High-Frequency Electrodynamics of Slow-Moving Limited Media Taking into Account the Additional Specular Reflection

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ABSTRACT The paper presents the results of constructing the physical and mathematical model of high-frequency electromagnetic waves propagation in slowly moving media of finite dimensions, which takes into account the phenomena of specular reflection of these waves. The constructed model is based on eqations designed to determine the speed of electromagnetic waves propagation in slowly moving media of finite dimensions, as well as on equations designed to describe these waves. The feature of these equations lies in the fact that they take into account the Fresnel drag coefficient for electromagnetic waves propagation speed, while the speed of the media isn't constant, it can depend on time and coordinates. An approach to these equations solving has been developed, as well as an approach to modeling the process of electromagnetic waves propagation in slowly moving media of finite dimensions, based on the use of a difference scheme, in which the motion of these media is taken into account using infinitesimal Lorentz transformations in a difference cell. It has been determined that the developed model and approaches can be used to solve problems associated with the construction of transmitting and receiving paths of radio communication systems and information transmission through moving plasma streams, gas clouds, macroscopic plasma clots, as well as in solving problems of aeroacoustics.

INDEX TERMS Propagation medium, Electrodynamics, Electromagnetic wave.

I. INTRODUCTION

THE relativity theory in combination with Maxwell's equations was the foundation for the electrodynamics of moving medium. Indeed, the equations for the electromagnetic field in a moving medium can be derived in two ways. On the one hand, they can be obtained by averaging the microscopic equations of the electron theory, when all particles that make up the medium have a speed of ordered motion. On the other hand, the equations of a macroscopic electromagnetic field in a moving medium can be obtained using the Lorentz transformations from the known field equations for a medium at rest. G. Minkowski took the second path, showing that the equations of the electromagnetic field for moving medium unambiguously follow from Maxwell's equations for medium at rest and the relativity principle [1].

From a practical point of view, the theory of electromagnetic phenomena in slowly moving medium, i.e., the theory that can be applied to solve problems in which $\frac{u}{c} << 1$ (u – medium speed, c – light speed in vacuum) and in which all processes, the parameters of which are proportional to the square and higher powers of the ratio $\frac{u}{c}$ can be neglected. These tasks belong to such areas of practical activity as the development, tuning and improvement of transceiver devices for radio communication systems, radio lines for transmitting information through streams of plasma and gas clouds created by jet rocket engines (solid and liquid propellants) [2]. At high-current plasma accelerators, it is already possible to obtain macroscopic plasma bunches moving as a whole with velocities of 108 cm/s and higher. To simulate electromagnetic waves interaction with these objects, it's necessary to use the electrodynamics of moving medium. Such phenomena as, for example, the motion of plasma in a magnetic field, the propagation of self-focusing beams of charged particles, the properties of a superhigh current discharge channel have become commonplace for the electric power industry (especially in connection with the problem of thermonuclear energy and are based on the concept of a macroscopic electromagnetic field in moving medium) [3].

In addition, in the so-called decelerating systems, electromagnetic waves propagation speed c' can be significantly less than the speed c in vacuum. An example of such systems is a waveguide partially filled with a dielectric, a spiral waveguide, etc. In this case, the determining relativistic effect of the motion of a medium is not a relation $\frac{u}{c}$, but a relation $\frac{u}{c'}$. The indicated effects are important for diagnostics of moving medium by their interaction with electromagnetic waves, for example, generation, reflection and refraction of waves in the presence of moving layers of the ionosphere [4].

These tasks are also relevant for such area of practical activity as aerodynamic sound generation.

In connection with the above, the aim of this work was to build a consistent physical and mathematical model of high-frequency electromagnetic waves propagation in slowly moving medium of finite dimensions, when the speed of the latter depends on coordinates and time.

To achieve this aim, the following tasks have been solved:

- the list of conditions and basic equations for building the model have been determined;
- the numerical algorithm for determining of electromagnetic waves propagation speed in slowly moving medium of finite dimensions has been proposed;
- equations to describe electromagnetic waves propagating in slowly moving medium of finite dimensions, taking into account the Fresnel drag coefficient for the speed of such waves have been proposed;
- the approach to the numerical solution of the proposed equations has been developed;
- the approach to modeling the process of electromagnetic waves propagation in slowly moving medium of finite dimensions has been developed using the infinitesimal Lorentz transformations in the difference cell;
- recommendations for the practical application of the constructed model have been developed.

