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E ects of Neutrino Multipole Moments on Neutrino Oscillations

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The evolution of the neutrino ux traveling through dense matter and vortex inhomogeneous intensive magnetic eld is investigated. As the examples of the intensive magnetic eld the magnetic elds of the collapsar jets and the coupled sunspots being the sources of the solar ares are considered. It is assumed that the neutrinos possess the following electromagnetic characteristics: neutrino charge radii, the dipole magnetic and anapole moments.

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1. Introduction

The electromagnetic properties of neutrinos have attracted considerable attention from researchers for many decades (see [1] for a review). However, until recently, there was no indication in favor of nonzero electromagnetic properties of neutrinos either from laboratory experiments with ground-based neutrino sources or from observations of astrophysical neutrino uxes. The situation changed after the XENON collaboration reported [2] results of the search for new physics with low-energy electronic recoil data recorded with the XENON1T detector. The results show an excess of events over the known backgrounds in the recoil energy which, as one of the possible explanations, admit the presence of a sizable neutrino magnetic moment, the value of which is of the order of the existing laboratory limitations.

Neutrinos are neutral particles and their total Lagrangian does not contain any multipole moments (MM's). These moments are caused by the radiative corrections. Interaction of neutrinos with external electromagnetic elds is de ned by the MM's. Due to smallness of the neutrino MM's this interaction becomes essential in the case of intensive elds only. The examples of such elds are the Sun's magnetic elds. In that case of special interest are the magnetic elds of the sunspots which will be the source of the solar ares (SF's). The energy generated during the SF is about of 10^{28}–10^{33} erg. It is believed that the magnetic eld is the main energy source of the SF's [1],[2]. During the years of the active Sun, the magnetic ux $\sim 10^{24}$ Gauss $\cdot$ cm^2 [3] erupts from the solar interior and accumulates within the sunspots giving rise to the stored magnetic eld. The SF formation starts from pairing big sunspots of opposite polarity (coupled sunspots CS's). Then the process of magnetic energy storage of the CS's begins. The duration of this initial SF stage varies from several to dozens of hours. In so doing the magnetic eld value for the CS's B_{cs} could be increased from $\sim 10^4$ Gs up to $\sim 10^5$ Gs and upwards. The more powerful the SF is, the greater the magnetic eld strength of the CS's will be. For example, in the case of the super-SF's [4], which energy could be of order 10^{36} erg, B_{cs} may reach the values of 10^8 Gs. It is clear that when the electron neutrinos beam passing through the magnetic eld of the CS's will change its composition and we could detect this changing, then the problem of the SF's prediction will be resolved.

It might be well to point out that the are events are also at work in other Sun-like stars (rst-generation stars). Consequently, the study of these phenomena helps to elucidate the structure and evolution of the Universe. There are special cosmic projects [5] which are focussed on investigation of the SF's happening at the Sun-like stars. For example, by now the Kepler mission [6] surveying the $\sim 10^5$ stars has accumulated a great deal of data concerning the large ares with energies of order of 10^{33} erg.

Other example of intensive magnetic eld we shall be interested in is the eld appearing during the Gamma-ray bursts (GRB's) [7 10]. Short GRB's seem to be the result of the nal merger of two compact objects, whereas long GRB's are probably associated with the gravitational collapse of very massive stars. These imploding stars are called collapsars. The collapse of the stellar core produces a black hole, which accretes material from the inner layers of the star. An ultradense magnetized accretion disk is formed during the accretion process. Part of the plasma that surrounds the black hole is ejected producing two relativistic jets. The magnetic elds within the jets of collapsars could be as high as $10^7 - 10^8$ Gs. Each jet pushes the stellar material outwards. The energy radiated during the GRB's could be very large $10^{51} - 10^{54}$ erg. Besides producing electromagnetic emission the GRB's could also be sources of three important non-electromagnetic signals: cosmic rays, neutrinos, and gravitational waves. The neutrino energies are monstrous large. They lie in the range PeV to EeV ($10^{15} - 10^{18}$ eV). A lot of works are available which investigate the neutrino production in di erent scenarios of the GRB's. Analysis of the combined IceCube40 (IC with 40 strings) and IC59 dataset [11] provides no high-energetic muon neutrino candidates associated with any of the 225 GRBs which happened in the course of data taking, although the detectors reached sensitivities which are close to model predictions. However, the existing models forecast the value of high-energetic muon neutrinos di ering from zero. For example, the model [8] predicts 8.4 events. This could lead to the conclusion that either the model picture of GRBs is wrong or the chosen parameter values are not correct. Note, that important model parameters are the Lorentz boost factor of the collimated out ow of the exploding star and the typical time scale of subsequent collisions of internal shocks. However, it is not inconceivable that neutrino interactions with the collapsar jet medium could explain this muon neutrino de cit [12].

