MAXWELL ELECTRODYNAMICS IN MEDIA, GEOMETRY EFFECTS ON CONSTITUTIVE RELATIONS

N.G. Krylova[1], E.M. Ovsiyuk[2], A.V. Ivashkevich[3], V.M. Red'kov[3]
1 Belarusian state agrarian technical university, Minsk, Belarus
2 Mozyr State Pedagogical University, Mozyr, Belarus
3 B.I. Stepanov Inst. of Physics of the National Academy of Science of Belarus, Minsk, Belarus

I. INTRODUCTION

The problems of constitutive relations in Maxwell electrodynamics, their possible form, its behavior under the motion of the reference frame, its connection with Special Relativity theory, interplay between constitutive relations and gravity are reviewed. The main accent in our treatment is the known possibility to simulate material media by geometrical methods.

These problems have a long history. We can track interest to the problem in the huge literature that has been produced on this issue. Note that Gordon [2] was first seemed largely interested in trying to describe dielectric media by an effective metrics; Gordon tried to use a gravitational field to mimic a dielectric medium. The idea was taken up and developed by Tamm and Mandel'stam [3, 4]; also see [5-20].

II. GEOMETRICAL MODELING OF THE CONSTITUTIVE RELATIONS IN ELECTRODYNAMICS

The basic relations are as follows. Effective constitutive equation generated by the Riemannian geometry with metric $g_{\alpha\beta}(x)$ have the form

$$D^i = \epsilon_0\, \epsilon^{ik}(x)\, E_k + \epsilon_0 c\, \alpha^{ik}(x)\, B_k\,, \quad H^i = \epsilon_0 c\, \beta^{ik}(x)\, E_k + \mu_0^{-1}\, (\mu^{-1})^{ik}(x)\, B_k\,.$$

Four dimensionless (3×3)-matrices $\epsilon, \alpha, \beta, \mu^{-1}$ are not independent because they are bilinear functions of only 10 components of the symmetrical tensor $g_{\alpha\beta}(x)$:

$$\epsilon^{ik} = \sqrt{-g}(g^{00}g^{ik} - g^{0i}g^{0k}), \quad \alpha^{ik} = +\sqrt{-g}g^{ij}g^{0l}\epsilon_{ljk},$$

$$\beta^{ik} = -\sqrt{-g}g^{0j}\epsilon_{jil}g^{lk}(x), \quad (\mu^{-1})^{ik} = 12\sqrt{-g}\epsilon_{imn}g^{ml}g^{nj}\epsilon_{ljk}(\mu^{-1})^{ik}.$$

These tensors obey the following symmetry constraints: $\epsilon^{ik} = +\epsilon^{ki}$, $(\mu^{-1})^{ik} = +(\mu^{-1})^{ki}$, $\beta^{ki}(x) = \alpha^{ik}$; so the (6×6)-matrix defining constitutive equations is symmetrical. Metrical tensors which are the most interesting in the General relativity have a quasi-diagonal structure $g_{0i}(x) = 0$, and the corresponding constitutive relations simplify

$$\epsilon^{ik} = \sqrt{-g}g^{00}\begin{vmatrix} g^{11} & g^{12} & g^{13} \\ g^{21} & g^{22} & g^{23} \\ g^{31} & g^{32} & g^{33} \end{vmatrix}, \qquad (\mu^{-1})^{ik} = \sqrt{-g}\begin{vmatrix} G^{11} & G^{12} & G^{13} \\ G^{21} & G^{22} & G^{23} \\ G^{31} & G^{32} & G^{33} \end{vmatrix},$$

where $G^{ik}(x)$ stand for (algebraic) co-factors to the elements $g^{ik}(x)$. According to this, two tensor $\epsilon(x)$ and $\mu^{-1}(x)$ obey the following constraint

$$\epsilon(x)\mu^{-1}(x) = I.$$

Thus, the metric tensors with quasi-diagonal structure effectively describe media with following constitutive relations

$$D = -\epsilon_0\epsilon(x)\, E, \qquad B = \mu_0\mu(x)\, H, \qquad \mu(x) = -\epsilon(x)\,,$$

$$(\epsilon^{ik})(x) = \sqrt{-g(x)}\, g^{00}(x)\begin{vmatrix} g^{11}(x) & g^{12}(x) & g^{13}(x) \\ g^{21}(x) & g^{22}(x) & g^{23}(x) \\ g^{31}(x) & g^{32}(x) & g^{33}(x) \end{vmatrix}.$$

III. CONCLUSIONS

Application of the Riemannian geometry permits to simulate effective media which constitutive equations are determined by the metrical structure of the Riemann spaces. Because there are known numerous Riemannian geometries, the number of such effective media is enormous as well.