Consideration of some nonrelativistic effects, for example, the phenomena of generation, reflection, and refraction of waves in the presence of moving layers in the propagation medium of these waves, is also of considerable practical interest.

II. ANALYSIS OF APPLICABILITY OF MAXWELL'S EQUATIONS AND MINKOWSKI'S MATERIAL EQUATIONS OF ELECTRODYNAMICS OF MOVING BOUNDARY MEDIUM IN THE ELECTRODYNAMICS OF MOVING INHOMOGENEOUS LAYERED STRUCTURES

As it's known, to use the basic relations of electrodynamics of unlimited medium moving at a constant speed, it's necessary to fulfill a number of conditions, which are given below [5].

Condition 1. The independence of the medium properties in the rest frame from coordinates and time (i.e., the medium is stationary and spatially homogeneous).

Condition 2. The constancy of the of medium movement speed and the independence of this speed from coordinates and time.

Condition 3. Absence of surface currents and charges at the boundary of a slowly moving medium.

Condition 4. Neglecting the effects of heat and mass on the electromagnetic waves propagation.

To construct the considered model, the following equations have been used.

1. Maxwell's equations used both to describe the electromagnetic field in medium at rest, and to describe the electromagnetic field in slowly moving medium of finite dimensions:

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j},\ \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \qquad (1)$$
$$\operatorname{div}\mathbf{D} = 4\pi\rho,\ \operatorname{div}\mathbf{B} = 0,$$

where **E** and **H** – electric and magnetic fields respectively; **D** and **B** – respectively, electric and magnetic induction in a slowly moving medium of limited dimensions; ρ and **j** – the density of external charges and currents in a slowly moving medium of limited dimensions.

2. The Minkowski equations expressing the relationship between the electric field and electric induction, magnetic field and magnetic induction and obtained on the basis of Lorentz transformations:

$$\mathbf{D} + \left[\frac{\mathbf{u}}{c}, \mathbf{H}\right] = \varepsilon\left(\mathbf{E} + \left[\frac{\mathbf{u}}{c}, \mathbf{B}\right]\right),$$
$$\mathbf{B} + \left[\mathbf{E}, \frac{\mathbf{u}}{c}\right] = \mu\left(\mathbf{H} + \left[\mathbf{D}, \frac{\mathbf{u}}{c}\right]\right), \qquad (2)$$

where ε and μ – respectively, the values of the dielectric and magnetic permeability of the medium at rest.

It should be noted that Equations (2) correspond to the above *Condition* 2. It should also be noted that the well-known Lorentz transformations for the cases corresponding to translational motion with a constant speed of one medium relative to another medium are valid only when these media are infinite, in connection with than there is no need to take into account the additional specular reflection of the signal [6, p. 314-315]. Thus, Minskovsky's equations have a limited application area.

3. Equations expressing the relationship between the electric field and electric induction, magnetic field and magnetic induction at the boundary of a slowly moving medium of finite dimensions:

$$
\begin{aligned}
&\left[\mathbf{n}, \mathbf{E}_2-\mathbf{E}_1\right]=\frac{v_n}{c}\left(\mathbf{B}_2-\mathbf{B}_1\right),\\
&\left[\mathbf{n}, \mathbf{H}_2-\mathbf{H}_1\right]=i_\tau-\frac{v_n}{c}\left(\mathbf{D}_2-\mathbf{D}_1\right),\\
&\operatorname{div}\mathbf{B}=B_{2n}-B_{1n}=0,\\
&\operatorname{div}\mathbf{D}=D_{2n}-D_{1n}=\sigma,
\end{aligned}
\tag{3}
$$

where **n** – normal to the boundary of a slowly moving medium of finite dimensions; $\mathbf{E}_1$, $\mathbf{H}_1$, $\mathbf{B}_1$, $\mathbf{D}_1$ – respectively, electric field, magnetic field, electric induction, magnetic induction on one side of the boundary of a slowly moving medium of finite dimensions; $\mathbf{E}_2$, $\mathbf{H}_2$, $\mathbf{B}_2$, $\mathbf{D}_2$ – respectively, electric field, magnetic field, electric induction, magnetic induction on the other side of the boundary of a slowly moving medium of finite dimensions; i_τ – surface current; v_n – the projection of the speed of the medium boundary on the normal of this boundary [1]; σ – surface charge.