The purpose of the present work is to consider the in uence of the MM's on evolution of the neutrino ux in an intensive magnetic eld and in a dense matter. In so doing we shall be interested in the depletion of the electron neutrino ux in the case of the coupled sunspot magnetic elds while in the case of the collapsar magnetic elds the subject of our interest will be decreasing the muon neutrino ux.

2. Neutrinos resonanse restrictions

In the one-photon approximation, the electromagnetic interactions of a neutrino field could be described by the effective interaction Hamiltonian

$$\mathrm{H}^{(\nu)}_{em}(x) = J^{(\nu)}_{\mu}(x)A^{\mu}(x) = \sum_{i,f} \overline{\nu_k}(x)\Lambda^{if}_{\mu}\nu_j(x)A^{\mu}(x). \quad (1)$$

So, the physical effect of $\mathrm{H}^{(\nu)}_{em}(x)$ is determined by the effective electromagnetic vertex, with the neutrino matrix element $\overline{\nu}_f(p_f)\Lambda^{if}_{\mu}\nu_i(p_i)$. In the most general case the vertex function $\Lambda^{if}_{\mu}(p_f, p_i)$ consistent with Lorentz and electromagnetic gauge invariance is defined in terms of four form factors [13, 14]

$$\Lambda^{if}_{\mu}(p_f, p_i) = i\sigma_{\mu\lambda}q^{\lambda}[F^{if}_{M}(q^2) + iF^{if}_{E}(q^2)\gamma_5] + (\gamma_{\mu} - q_{\mu}q^{\lambda}\gamma_{\lambda}/q^2)[F^{if}_{Q}(q^2) + F^{if}_{A}(q^2)q^2\gamma_5], \quad (2)$$

where $q_{\mu} = p'_{\mu} - p_{\mu}$ is the transferred 4-momentum, while $F^{if}_{Q}, F^{if}_{M}, F^{if}_{E}$, and F^{if}_{A} are the real charge, dipole magnetic, dipole electric, and anapole neutrino form factors. In the a static limit ($q^2 = 0$), $F^{if}_{M}(q^2)$, $F^{if}_{E}(q^2)$ and $F^{if}_{A}(q^2)$ define the dipole magnetic, dipole electric and anapole moments, respectively. Even if the electric charge of a neutrino is zero, the neutrino can be characterized by a (real or virtual) superposition of two different charge distributions of opposite signs, which is described by an electric form factor. Then the second term in the expansion of the form factor $F^{if}_{Q}(q^2)$ in series of powers of q^2 is connected with the neutrino charge radius (NCR) $< r^2_{ij} >$ $(i, j = \nu_e, \nu_{\mu}, \nu_{\tau})$

$$< r^2_{if} >= 6 \left.\frac{dF^{if}_{Q}(q^2)}{dq^2}\right|_{q^2=0}. \quad (3)$$

Amongst the neutrino electromagnetic characteristics, we shall be interested in the dipole magnetic moments, the anapole moments and the charge radii.

The neutrino DMM's are being searched in reactor (MUNU, TEXONO and GEMMA) [21–23], accelerator (LSND) [24, 25], and solar (Super-Kamiokande and Borexino) [26, 27] experiments. The current best sensitivity limits on the DMM's obtained in laboratory measurements are

$$\mu^{exp}_{\nu_e} \le 2.9 \times 10^{-11}\mu_B, \qquad 90\%\ C.L. \qquad [23],$$

$$\mu^{exp}_{\nu_{\mu}} \le 6.8 \times 10^{-10}\mu_B, \qquad 90\%\ C.L. \qquad [LSND]\ [24].$$

For the τ-neutrino, the bounds on $\mu_{\nu_{\tau}}$ are less restrictive (see, for example [28]), and the current upper limit on that is $3.9 \times 10^{-7}\mu_B$.

In the SM the introduction of the NCR has a long history, with some controversies. In one of the first studies [32], it was claimed that in the SM the NCR is ultraviolet-divergent and gauge-dependent. As a result, the authors concluded that this quantity is not a physical observable. However, later on, in the works [33–36] it was demonstrated that a more careful consideration of all divergent diagrams allows to obtain the finite and gauge-independent result, which made it possible to introduce the NCR as a physical observable.