REFERENCES

[1] Minkowski, H., Die Grundlagen für die electromagnetischen Vorgänge in bewegten Kërpern. Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, mathematisch-physikalische Klasse. 1908. 53–111; reprint in Math. Ann. 1910. 68. 472–525.

[2] Gordon W. Zur Lichtfortpanzungnach der Relativitätstheorie. Ann. Phys. (Leipzig): 1923;72: 421–456.

[3] Tamm IE. Electrodinamika anizotropnoi sredy v spetsialnonoi teorii otnositelnosti. Zh. R, F, Kh. O, Fiz. dep.: 1924;56(2–3): 248–262. Tamm IE. Kristallooptika teorii otnositelbnosti v svyazi s geometriei bikvadratichnoi formuy. Zh. R, F, Kh. O, Fiz. dep.: 1925;54(3–4): 1.

[4] Mandelstam LI, Tamm IE. Elektrodynamik der anisotropen Medien und der speziallen Relativitatstheorie. Math. Annalen: 1925;95: 154–160.

[5] Landau LD, Lifsjitz EM. Theoretical physics. Vol. 2. Field theory. Moscow: Science; 1973.

[6] Balazs NL. Effect of a gravitational field, due to a rotating body, on the plane of polarization of an electromagnetic wave. Phys. Rev.: 1958;110: 236–239.

[7] Tomil'chik LM, Fedorov FI. Magnetic anisotropy as metrical property of space. Crystalography.: 1959;4(4): 498–504.

[8] Post E. Formal Structure of Electrodynamics. General Covariance and Electromagnetics. Amsterdam: North–Holland; 1962.

[9] De Felice F. On the gravitational field acting as an optical medium. Gen. Relat. Grav.: 1971;2: 347–357.

[10] Bolotovskiy BM, Stolyarov SN. Modern state of electrodynamics of moving media. Uspekhi Fizicheskikh Nauk: 1974;114(4): 489–529.

[11] Schleich W, Scully MO. General relativity and modern optics. In: New Trends in Atomic Physics, Les Houches, Session XXXVIII, 1982. Eds.: G. Grynberg, R. Stora. Amsterdam: North–Holland, 1984.

[12] Hillion P. Constitutive relations and Clifford algebra in electromagnetism. Adv. Appl. Clifford Alg.: 1995;5: 141–158.

[13] Leonhardt U. Space–time geometry of quantum dielectrics. Phys. Rev. A.: 2000;62: 012111.

[14] De Lorenci WA, Klippert R, Obukhov YN. On optical black holes in moving dielectrics. Phys. Rev. D.: 2003;68: 061502.

[15] Hehl FW, Obukhov YN. Linear media in classical electrodynamics and the Post constraint. Phys. Lett. A.: 2005;334: 249–259.

[16] Novello M, Salim JM. Effective electromagnetic geometry. Phys. Rev. D.: 2001;63: 083511.

[17] Novello M, Perez Bergliaffa S, Salim J. Analog black holes in flowing dielectrics. Class. Quant. Grav.: 2003;20: 859–872.

[18] Nandi KK et al. Analog of the Fizeau effect in an effective optical medium. Phys. Rev. D.: 2003;67: 025002.

[19] Red'kov VM. Particle fields in Riemannian space and the Lorentz Group. Minsk: Belarusian Science; 2009.

[20] A.V. Ivashkevich, E.M. Ovsiyuk, V.V. Kisel, V.M. Red'kov. Spinor Maxwell equations in Riemannian space-time and the geometrical modeling of constitutive relations in electrodynamics. Materials Physics and Mechanics. 2020. Vol. 45, no 1, P. 104–131.