Appendix A to this article describes the equations that can be used to take into account the surface current and charge at the interface between adjacent medium when simulating the process of propagation of electromagnetic waves in slowly moving medium. This issue is described in more detail in [7]. In most works on the electrodynamics of inhomogeneous layered structures, the surface current and charge at the interface between adjacent medium are not taken into account [4].

Thus, the theory of electrodynamics of slowly moving medium in inhomogeneous layered structures of a finite size is far from complete.

III. ELECTROMAGNETIC WAVES PROPAGATION SPEED IN SLOW MOVING MEDIA

Consider a plane light wave with frequency ω, propagating in a uniform isotropic nonmagnetic dielectric moving with speed u along the z axis, provided that the direction of motion of the dielectric coincides with the direction of wave propagation, i.e. $u_x=u_y=0$, $u_z=\pm u$:

$$\mathbf{E}=\mathbf{E}_0 e^{i(\omega t-uz)}.$$

In this case, taking into account that $v=\frac{\omega}{k}$, where v – wave speed, we have the following [8, p. 84-87]:

$$
\begin{aligned}
&vE_{0x}=\frac{c}{\varepsilon}H_{0y}+u_z\left(1-\frac{1}{\varepsilon}\right)E_{0x}\\
&vE_{0y}=\frac{c}{\varepsilon}H_{0x}+u_z\left(1-\frac{1}{\varepsilon}\right)E_{0y},\ E_{0z}=0,\\
&vH_{0x}=-cE_{0y}+u_z\left(1-\frac{1}{\varepsilon}\right)H_{0x},\\
&vH_{0y}=cE_{0x}+u_z\left(1-\frac{1}{\varepsilon}\right)H_{0y},\ u_{0z}=0
\end{aligned}
\tag{4}
$$

As follows from Equations (4), if E_{0x}, E_{0y}, u_{0x}, u_{0y} are different from zero, then $v=u_z\left[1-\frac{1}{\varepsilon}\right]=\frac{c}{\sqrt{\varepsilon\varepsilon_0}}$. So as $\sqrt{\varepsilon}=n$ – refractive index of the medium, we finally write down the following equation:

$$v=\frac{c}{n}+\left(1-\frac{1}{n^2}\right)u_z. \tag{5}$$

Performing calculations for the case of an arbitrary angle between v and u, we can make sure that Equation (5) remains valid in the general case. Equation (5) was first obtained by O. Fresnel in 1818. According to the studies carried out by H. Lorentz (1895), some correction must be introduced into the Fresnel formula, taking into account the medium dispersion. Fresnel's formula is confirmed by the experimental data of A. Fizeau (1851) and with particular accuracy – by P. Zeeman (1914).

IV. EQUATIONS FOR DESCRIPTION OF ELECTROMAGNETIC WAVES PROPAGATING IN SLOWLY MOVING MEDIUM

In what follows, the electromagnetic wave speed v will be determined by the Fresnel formula. In high-frequency electrodynamics, to take into account the medium motion, the state changes of which are sufficiently slow compared to the frequency of an electromagnetic wave propagating through it, when $v(t)$ and $\frac{v}{c}<<1$ it is possible to recommend using the hyperbolic "aeroacoustics" equation derived by D. Blokhintsev [8, p. 40, 84-87] for electrodynamics [7, 9, 12]. In this case, the telegraph equation will be as follows:

$$
\begin{aligned}
&\frac{\varepsilon\mu}{c^2}\left(\frac{\partial^2\mathbf{E}}{\partial t^2}-2(\mathbf{v},\nabla)\frac{\partial\mathbf{E}}{\partial t}-(\mathbf{v},\nabla)(\mathbf{v},\nabla)\mathbf{E}+\right.\\
&\left.\left(\frac{\partial\mathbf{v}}{\partial t},\nabla\mathbf{E}\right)+\mu_0\left(\frac{\partial\mathbf{J}}{\partial t}+(\mathbf{v},\nabla\mathbf{J})\right)\right)=\\
&=\frac{1}{\mu(n)}(\Delta\mathbf{E}-\operatorname{grad}\operatorname{div}\mathbf{E})-\nabla\left(\frac{1}{\mu(n)}\right)\times\operatorname{rot}\mathbf{E}.
\end{aligned}
\tag{6}
$$

In the presence of convective motion with constant speed v in the one-dimensional case, if $\varepsilon=\text{const}$, $\mu=\text{const}$, $\lambda=\text{const}$, we receive the following:

$$
\begin{aligned}
&\frac{\varepsilon\mu}{c^2}\left(\frac{\partial^2E}{\partial t^2}-2v\frac{\partial^2E}{\partial t\partial x}\right)+\mu\mu_0\left(\frac{\partial J}{\partial t}+v\frac{\partial J}{\partial t}\right)=\\
&=\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2E}{\partial x^2}.
\end{aligned}
\tag{7}
$$

