{2,t} = (x_{2,t}, y_{2,t})$, …, $p_{m_t} = (x_{m_t}, y_{m_t})$. We represent the template contour curve in parametric form $x = x(i)$, $y = y(i)$ as the quadratic parabolas

$$x(i) = a_{x,1} + a_{x,2} i + a_{x,3} i^2, \tag{3}$$

$$y(i) = a_{y,1} + a_{y,2} i + a_{y,3} i^2, \tag{4}$$

where i is the point number at the contour as the parameter, $i = 1,2,...,m_t$. The coefficients $a_{x,1}, a_{x,2}, a_{x,3}$, $a_{y,1}, a_{y,2}, a_{y,3}$ of the parabolas can be found by interpolation of the functions $x(i)$, $y(i)$ on three nodes (x_i, y_i), (x_{i+1}, y_{i+1}), (x_{i+2}, y_{i+2}), where $x_i = x(i)$, $y_i = y(i)$, $i = 1,2,...,m_t$. So the values of the functions (3), (4) under values of the parameter i, $i+1$, $i+2$ are x_i, x_{i+1}, x_{i+2}, y_i, y_{i+1}, y_{i+2} and obtained coefficients are $a_{x,1,i+1}, a_{x,2,i+1}, a_{x,3,i+1}$, $a_{y,1,i+1}, a_{y,2,i+1}, a_{y,3,i+1}$ for every fixed i.

In can be shown, that the points $(x_{n,i+1}, y_{n,i+1})$ of the vertexes of the normals are defined by the following formulae:

$$x_{n,i+1} = x_{i+1} + \Delta x_{i+1}, \tag{5}$$

$$y_{n,i+1} = y_{i+1} - \frac{a_{x,2,i+1} + 2a_{x,3,i+1}(i+1)}{a_{y,2,i+1} + 2a_{y,3,i+1}(i+1)} \Delta x_{i+1}, \; i = 1,2,...,m_t, \tag{6}$$

where $|\Delta x_{i+1}|$ is defined the normal length. The bases of the normals are the given points $p_i = (x_i, y_i)$, $i = 1,2,...,m_t$, on the template contour.

The directions of the normal must be agreed. We need that the normal have directions to the workpiece outline. It can be performed using the following derivatives of the functions $x = x(i)$, $y = y(i)$ by the parameter i at the points $x_{i+1} = x(i+1)$, $y_{i+1} = y(i+1)$:

$$x'_{i+1} = a_{x,2,i+1} + 2a_{x,3,i+1}(i+1),$$

$$y'_{i+1} = a_{y,2,i+1} + 2a_{y,3,i+1}(i+1), \; i = 1,2,...,m_t.$$

The allowance value at the template contour point p_i is the distance between the template contour point p_i and the workpiece contour point r_j nearest to the normal. The nearest point will be that point r_j at the workpiece contour which vector from the point p_i have the minimum angle with vector of the normal from the point p_i. Let $p_i = (x_i, y_i)$ be the template contour point, $p_{n,i} = (x_{n,i}, y_{n,i})$ be the vertex of the normal at the point $p_i = (x_i, y_i)$ and $r_j = (x_{w,j}, y_{w,j})$ be the workpiece contour point. It can be shown, that the cosine of the angle $\gamma_{i,j}$ between the vectors $(p_i, p_{n,i})$, (p_i, r_j) is defined by the following formula [4]:

$$\cos\gamma_{i,j} = \frac{(x_{n,i} - x_i)(x_{w,j} - x_i) + (y_{n,i} - y_i)(y_{w,j} - y_i)}{\sqrt{(x_{n,i} - x_i)^2 + (y_{n,i} - y_i)^2}\;\sqrt{(x_{w,j} - x_i)^2 + (y_{w,j} - y_i)^2}},$$

$$i = 1,2,...,m_t, \; j = 1,2,...,m_w. \qquad (7)$$

Further, we need to find the j^* for every fixed $i = 1,2,...,m_t$ from the condition

$$j^* = \arg\min_j (1 - \cos\gamma_{i,j}). \qquad (8)$$

The allowance value at the template contour point $p_i = (x_i, y_i)$ is defined then by the formula:

$$\delta_i = \sqrt{(x_{w,j^*} - x_i)^2 + (y_{w,j^*} - y_i)^2}, \; i = 1,2,...,m_t, \qquad (9)$$

where x_{w,j^*}, y_{w,j^*} are coordinates of the $j^{*\,\text{th}}$ point at the workpiece contour.