The NCR has an effect in the scattering of neutrinos with charged particles. The bounds on the NCR's could be obtained from observation of the elastic neutrino-electron scattering. For example, investigation of this process at the TEXONO experiment leads to the following bounds on the charge radius of the electron neutrino [37]

$$-2.1 \times 10^{-32}\ \mathrm{cm}^2 \le < r^2_{\nu_e} > \le 3.3 \times 10^{-32}\ \mathrm{cm}^2. \quad (4)$$

Coherent elastic neutrino-nucleus scattering (CENNS) is also a powerful tool to study the neutrino multipole moments. Investigation of CENNS ful lled in the TEXONO [38], LSND [39] and BNL-E734 [40] experiments allowed to get following bounds on the diagonal NCR's

$$-4.2\times10^{-32}\ \text{cm}^2 \leq (<r^2_{\nu_e}>) \leq 6.6\times10^{-32}\ \text{cm}^2, \qquad \text{[TEXONO]}$$

$$-5.94\times10^{-32}\ \text{cm}^2 \leq (<r^2_{\nu_e}>) \leq 8.28\times10^{-32}\ \text{cm}^2, \qquad \text{[LSND]}$$

$$-5.7\times10^{-32}\ \text{cm}^2 \leq (<r^2_{\nu_\mu}>) \leq 1.1\times10^{-32}\ \text{cm}^2, \qquad \text{[BNL-E734].}$$

In its turn the bounds on the transition NCR's

$$|<r^2_{\nu_e\nu_\mu}>| \leq 28\times10^{-32}\ \text{cm}^2, \qquad |<r^2_{\nu_e\nu_\tau}>| \leq 30\times10^{-32}\ \text{cm}^2,$$

$$|<r^2_{\nu_\mu\nu_\tau}>| \leq 35\times10^{-32}\ \text{cm}^2,$$

were obtained from analysis of the COHERENT data on CENNS [41].

The NCR has also some impact on astrophysical phenomena and on cosmology. For example, when neutrinos have the Dirac nature, e^+e^- annihilations could produce right-handed neutrino-antineutrino pairs through the coupling caused by the NCR. This process would a ect primordial Big-Bang Nucleosynthesis and the energy release of a core-collapse supernova.

The anapole moment of $1/2$-spin Dirac particle was introduced by Zel'dovich [42] for a T -invariant interaction which does not conserve P -parity and C-parity individually. Subsequently, a more general characteristic, the toroid dipole moment (TDM) [43], was proposed to describe of this kind of interaction. As was demonstrated, the TDM is a common case of the anapole and it coincides with an anapole on the mass-shell of the particle under consideration. The simplest model of TDM (anapole) represents a conventional solenoid folded into a torus and having only a poloidal current [42]. For such a stationary solenoid, having neither an azimuthal (toroidal) component of the current nor electric elds around the torus, there is only a nonzero azimuthal magnetic eld inside the torus. The toroid interactions of Dirac or Majorana neutrinos are exhibited by collisions of the neutrinos with charged particles (electrons, quarks and nuclei). As this takes place, this interaction conserves the neutrino helicity and results an extra contribution, as a part of the radiative corrections. In this respect, the anapole is similar to the neutrino charge radius (NCR). Both quantities conserve the helicity in coherent neutrino scattering, but have di erent natures. They determine the axial-vector (TDM) and the vector (NCR) contact interactions with an external electromagnetic eld, respectively. Such interactions are the subject of interest in low-energy scattering processes and place at our disposal one way to investigate the NCR and TDM (see, for example, Refs. [44, 45]). The toroid interactions of neutrinos may have a very interesting consequences in di erent media. The possible role of the anapole in investigations of neutrino oscillations was rst pointed out in Refs. [46]). A point that should be also mentioned is Ref. [47] where the behavior of neutrinos endowed with the anapole in a vortex magnetic eld was considered upon discussing the correlation between the electron neutrino ux and the solar are events.

Phenomenology of the anapole moment is similar to that of neutrino charge radii (NCR). In the SM for a zero-mass neutrino, the value of the anapole moment is connected with the charge radius through the relation (see, for example, [48])

$$a_\nu = \frac{1}{6}<r^2_\nu>. \qquad (5)$$

In the case of a massive neutrino, this relationship is violated [49].

The neutrino magnetic moment predicted by the standard model (SM) is proportional to the neutrino mass [51]

$$\mu_\nu = \frac{3eG_F m_\nu}{8\sqrt{2}\pi^2} = 10^{-19}\mu_B\left(\frac{m_\nu}{\text{eV}}\right), \tag{6}$$

and, as a consequence, cannot lead to any observable effects in real fields. Therefore, if one uses the values of the neutrino magnetic moments which are close to the upper experimental bounds ($\sim 10^{-11}\mu_B$), then one should employ the SM extension containing the right-handed charged currents and/or charged Higgs bosons. As an example of such models could be the left-right symmetric model (LRM) based on the $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ gauge group [52–54]. The Higgs sector structure of the LRM specifies the neutrino nature. When the Higgs sector of the LRM contains the bi-doublet $\Phi(1/2, 1/2, 0)$ and two triplets $\Delta_L(1, 0, 2)$, $\Delta_R(0, 1, 2)$ [55] (in brackets the values of S^W_L, S^W_R and $B-L$ are given, S^W_L (S^W_R) is the weak left (right) isospin while B and L are the baryon and lepton numbers), then the neutrino has a Majorana nature. For the neutrino to be a Dirac particle, the Higgs sector must hold the bidoublet $\Phi(1/2, 1/2, 0)$ and two doublets $\chi_L(1/2, 0, 1)$, $\chi_R(0, 1/2, 1)$ [56].

