COMPUTER SIMULATION OF MODAL CHARACTERISTICS OF PHOTONIC CRYSTALLINE FIBERS WITH A LAYERED COATING

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I. INTRODUCTION

Currently, the most spread methods for computer modeling of transmission properties of photonic-crystal fibers (PCFs) are the finite difference method and the finite element method. Their implementation requires a limited 2D computational window surrounding the fiber cross-section. But PCFs are open 3D dielectric structures. In this case, perfectly matching layers (PMLs) are used to model open space, placed at the boundaries of the computational window. However, only unlimited planar PMLs, which cannot be used to bound the 2D computational window, can completely absorb the radiation incident on them. This leads to uncontrollable errors in the calculation of the mode attenuation coefficients caused by confinement losses. This limitation makes it difficult to estimate the transmission spectra of PCFs, that are necessary for the design of fiber-optic sensors.

To correctly take into account the open nature of the electrodynamics problem, the method of Green's functions can be used [1]. In its initial form this method has been formulated for PCFs formed in a dielectric matrix by air channels of circular cross section. In this case mode field components are represented by rows in cylindrical functions and the standard Graph addition theorem for the two dimensional Green's function is used. But if the channels have more complicated cross sections this approach loses its applicability. In addition, the method [1] is not applicable in the case when a layered absorbing coating is applied to the outer surface of the PCF, which is a key element of the currently intensively studied fiber-optic sensors, using the lossy mode resonance effect [2].

This report presents the development of the method of Green's functions, performed with the aim of overcoming the above limitations. On the basis of the relations obtained, the modulation characteristics of a photonic crystal sensor of hydrogen concentration in the atmosphere are assessed.

II. DESIGN RATIOS

As the electrodynamics potential functions, we choose the longitudinal components of the electromagnetic field of a PCF mode E_z and H_z (0z is the PCF uniformity axis). Let the PCF consists of parallel air channels located in a dielectric matrix with permittivity ε_s. The cross-sectional perimeter of the *i-th* channel is described by a function $\rho_i(\varphi_i)$ (for a circular channel $\rho_i(\varphi_i) = const$), where ρ_i and φ_i are the local polar coordinates of the channel. The following Fourier series can be written on the perimeter

$$E_z = \sum_{\nu=-m}^{m} e_\nu^{(j)} \exp(i\nu\varphi_j)\,, \nabla_n E_z = \sum_{\nu=-m}^{m} e_\nu^{\prime(j)} \exp(i\nu\varphi_j),$$

$$H_z = \sum_{\nu=-m}^{m} h_\nu^{(j)} \exp(i\nu\varphi_j)\,, \nabla_n H_z = \sum_{\nu=-m}^{m} h_\nu^{\prime(j)} \exp(i\nu\varphi_j), \tag{1}$$

where ∇_n denotes the normal derivatives of the functions, $e_\nu^{(i)}$, $e_\nu^{\prime(i)}$, $h_\nu^{(i)}$, $h_\nu^{\prime(i)}$ are unknown coefficients, m is the order of the Fourier polynomials. The outer surface of the PCF is coated with n layers. The thickness and permittivity of *j-th* layer are d_j and ε_j, respectively. Series similar to (1) with unknowns $e_\nu^{(0)}$, $e_\nu^{\prime(0)}$, $h_\nu^{(0)}$, $h_\nu^{\prime(0)}$, take place on the perimeter of the PCF outer surface:

$$E_z = \sum_{\nu=-m}^{m} e_\nu^{(0)} \exp(i\nu\varphi)\,, \nabla_n E_z = \sum_{\nu=-m}^{m} e_\nu^{\prime(0)} \exp(i\nu\varphi),$$

$$H_z = \sum_{\nu=-m}^{m} h_\nu^{(0)} \exp(i\nu\varphi)\,, \nabla_n H_z = \sum_{\nu=-m}^{m} h_\nu^{\prime(0)} \exp(i\nu\varphi), \tag{2}$$

where φ is the global polar angle.