Note that Equation (6) is valid if the medium speed u depends on time, but doesn't depend on coordinates. When the medium speed is also determined by the coordinates, then Equation (6) doesn't hold. In this case, to solve the problem, we can recommend the proposed numerical method based on the use of local Einstein's invariants in the difference cell, simultaneously with the Courant condition, which relates the step in time and space:

$$\Delta\tau = \frac{h_0}{c[1-(u^2/c^2)]} = \frac{\Delta\tau_0}{[1-(u^2/c^2)]}, \qquad (8)$$

where c – speed of disturbance transmission in a medium at rest; h_0, $\Delta\tau_0$ – the length of the difference cell and the time step in the Lagrange frame of reference. In fact, Equation (7), if there is no conduction current, we solved numerically for acoustic tasks in this paper. For the one-dimensional case, instead of **E**, we used the sound pressure P [9]. The sound "audibility" against the wind decreases, and increases with the wind. These well-known experimental facts were confirmed by numerical calculations. Comparison of the data in the medium at rest and in the moving medium showed that only the sound intensity changes, but we didn't observe a phase shift.

Equation (7) can be used to simulate the occurrence of electric fields due to the movement of the medium. The idea of constructing a difference scheme for Equation (6) taking into account the medium motion is based on the papers of M. Laue [10] and A. Einstein [11], in which it is proved that a light pulse propagating in the form of a separate wave packet can be transformed as the speed of a material points. Due to the unusual and "little-known" of this position, we quote M. Laue from the paper [10]: "Imagine a light pulse propagating in the form of a wave packet and moving with a speed c relative to an observer stationary in this medium. Together with this packet, some object moves, which is always in the field of the light beam and, therefore, remains illuminated by this beam all the time. Obviously, for another observer, moving at some arbitrary speed relative to the first, this object will also appear to be illuminated all the time.

And in order for this condition to be fulfilled, it is necessary that the speed of the packet of light waves during the transition from the frame of reference of the first observer (in which he is stationary) to the "frame of the second observer" is transformed as the speed of a material point." We also present the reasoning of A. Einstein [11, p. 12]: "Let at the moment of time t_A (here "time" means "time of the resting system") a light beam leaves A, is reflected in B at the moment of time t and returns back to A at the moment of time t_B. Taking into account the principle of the light speed constancy, we find:

$$t_B - t_A = \frac{r_{AB}}{V-v},\ t'_A - t_B = \frac{r_{AB}}{V+v},$$

where r_{AB} – the length of the moving rod, measured in the frame at rest." Here V is the light speed, designations are preserved according to [6, p. 12]. Further: "Let a light beam be sent from the origin of coordinates of the system k at time τ_0 along the x axis to point x' and reflected from there at time τ_1 back to the origin where it arrives at time τ_2, then there must be ratio

$$\frac{1}{2}(\tau_0 + \tau_2) = \tau_1$$

or, writing out the arguments of the function τ and applying the principle of constancy of the speed of light in a stationary system, we have

$$\frac{1}{2}\left(\tau_0(0,0,0,t) + \tau_2\left(0,0,0,\left\{t + \frac{x'}{V-v} + \frac{x'}{V+v}\right\}\right)\right) =$$

$$= \tau_1\left(x,0,0,t + \frac{x}{V-v}\right).$$

If x' is taken to be infinitely small, then this implies

$$\frac{1}{2}\left(\frac{1}{\mathbf{V-v}} + \frac{1}{\mathbf{V-v}}\right)\frac{\partial\tau}{\partial t} = \frac{\partial\tau}{\partial x'} + \frac{1}{\mathbf{V-v}}\frac{\partial\tau}{\partial t}$$

or

$$\frac{\partial\tau}{\partial x'} + \frac{v}{V^2 - v^2}\frac{\partial\tau}{\partial t} = 0.$$

It should be noted that, instead of the origin, we could have chosen any other point as the starting point of the light beam, and therefore the equation just obtained is valid for all values x', y, z.