In the LRM the Lagrangian describing neutrino interaction with the $W^\pm$, Z gauge bosons must be added by the Lagrangians which are responsible for neutrino interactions with additional gauge bosons $W'^\pm$, Z' and singly charged Higgs bosons $h^{(\pm)}$, $\tilde{\delta}^{(\pm)}$ [57]. Inasmuch as the masses of $W'^\pm$, Z', and $h^{(\pm)}$ lay at the TeV scale [58], then one may neglect their contributions in the neutrino Lagrangian. On the other hand, the $\tilde{\delta}^{(\pm)}$ boson does not interact with the quarks, and as a result, the more firm data for deriving the bounds on the $m_{\tilde{\delta}}$ follow from the electroweak processes. For example, results from LEP experiments (ALEPH, DELPHI, L3, and OPAL) gave the bound $m_{H^+} > 80$ GeV [58]. In the LRM, the interaction between neutrino and $\tilde{\delta}^{(\pm)}$ boson is described by the Lagrangian

$$\mathcal{L}_{\tilde{\delta}} = \frac{f_{ll'}}{\sqrt{2}}\bar{l}^c(x)(1-\gamma_5)\nu_{l'}(x)\tilde{\delta}^+(x), \tag{7}$$

where $f_{ll'}$ is a triplet Yukawa coupling constant (TYCC), $l, l' = e, \mu, \tau$ and the upper index c means the charge conjugation operation. Then this interaction leads to changing of the matter potential on the value

$$V^{\tilde{\delta}}_{ll'} = -\frac{f_{el}f_{el'}}{m_{\tilde{\delta}}}n_e, \tag{8}$$

(n_e is an electron density), which could be as large as few $\times 10\%$ from its SM value [59]. For the sake of simplicity, we shall assume that only the diagonal TYCC are different from zero.

As the magnetic field is concerned, we shall reason that it is vortex (rot $\mathbf{B} = 0$) and exhibits the geometrical phase $\Phi(z)$

$$B_x \pm iB_y = B_\perp e^{\pm i\Phi(z)}. \tag{9}$$

For $\Phi(z)$ we shall adopt a simple model in which the magnetic field exists over a distance L_{mf} and twists by an angle $\alpha\pi$ (α is a constant). i.e.

$$\Phi(z) = \frac{\alpha\pi}{L_{mf}}z. \tag{10}$$

Then for the Majorana neutrino the evolution equation in the flavor basis will look like

$$i\frac{d}{dz}\begin{pmatrix}\nu_{eL}\\ \nu_{\mu L}\\ \nu_{\tau L}\\ \overline{\nu}_{eR}\\ \overline{\nu}_{\mu R}\\ \overline{\nu}_{\tau R}\end{pmatrix} = \mathcal{H}^M\begin{pmatrix}\nu_{eL}\\ \nu_{\mu L}\\ \nu_{\tau L}\\ \overline{\nu}_{eR}\\ \overline{\nu}_{\mu R}\\ \overline{\nu}_{\tau R}\end{pmatrix}, \tag{11}$$

where

$$\mathcal{H}^M = \begin{pmatrix} -\delta_c^{12} + V'_{eL} + \mathcal{A}^L_{ee} & \delta_s^{12} + \mathcal{A}^L_{e\mu} & 0 & \mu_{e\mu}B_\perp \\ \delta_s^{12} + \mathcal{A}^L_{\mu e} & \delta_c^{12} + V_{\mu L} + \mathcal{A}^L_{\mu\mu} & -\mu_{e\mu}B_\perp & 0 \\ 0 & -\mu_{e\mu}B_\perp & -\delta_c^{12} - V'_{eL} + \mathcal{A}^R_{ee} & \delta_s^{12} + \mathcal{A}^R_{e\mu} \\ \mu_{e\mu}B_\perp & 0 & \delta_s^{12} + \mathcal{A}^R_{\mu e} & \delta_c^{12} - V_{\mu L} + \mathcal{A}^R_{\mu\mu} \end{pmatrix}, \tag{12}$$