As a result we obtain the following algorithm for allowance calculation:

1) calculate the coefficients $a_{x,1,i+1}, a_{x,2,i+1}, a_{x,3,i+1}$, $a_{y,1,i+1}, a_{y,2,i+1}, a_{y,3,i+1}$ of the parabolas (3), (4) by interpolation of the functions $x(i)$, $y(i)$ on three nodes (x_i, y_i), (x_{i+1}, y_{i+1}), (x_{i+2}, y_{i+2}) for all $i = 1,2,...,m_t$;

2) assume $k_1 = 20$; (number k_1 determines the normal length)

if $y'_{i+1} < 0$ and $x'_{i+1} \geq 0$, then assume $\Delta x_{i+1} = -k_1$;

if $y'_{i+1} < 0$ and $x'_{i+1} \leq 0$, then assume $\Delta x_{i+1} = -k_1$;

if $y'_{i+1} = 0$ and $x'_{i+1} \leq 0$, then assume $\Delta x_{i+1} = 0$, $\Delta y_{i+1} = k_1$;

if $y'_{i+1} = 0$ and $x'_{i+1} \geq 0$, then assume $\Delta x_{i+1} = 0$, $\Delta x_{i+1} = -k_1$;

if $y'_{i+1} \leq 0$ and $x'_{i+1} \geq 0$, then assume $\Delta x_{i+1} = k_1$;

3) calculate the coordinates $x_{n,i+1}, y_{n,i+1}$ of the normal vertex by the formulae (5), (6) for all $i = 1,2,...,m_t$;

4) calculate the $\cos\gamma_{i,j}$ in accordance with the formula (7) and find the j^* from the condition (8);

5) calculate the allowance value at the template contour point $p_i = (x_i, y_i)$ in accordance with the formulae (8), (9).

Computer simulation

The presented above algorithm was programmed in Matlab for the workpiece and template in the Fig. 2, 3 defined by 60 points on their contours. The built normals to the template contour at each point are shown in the Fig. 4. They allow us to visually evaluate the allowance value at each point of the template contour. The calculated allowance values at all points of the template contour are shown in the Fig. 5 in the form of graph from the first to the last point. This graph corresponds to the Fig. 4. The minimal allowance value there is at the 41$^{\text{th}}$ point and the maximal one at the 2$^{\text{nd}}$ point. In general, the computer simulation completely confirmed the correctness of the developed algorithm.

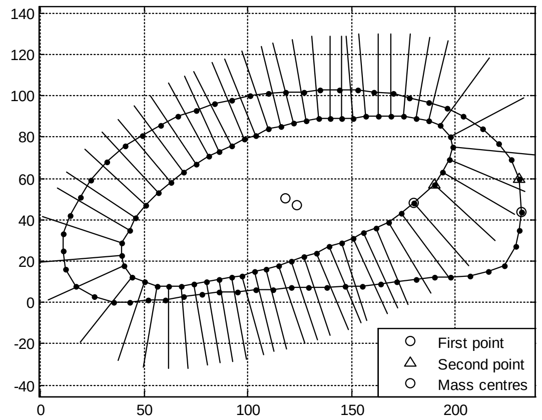

Fig. 4. The normals at the template contour points

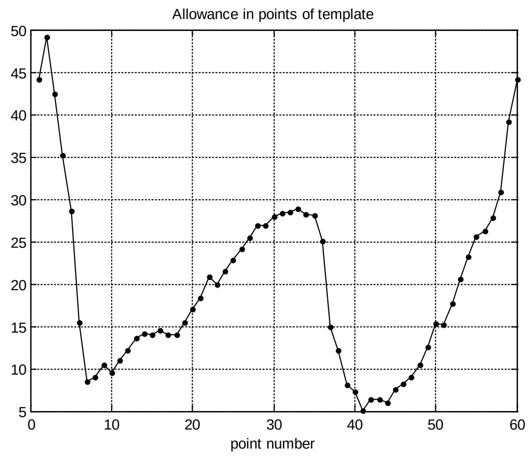

Fig. 5. The allowance values at the template contour points

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