If we take into account that light along the y and z axes, when observed from a stationary frame, always propagates with a speed $\sqrt{V^2 - v^2}$, then a similar reasoning applied to these axes gives

$$\frac{\partial\tau}{\partial y} = 0,\ \frac{\partial\tau}{\partial z} = 0.$$

Since t is a linear function, it follows from these equations

$$\tau = a\left(t - \frac{v}{V^2 - v^2}x\right),$$

where a – yet unknown function $\varphi(v)$ and for the sake of brevity it is assumed that at the origin of coordinates of the system k at $\tau = 0$ also $t = 0$."

Note, that A. Einstein used his transformations only for infinitely small segments, therefore, it is logical to use them to take into account the motion of the medium in the difference cell. The need to present these considerations is due to the fact that in [11, p. 12], the case of two moving bodies, but not a continuous medium, is proposed. For a continuous medium, as shown in the paper [9], it is necessary to introduce at least one more point and use two-layer time difference schemes for the hyperbolic equation.

The concepts of space and time, which were developed by A. Einstein, arose on an experimental-physical basis. However, in recent publications and textbooks on field theory, the above reasoning [11, p. 12] are virtually absent. Considering that the concept of simultaneity of non-uniform events is relative, A. Einstein introduced a virtually new concept of time, which is used in the relativity theory.

V. NUMERICAL SIMULATION OF ELECTROMAGNETIC WAVE PROPAGATION IN SLOW MOVING BOUNDED MEDIA USING INFINITELY SMALL LORENTZ'S TRANSFORMATIONS

To simulate the process of electromagnetic waves propagation in slowly moving medium, the authors propose to use an approach based on the construction and use of a difference scheme, which takes into account of these medium motion.

We used a similar approach to simulate the sound waves propagation in a moving inhomogeneous medium [9]. Earlier, we showed [12] that to solve the problem of an electromagnetic pulse propagation in an inhomogeneous medium, taking into account the induced surface charge and surface current, it is necessary to reduce Maxwell's equations to hyperbolic equations for **E** and take into account the matching conditions at the interface between adjacent medium.

We have considered in more detail the issues of agreement in work [7] and presented in Appendix B to this article. Note that in order to fulfill the matching conditions, we proposed to use a soliton-like electromagnetic wave packet, which can be used to take into account the broadening of spectral lines, the temporal and spatial dispersion of signals in a material moving medium.

Using the known characteristic of the electric field **E** (r, τ), it is easy to determine the characteristic of the magnetic field **H** (r, τ) from the second Maxwell's equation. Note that at present, when considering problems of mathematical physics, which are described using hyperbolic equations, as a rule, the effect of convective transport is not taken into account. For example, when deriving the string vibration equation, it is assumed that the string length is constant.

When modeling the process of propagation of sound waves in a moving inhomogeneous medium, we used a wave equation of the form:

$$\eta(P,x,t)\frac{\partial^2 P}{\partial t^2} = \frac{\partial}{\partial x}\left(k(P,x,t)\frac{\partial P}{\partial x}\right), \qquad (9)$$

where P – the pressure exerted by sound waves on the particles of the medium in which they propagate.

The presented equation takes into account the fact of medium motion in which sound waves propagate.

It is known that upstream and upstream disturbances occur at different speeds: $v\left(1+\frac{u}{v}\right)$ and $v\left(1-\frac{u}{v}\right)$, where v – in the case under consideration, the speed of sound waves, u – the speed of movement of the medium in which these waves propagate. Multipliers $\left(1\pm\frac{u}{v}\right)$ in what follows we will define it as a nonreciprocity relation. We will take this effect into account when constructing a difference scheme for the wave Equation (6) using deformations of the difference cell, which reflect the influence of the motion of the medium.

Let's define the relationship of spatial and temporal dimensions in a moving medium. Let us introduce a uniform mesh in a medium at rest: $\Omega_{h_0,\tau} = \Omega_{h_0} \times \Omega_\tau$, where $\Omega_{h_0} = \{x_i = ih_0,\ i = 0, 1, 2, ..., N_x,\ h_0 N_x = \ell\}$ and $\Omega_\tau = \{t_j = j\tau, j = 0, 1, 2, ..., N_\tau, \tau N_\tau = t_m\}$.

In the case of the medium motion, each node moves relative to a stationary observer with a speed v_i, therefore, during the sound signals propagation, during which the observer constantly registers the amplitude and frequency of changes in the pressure exerted by sound waves on the particles of the medium in which they propagate, or the speed of motion of this medium, the node moves some additional distance. For a stationary observer at node i, sending or receiving sound waves (change in pressure or speed) in the direction of motion of the medium and against the direction of motion of the medium (specular reflection), the paths of its passage for a cell of length h_0 (h_0 are cells in the Lagrange frame of reference) will be different.