Application of the Green's theorem to the known second-order differential equations, to which potential functions E_z and H_z obey in the whole space, leads to functional equations

$$\int_0^{2\pi} \left[\Phi\nabla_n G - G\nabla_n\Phi \sqrt{\left(\frac{d\rho_0}{d\varphi}\right)^2 - \rho_0^2} \right] d\varphi - \sum_{j=1}^{N} \int_0^{2\pi} \left[\Phi\nabla_n G - G\nabla_n\Phi \sqrt{\left(\frac{d\rho_j}{d\varphi}\right)^2 - \rho_j^2} \right] d\varphi_j = 0, \tag{3}$$

where $\Phi = E_z(\boldsymbol{r})$ or $\Phi = H_z(\boldsymbol{r})$, N is a number of air channels in the PCF,

$$G = 0.25 i\pi H_0^{(2)}\left(\frac{2\pi}{\lambda} k_s |\boldsymbol{r}' - \boldsymbol{r}|\right), \tag{4}$$

is the two-dimensional electrodynamics Green's function, $k_s = \sqrt{\varepsilon_s - \beta^2}$, β is the dimensionless mode propagation constant, the radius vector $\boldsymbol{r}$ runs through the perimeters of the outer boundary of the PCF and the cross sections of the air channels with the variables of integration, $\boldsymbol{r}'$ is the radius vector of the observation point that can be located in any of the air channels or outside the PCF.

Substitution of series (1), (2) into Eqs (3) and application of the Graph's addition theorem to the function (4) leads to a homogeneous algebraic system

$$MX = 0, \tag{5}$$

where *M* is a matrix of dimensions *(N+1)(4m+2)x(N+1)4m+2), X* is a column vector composed of the unknown coefficients $e_\nu^{(i)}$, $h_\nu^{(i)}$ $(i = \overline{0,N})$.The possible values of the mode propagation constant are found from the equation

$$detM(\beta) = 0. \tag{6}$$

Fig. 1 illustrates the application of the developed method to the design of a photonic crystal sensor for detecting hydrogen in the atmosphere. Sensor action is based on the lossy mode resonance, in which the fundamental mode of the fiber interacts with the mode of the absorbing coating. Calculations are performed for two - layered cover consists of 1 μm TiO_2 layer and 8nm Pd layer. The PCF is supposed to be formed by two hexagonal rings of air channels of diameter 3 μm and pitch 8.8 μm in quartz glass. Outer radius of the PCF is 61.7 μm.

The arrow in Fig.1a indicates the main direction of the mode electric field. Fig1a refers to the wavelength λ=1.2178 μm, at which the maximal mode attenuation is observed. According to Fig.1a, the lossy mode resonance is due to the formation of standing waves in the PCF cladding.

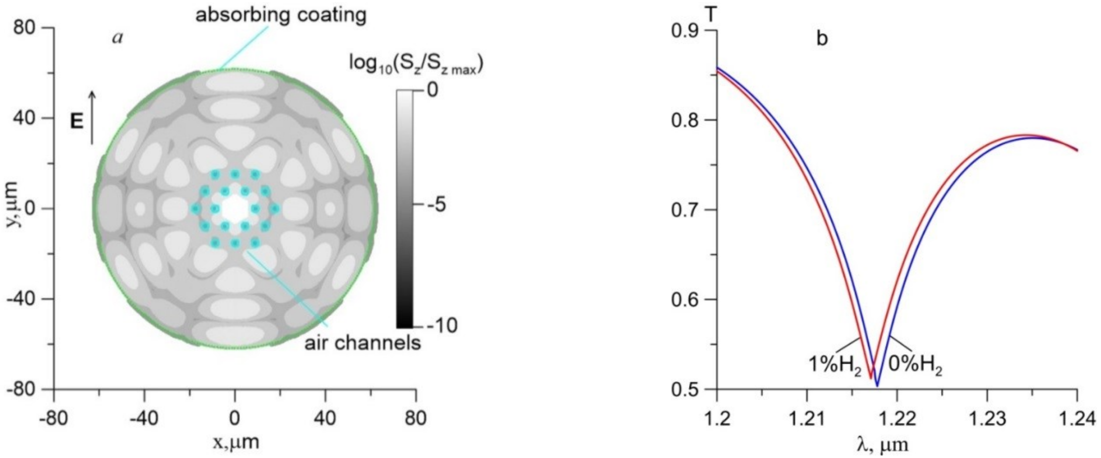

Figure1. Lossy mode resonance in PCF hydrogen sensor: the fundamental mode intensity distribution over the PCF cross section (a) and the resonance transmission of the sensor due to the lossy mode resonance when the concentration of hydrogen in the atmosphere changes (b)

III. CONCLUSIONS

The developed method of Green's functions is effective for calculating PCF sensors using the lossy mode resonance phenomenon.

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