V'_{eL} ($V_{\mu L}$) is a matter potential describing interaction of the ν_{eL} ($\nu_{\mu L}$) neutrinos with a dense matter,

$$V'_{eL} = V_{eL} + \tilde{V}_{ee}, \qquad V_{eL} = \sqrt{2}G_F(n_e - n_n/2), \qquad V_{\mu L} = V_{\tau L} = -\sqrt{2}G_F n_n/2, \tag{13}$$

$$\delta^{12}_{c(s)} = \frac{m_1^2 - m_2^2}{4E}\cos 2\vartheta_v(\sin 2\vartheta_v), \tag{14}$$

$$\mathcal{A}^L_{ll'} = e\left\{[1-\delta_{ll'}]\frac{< r^2_{\nu_{lL}\nu_{l'L}} >}{6} + a_{\nu_{lL}\nu_{l'L}}\right\}[\mathrm{rot}\,\mathbf{H}(z)]_z - \dot{\Phi}/2,$$
$$\mathcal{A}^R_{ll'} = e\left\{[1-\delta_{ll'}]\frac{< r^2_{\nu_{lR}\nu_{l'R}} >}{6} - a_{\nu_{lR}\nu_{l'R}}\right\}[\mathrm{rot}\,\mathbf{H}(z)]_z + \dot{\Phi}/2, \tag{15}$$

and n_n is a neutron density. When writing the Hamiltonian we have taken into account that the anapole and NCR interactions are different from zero in the presence of the inhomogeneous vortex magnetic field. In a concrete experimental situation this field could be realized owing to Maxwell's equations as the displacement and conduction currents.

When the neutrinos are the Dirac particles we have deal with the following Hamiltonian

$$\mathcal{H}^D = \begin{pmatrix} -\delta_c^{12} + V_{eL} + \mathcal{A}^{DL}_{ee} & \delta_s^{12} + \mathcal{A}^{DL}_{e\mu} & \mu_{ee}B_\perp & \mu_{e\mu}B_\perp \\ \delta_s^{12} + \mathcal{A}^{DL}_{\mu e} & \delta_c^{12} + V_{\mu L} + \mathcal{A}^{DL}_{\mu\mu} & \mu_{e\mu}B_\perp & \mu_{\mu\mu}B_\perp \\ \mu_{ee}B_\perp & \mu_{e\mu}B_\perp & -\delta_c^{12} + \mathcal{A}^{DR}_{ee} & \delta_s^{12} + \mathcal{A}^{DR}_{e\mu} \\ \mu_{e\mu}B_\perp & \mu_{\mu\mu}B_\perp & \delta_s^{12} + \mathcal{A}^{DR}_{\mu e} & \delta_c^{12} + \mathcal{A}^{DR}_{\mu\mu} \end{pmatrix}, \tag{16}$$

where

$$\mathcal{A}^{DL}_{ll'} = e\left\{\frac{< r^2_{\nu_{lL}\nu_{l'L}} >}{6} + a_{\nu_{lL}\nu_{l'L}}\right\}[\mathrm{rot}\,\mathbf{H}(z)]_z - \dot{\Phi}/2, \qquad \mathcal{A}^{DR}_{ll'} = \dot{\Phi}/2.$$

Our next task is to identify possible resonance conversions of the neutrino beam which travels both in the region of the coupled sunspots being the source of the solar flares and in the region of the collapsar jets. Remember, that for the resonance conversion to take place, there is a need to comply with the following requirements: (i) the resonance condition must be fulfilled; (ii) the resonance width must be nonzero; (iii) the neutrino beam must pass a distance comparable with the oscillation length. We shall also assume that the resonance localization place are situated rather far from one another what allows us to consider them as independent ones.

We start with resonant conversions of the electron neutrinos in the Sun's conditions. In the Majorana neutrino case the ν_{eL} may exhibit two resonance conversions. The $\nu_{eL} \leftrightarrow \nu_{\mu L}$ (Micheev-Smirnov-Wolfenstein MSW [60, 61]) resonance is the rst. The corresponding resonance condition, the transition width and the oscillation length are de ned by the expressions

$$H^M_{22} - H^M_{11} = -2\delta^{12}_c + V'_{eL} - V_{\mu L} + A^L_{ee} - A^L_{\mu\mu} = 0, \tag{17}$$

$$\Gamma_{\nu_{eL}\nu_{\mu L}} \simeq \frac{\sqrt{2}H^M_{12}}{G_F} = \frac{\sqrt{2}(\delta^{12}_s + A^L_{e\mu})}{G_F}, \tag{18}$$

$$L_{\nu_{eL}\nu_{\mu L}} = \frac{2\pi}{\sqrt{(H^M_{22} - H^M_{11})^2 + (H^M_{12})^2}} =$$

$$= \frac{2\pi}{\sqrt{[2\delta^{12}_c - (V'_{eL} - V_{\mu L} + A^L_{ee} - A^L_{\mu\mu})]^2 + (\delta^{12}_s + A^L_{e\mu})^2}}. \tag{19}$$