Let's consider two cases of sound wave propagation: A – to the right of node i; B – to the left of node i. We assume that the medium speed within the difference cell, as well as the sound waves speed, are constant values. The dependence of the medium motion speed u and the sound propagation speed v on the coordinate and time will be taken into account when constructing the difference scheme.

According to [8], for the general case when the convective term $u(x, \tau)$ depends on both the coordinate and time, there is no solution to Equation (9) in differential form.

A. The stationary observer is at node i, then the sound waves, the direction of propagation of which coincides with the direction of the medium motion, has a total speed $v+u$, and during the time Δt of the signal from node i to node i+1, node i+1 itself moves to some additional distance $u\Delta t$, therefore $h^+ = h_0 + u\Delta t$, where $\Delta t = h^+/(v+u)$ and $h^+ = h_0 + uh^+/(v+u)$.

B. A stationary observer is at node i, then sound waves, the propagation direction of which doesn't coincide with the medium motion direction, have a total speed v–u, and during the time of signal propagation Δt from node i to node i–1, the last one, that is, node i–1, approaches node i at a distance $u\Delta t$, therefore $h^- = h_0 - u\Delta t$, where $\Delta t = h^-/(v-u)$ and $h^- = h_0 - uh^-/(v-u)$.

Then

$$h^+ = h_0\left(1+\frac{u}{v}\right),\ h^- = h_0\left(1-\frac{u}{v}\right), \qquad (10)$$

where h^+, h^- – sizes of cells in the Euler frame of reference. Consequently, for a stationary observer located at node i, an initially uniform grid is deformed in the Euler coordinate system.

When deriving Equation (10), we actually used in the difference cell the general d'Alembert solution for the one-dimensional wave Equation (9), which represents a pair of traveling waves propagating, respectively, to the right and left along the x axis at a constant speed: $P(x, t) = P_1(x-ct)+P_2(x+ct)$. These waves are the superposition of direct and reflected waves [13, p. 309-310] (specular reflection).

In the difference scheme, we will use Equation (10), which actually lead to deformations of the Euler computational domain due to the motion of the medium. The general d'Alembert solution for the one-dimensional wave Equation (9) was generalized by us to a moving medium. The medium motion is taken into account directly in the difference cell when constructing the difference algorithm. The size of this cell should not depend on the speed sign; therefore, when constructing the difference scheme, we use the condition that the dimensions of the cell are independent of the direction and sign of the moving medium speed [11]. This means that the sound waves sent by the observer at time j–1 in the direction of medium propagation (or against this direction) should be perceived at time j+1 at the same node i, when the direction of the signal is reversed, i.e. that is, measurements are averaged over time. In other words, this procedure of averaging measurements in time reflects the fact that the same node at the same time is the emitter and receiver of sound waves.

When constructing the difference scheme, we assume that the wave Equation (9) is valid in a medium at rest and in a moving medium in the local instantly accompanying Lagrange frame of reference for a given individual difference cell with a sufficiently small step h_0. Having solved Equation (10) with respect to h_0, we pass to the Euler variables and, for convenience, choosing the step in space $h = \ell / N_x$, we have $h_0 = h/(1 \pm u/v)$,

$$\bar{h}_0 = \frac{1}{2}\left(\frac{h}{1-\frac{u}{v}} + \frac{h}{1+\frac{u}{v}}\right) = \frac{h}{1-\frac{u^2}{v^2}}. \qquad (11)$$

Let us first consider the case when $\eta = 1/v^2$ and k = 1. We write the initial wave equation not for a segment, but for the difference spatial nodes of the grid:

$$\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} = \left.\frac{\partial^2 P}{\partial x^2}\right|_{x=x_*}, \ t_{j-1} \le t \le t_{j+1}, \qquad (12)$$

where x_* – deformed nodes constructed in accordance with Equation (11).

Boundary and initial conditions:

$$P(0,t) = P_1(t), \quad P(\ell,t) = P_2(t), \quad P(x,t_0) = P_0(x) \qquad (13)$$

$$\frac{\partial P(x,t_0)}{\partial t} = P^*(x), \ 0 \le x \le \ell. \qquad (14)$$

A difference algorithm for solving the initial-boundary value problem (12)–(14) is given in Appendix B to this article.