From Eqs.(18) and (19) it follows that the oscillation length achieves maximum value at the resonance and the relation

$$\Gamma_{\nu_{eL}\nu_{\mu L}} = \frac{2\sqrt{2}\pi}{G_F [L_{\nu_{eL}\nu_{\mu L}}]_{max}} \tag{20}$$

takes place. With a help of the relations (17)-(19) one could obtain the probability of the $\nu_{eL} \leftrightarrow \nu_{\mu L}$ resonance transition. In the most simple case, when the neutrino system consists only from ν_{eL} and $\nu_{\mu L}$ while the Hamiltonian is not a distance function, this quantity is de ned by the expression

$$P_{\nu_{eL}\nu_{\mu L}}(z) = \sin^2 2\vartheta_m \sin^2\left(\frac{z}{L_{\nu_{eL}\nu_{\mu L}}}\right), \tag{21}$$

where

$$\sin^2 2\vartheta_m = \frac{4(\delta^{12}_s + A^L_{e\mu})^2}{[2\delta^{12}_c - (V'_{eL} - V_{\mu L} + A^L_{ee} - A^L_{\mu\mu})]^2 + 4(\delta^{12}_s + A^L_{e\mu})^2} \tag{22},$$

and ϑ_m is a mixing angle in a matter and a magnetic eld.

Taking into account

$$\Delta m^2_{12} = 7.37 \times 10^{-5}\ \text{eV}^2, \qquad \sin^2\vartheta_{12} = 0.297 \tag{23}$$

and considering for the solar neutrinos $E_\nu = 10$ MeV, we get $2\delta^{12}_s \simeq$ few $\times 10^{-12}$ eV. Then the maximum value of the oscillation length will have the order of 3.5×10^7 cm. Consequently, this resonance transition is ful lled before the convective zone and has no bearing on the solar ares which take place in the solar atmosphere. To put this another way, in the case of the MSW resonance the quantities A^L_{ee}, $A^L_{\mu\mu}$ and $A^L_{e\mu}$ do not play any role.

Further we shall consider the $\nu_{eL} \leftrightarrow \overline{\nu}_{\mu R}$ resonance. The resonance condition and the maximum value of the oscillation length are as follows

$$-2\delta^{12}_c + V'_{eL} + V_{\mu L} + e(a^L_{\nu_e\nu_e} + a^R_{\nu_\mu\nu_\mu})(\text{rot}\,\mathbf{H})_z - \dot{\Phi} = 0 \tag{24}$$

$$(L_{\nu_{eL}\overline{\nu}_{\mu R}})_{max} \simeq \frac{2\pi}{\mu_{e\mu}B_\perp}. \tag{25}$$

For the solar neutrinos the terms V'_{eL} and $V_{\mu L}$ in Eq. (24) are more less than δ^{12}_c and do not play any part. Moreover, since the conduction current existing above big sunspots are constrained by the value of 10^{12} A, then, as the analysis shows, the forth term in Eq. (24) appears to be negligibly small compared with δ_c^{12} too. Therefore, the resonance $\nu_{eL} \leftrightarrow \overline{\nu}_{\mu R}$ may occur only at the cost of magnetic eld twisting, that is, when the relation

$$2\delta_c^{12} + \dot{\Phi} \simeq 0. \qquad (26)$$

will be ful lled. Using $\mu_{e\mu} = 10^{-11}\mu_B$ and $B_\perp = 10^5$ Gs, we obtain $(L_{\nu_{eL}\overline{\nu}_{\mu R}})_{max} \simeq 10^9$ cm. Then the condition (24) and the equality $L_{mf} = (L_{\nu_{eL}\nu_{\overline{\mu}R}})_{max}$ will be ful lled provided the twist frequency $\dot{\Phi}$ is equal to $-10\pi/L_{mf}$. On the other hand when the magnetic eld over the CS's reaches the value of 10^6 Gs, the ful llment above mentioned requirements will be e ected at the twist frequency being equal to $-\pi/L_{mf}$. So, we see that under the speci c conditions the $\nu_{eL} \leftrightarrow \overline{\nu}_{\mu R}$ resonance may be in existence in the Sun conditions. Because the resonance condition (24) does not depend on n_e and n_n, then the resonance $\nu_{eL} \leftrightarrow \overline{\nu}_{\mu R}$ may take place both in the chromosphere and in the corona.

We shall be also interested in the resonance conversions of the $\nu_{\mu L}$ neutrinos in the jet conditions. Let us start with the $\nu_{\mu L} \to \nu_{eL}$ transition. The expressions for the resonance conditions, maximal value of the oscillation length are the same as in the case of $\nu_{eL} \to \nu_{\mu L}$ transition. For the collapsar jets the matter potential has the order of 10^{-21} eV while the neutrinos energy could be as high as 10^{18} eV. Then, since the minimal value of δ_c^{12} is 10^{-23} eV the $\nu_{eL} \leftrightarrow \nu_{\mu L}$ resonance transition seemingly may occur for the neutrinos with the energies of the order of 10^{15} eV. However, the oscillation length proves to be equal to $\sim 10^{15}$ cm while the size of the collapsar jet is as short as $\sim 10^8$ cm. So under the jet conditions this resonance is not observed.

For the $\nu_{\mu L} \to \overline{\nu}_{eR}$ resonance the corresponding resonance condition and the maximum value of the oscillation length are determined by the expressions

$$2\delta_c^{12} + V_{\mu L} + V'_{eL} + (a^R_{\nu_e\nu_e} + a^L_{\nu_\mu nu_\mu})(\text{rot}\,\mathbf{H})_z - \dot{\Phi} = 0 \qquad (27)$$

$$(L_{\nu_{\mu L}\nu_{eR}})_{max} \simeq \frac{2\pi}{\mu_{e\mu}B_\perp}. \qquad (28)$$

From Eq. (27) it follows, that in the case under consideration the condition (27) could not be satis ed and, as a result, the $\nu_{\mu L} \to \overline{\nu}_{eR}$ resonance is forbidden.

Now we proceed to the Dirac neutrino case. Here the solar electron neutrinos could undergo three following resonance conversions

$$\nu_{eL} \leftrightarrow \nu_{\mu L}, \qquad \nu_{eL} \leftrightarrow \overline{\nu}_{eR}, \qquad \nu_{eL} \leftrightarrow \overline{\nu}_{\mu R}. \qquad (29)$$

The $\nu_{eL} \leftrightarrow \nu_{\mu L}$ resonance (MSW-resonance) is of little interest. As in the Majorana neutrino case it occurs before the convective zone.

The resonance condition and the maximal value of the oscillation length for the $\nu_{eL} \leftrightarrow \overline{\nu}_{eR}$ resonance are given by the expressions

$$V_{eL} + e\left\{\frac{< r^2_{\nu_{eL}\nu_{eL}} >}{6} + a_{\nu_{eL}\nu_{eL}}\right\}[\text{rot}\,\mathbf{H}(z)]_z - \dot{\Phi} = 0. \qquad (30)$$

$$(L_{\nu_{eL}\nu_{eR}})_{max} \simeq \frac{2\pi}{\mu_{ee}B_\perp}. \qquad (31)$$

For the solar neutrinos traveling through the coupled sunspots the situation, when the term proportional to $(\text{rot}\,\mathbf{H})_z$ is negligibly small compared with $\dot{\Phi}$ whereas the resonance condition reduces to

$$V_{eL} \simeq \dot{\Phi}, \qquad (32)$$

is not realistic. Really, in order to met Eq. (32) it is necessary that the twisting magnetic eld exists over the distance being much bigger than the solar radius. Therefore, the $\nu_{eL} \rightarrow \overline{\nu}_{eR}$ resonance in the Sun conditions is forbidden.

Further we consider the $\nu_{eL} \rightarrow \overline{\nu}_{\mu R}$ resonance. In this case the resonance condition and the maximum value of the oscillation length are as follows

$$-2\delta_c^{12} + V_{eL} + A_{ee}^{DL} - A_{\mu\mu}^{DR} = 0, \quad (33)$$

$$(L_{\nu_{eL}\nu_{\mu R}})_{max} \simeq \frac{2\pi}{\mu_{e\mu}B_\perp}. \quad (34)$$

From comparing the foregoing expressions with (24) and (25) one may make the conclusion that the conditions of observing the $\nu_{eL} \leftrightarrow \overline{\nu}_{\mu R}$ resonance in the Dirac and Majorana cases are only little di erent from each other. Then, considering this resonance in the region of the CS's we may argue, as in the Majorana case, that $\nu_{eL} \leftrightarrow \overline{\nu}_{\mu R}$ resonance may also occur only at the cost of magnetic eld. The value of $\delta^{12}_c \simeq 10^{-12}$ eV entering into the resonance condition (34) could be compensated by the twisting frequency $\dot{\Phi}$ only.

Now we discuss the resonance transitions of the $\nu_{\mu L}$ neutrino in the jet conditions. These neutrinos could exhibit $\nu_{\mu L} \rightarrow \nu_{eR}$, $\nu_{\mu L} \rightarrow \overline{\nu}_{eR}$ and $\nu_{\mu L} \rightarrow \overline{\nu}_{\mu R}$ resonance conversions.