The proposed method for the numerical solution of the wave equation in a moving medium using infinitely small Lorentz transformations is also applicable for cases when the medium has ionic conductivity (electrolyte, plasma, ionosphere). It should be noted that when using this method in this case, it is necessary:

- take into account the effect of heat and mass fluxes in the expression for the total current [12];
- determine the speed of the medium taking into account the Fresnel increase factor.

VI. CONCLUSION

When using the developed method in the course of modeling of electromagnetic waves propagation process in slowly moving bounded medium, it's possible to take into account the influence of the medium motion on the electromagnetic waves propagation, as well as additional specular reflection, when the velocity of the medium depends not only on time, but also on coordinates. The proposed method for the numerical solution of the wave equation in a moving medium is applicable for cases when the medium has ionic conductivity (electrolyte, plasma, ionosphere). It should be noted that when using this method in this case, it is necessary:

- take into account the effect of heat and mass fluxes in the expression for the total current [7];
- determine the medium speed taking into account the Fresnel increase factor.

Also, the proposed method for the numerical solution of the wave equation in a moving medium can be generalized to phenomena associated with the propagation of electromagnetic waves with a speed exceeding the speed of light. However, it should be noted that such waves cannot be used in practice in order, for example, to remotely activate an electronic device or to transmit information over a wireless medium, that is, in essence, such waves are not signals. A. Sommerfeld in the paper [14] has substantiated the signals properties. In accordance with [14], the most significant property of the signal is that its front under all circumstances and for any medium features propagates at the light speed.

In accordance with the reasoning presented in Sections IV and V of this paper, the proposed principle of discrete consideration of wave propagation can be used to solve problems both in the field of electrodynamics and in the field of aeroacoustics, as well as when constructing theories of diffusion elasticity, wave equations of heat conduction, and other phenomena that are described by hyperbolic equations and in which it is necessary to take into account the effect of convective motion of the medium.

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APPEDINX A

Maxwell's boundary conditions do not contain any closing relations for the induced surface charge and surface current. The use of known Leontovich – Shchukin conditions at interfaces, for example, for nanoobjects, is incorrect, since these conditions assume that the electrophysical properties and characteristics of the object vary little over distances of the order of a wavelength. The surface charge due to the passage of a direct current through the interface of adjacent medium can be determined according to [9, 12]

$$\sigma = \frac{U}{RS}\varepsilon_0\left(\frac{\varepsilon_1}{\lambda_1} - \frac{\varepsilon_2}{\lambda_2}\right). \qquad \text{(A1)}$$

As follows from Equation (A1), the induced electric charge σ is determined by the magnitude of the current, and also by a factor that takes into account the properties of the medium. It is assumed that the surface charge is zero and is associated with electrostatics. As can be seen from Equation (A1), this assertion is valid only if

$$\frac{\varepsilon_1}{\lambda_1} - \frac{\varepsilon_2}{\lambda_2} = 0.$$

It is usually believed that the interface between adjacent medium isn't capable of carrying a surface charge, and it should be taken into account only for electrostatic problems. In fact, as shown in [9, 12], the surface charge can arise in the presence of a normal component of the conduction current.

The reformulation of Maxwell's equations and their reduction to a wave equation make it possible to exclude from consideration surface charge and current, but in this case the number of boundary conditions becomes significantly larger, since at each boundary it is required to specify nine first derivatives now with respect to coordinates, as well as E_x, E_y and E_z, and their derivatives with respect to time, i. e. 15 conditions, and the problem also becomes technically unrealizable. The way out is to use the continuity condition of the total current at the interface between adjacent medium and the use of end-to-end counting schemes. The calculation method consists in that the investigated region containing various inhomogeneous (angular, layered) structures is conventionally placed in the "casing" and we specify the boundary conditions only on it [7, 9].

APPENDIX B

For clarity, we consider the one-dimensional wave equation of oscillations of a string:

$$\frac{\partial^2 u}{\partial t^2} - a^2\frac{\partial^2 u}{\partial x^2} = f(x,t),\ (x,t)\in \mathbf{D}, \qquad \text{(B1)}$$

$$u\,|_{t=0} = \varphi(x), \left.\frac{\partial u}{\partial t}\right|_{t=0} = \psi(x), 0 \le x \le l, \qquad \text{(B2)}$$