To investigate the $\nu_{\mu L} \rightarrow \nu_{eR}$ resonance we should address to Eqs.(17)-(19). The resonance condition might be ful lled but in this case the magnetic eld must exists over the distance which is vastly more than the oscillation length. Therefore, this resonance is forbidden.

The $\nu_{\mu L} \rightarrow \overline{\nu}_{eR}$ transition is characterized by the expressions

$$2\delta_c^{12} + V_{\mu L} + A_{\mu\mu}^{DL} - A_{ee}^{DR} = 0, \quad (35)$$

$$(L_{\nu_{\mu L}\overline{\nu}_{eR}})_{max} \simeq \frac{2\pi}{\mu_{e\mu}B_\perp}. \quad (36)$$

Since the condition (35) could be satis ed for the muon neutrinos possessing the energy in the region of 10^{15} eV, then the resonance under consideration is allowed and could be attributed to the matter-induced resonances.

In the case of the $\nu_{\mu L} \rightarrow \overline{\nu}_{\mu R}$ the resonance condition and the maximum value of the oscillation length are given by the expressions

$$V_{\mu L} + A_{\mu\mu}^{DL} - A_{\mu\mu}^{DR} = 0, \quad (37)$$

$$(L_{\nu_{\mu L}\nu_{eR}})_{max} \simeq \frac{2\pi}{\mu_{\mu\mu}B_\perp}. \quad (38)$$

It is clear that the resonance condition (37) could not be ful lled and, as a result, this resonance is forbidden.

3. Conclusions

In this work we have considered the behavior of the neutrino ux in dense matter and intensive magnetic eld within three neutrino generations. The investigations have be ful lled for the Majorana and Dirac neutrinos. One was assumed that the neutrinos possess both the dipole magnetic moment and the anapole moment. As far as the magnetic eld is concerned, it has a twisting nature and displays nonpotential character. For the description of the magnetic eld twisting the simple model with the geometrical phase

$\Phi(z)$ being equal to $\exp\{\alpha\pi/L_{mf}\}$ has been used. As the examples of magnetic elds we have covered elds of the coupled sunspots being the source of the solar ares and elds of the collapsar jets. To make the results physically more transparent we have passed from the avor basis to the new one in which the resonance conditions do not depend on the angle ϑ_{23} while the ϑ_{23}-dependence has been transported on the resonance widths and the oscillation lengths.

In the Sun's and collapsar jet conditions the possible resonance conversions of the active neutrinos have been examined. In spite of similar behavior of the neutrino beam in the Majorana and Dirac pictures there is the principal di erence between these cases. It lies in the fact that in the Dirac neutrino case all magnetic-induced resonances transfer active neutrinos into sterile ones while in the Majorana neutrino case we deal with active neutrinos only. So, if the neutrino exhibits the Majorana nature, then the solar electron neutrino ux traveling through the region of the coupled sunspots could be converted into the active right-handed neutrinos $(\bar{\nu}_{eR}, \bar{\nu}_{\mu R}, \bar{\nu}_{\tau R})$. Emergence of the $\bar{\nu}_{eL}$ neutrinos, as an example, could be recorded by terrestrial detectors through the inverse β-decay reaction $\bar{\nu}_{eL} + p \to n + e^+$ having a threshold $E_{th} = 1.8$ MeV. Note, that this reaction is at the heart of the antineutrino detectors used for nuclear reactor monitoring in the on-line regime. On the other hand, since in the Dirac neutrino case the magnetic-induced resonances convert the ν_{eL} neutrinos into the sterile ν_{lR} neutrinos then only decreasing the number of ν_{eL} could be observed when the solar neutrino ux passes the coupled sunspots region. As regards the phenomena of depleting the solar electron neutrino ux, the observation of decreasing the β-decay rates of some elements during the SF's [64 67] may be speculated to be its experimental con rmation. It should be stressed that the existence of such depletion of the electron neutrino ux must be supported by other experiments.

It was shown that the beam, which consists of the muon neutrinos and muon antineutrinos, passing through the collapsar jet medium could undergo the matter-induced resonances, and, as a result, the terrestrial observer may detect decreasing the intensity of this beam.

We have also demonstrated that the expressions for the survival probability of electron and muon neutrinos found in the three neutrino generations convert into the well known expressions of the two FA provided $\phi = \psi = 0$.

It should be stressed that investigation of the neutrino uxes which are emitted from the stellar objects will enable us to deduce information not only about such neutrino properties as multipole moment values and their nature (Dirac or Majorana) but about stellar object structure too.

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