$$u\,|_{x=0} = \mu_1(t), u\,|_{x=l} = \mu_2(t),\ t \ge 0. \qquad \text{(B3)}$$

With given functions $f''_{xx} \in C(\mathbf{D})$, $\varphi \in C^2(0 \le x \le l)$, $\psi \in C^1(0 \le x \le l)$, and $\mu_1 \in C^2(0 \le x \le \infty)$, you need to find a function $u \in C^2(\mathbf{D})$, that satisfies Equation (B1) in the region **D**, initial conditions (B2), and boundary conditions of the first kind (B3). Problem (B1) – (B3) describes the process of oscillations of a homogeneous string of length l stretched along a segment $0 \le x \le l$. The first initial condition of (B2) defines the graph $u = \varphi(x)$ of the string at the initial time t=0, and the quantity $\psi(x)$ from the second initial condition of (B2) is the initial velocity of the string at the point with a coordinate x.

Note that when formulating the problem (B1) – (B3), certain restrictions must be imposed on the given functions φ, ψ, and μ_1. In particular, matching conditions must be satisfied at the angular points of the region:

$$\begin{gathered}\varphi(0)=\mu_1(0),\ \varphi(l)=\mu_2(0),\\ \mu'_1(0)=\psi(0),\ \mu'_2(0)=\psi(l)\end{gathered} \tag{B4}$$

These conditions are necessary conditions for the continuous differentiability of the solution $u(x, t)$ in a closed region **D**. Since the solution is $u \in C^2(\mathbf{D})$, then in addition to conditions (B4), second-order conditions must be satisfied:

$$\begin{gathered}\mu''_1(0)-a^2\varphi''(0)=f(0,0),\\ \mu''_2(0)-a^2\varphi''(l)=f(l,0)\end{gathered}. \tag{B5}$$

Indeed, we differentiate conditions (B3) twice with respect to t, and the first condition of (B4), twice with respect to x, then

$$\begin{gathered}\left.\frac{\partial^2 u}{\partial t^2}\right|_{x=0,t=0}=\mu''_1(0),\ \left.\frac{\partial^2 u}{\partial t^2}\right|_{x=l,t=0}=\mu''_2(0),\\ \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=0,t=0}=\varphi''(0),\ \left.\frac{\partial^2 u}{\partial x^2}\right|_{x=l,t=0}=\varphi''(0),\end{gathered} \tag{B6}$$

Substituting the values of the derivatives at the corresponding points to Equation (B1), we obtain the required conditions (B5).

In the future, we will be interested in waves generated by sources that perform modulated oscillations:

$$A\cos wt \pm \frac{mA}{2}\cos w_1 t \pm \frac{mA}{2}\cos w_2 t, \tag{B7}$$

where $w_1 = w+\Omega$ and $w_2 = w+\Omega$.

Equation (B7) is easily transformed into

$$S=A[1\pm m\cos(\Omega t-Kx)]\cos(wt-Kx) \tag{B8}$$

or

$$S=A[1\pm m\cos(\Omega t-Kx)]\sin(wt-Kx), \tag{B9}$$

where $K = 0.5(k_1-k_2)$, $K' = 0.5(k_1+k_2)$, and $\Omega \ll w$, Ω is the frequency and k_1 and k_2 are the wave numbers.

In fact, the wave generated by our modulated source does not differ from the superposition of the three waves that would be created by three independent sources. Let us imagine that waves of the form (B8) and (B9), which were formed with the help of special "tuning forks", i.e. technical means, begin to propagate in a continuous medium. In this case, because of the interaction of a "direct" wave of the form (B8) and (B9) with the backward wave, the spectral line broadens. The broadening of the spectral line is explained by many effects: spin-spin interaction, signal absorption by the medium, exchange interaction, collisional broadening, Doppler effect, etc. As is known, the general solution of the wave Equation (B1) is the d'Alembert solution: the sum of the forward and backward waves, with the backward wave having a broadening of ±2Δ. As a result, we obtain for an electromagnetic wave

$$\begin{aligned}&E_x(t)=A[1+m\cos\Omega t]\times\\ &\times[2\sin wt-\sin(w-2\Delta)t-\sin(w+2\Delta)t]=\\ &=4A[1\pm m\cos\Omega t]\sin^2\Delta t\sin wt,\end{aligned} \tag{B10}$$

where the frequency w corresponds to a wave moving to the right, and the frequency $w\pm2\Delta$ corresponds to a wave moving to the left. A similar expression can also be obtained for E_y:

$$\begin{aligned}&E_y(t)=A[1\pm m\cos\Omega t]\times\\ &\times[2\cos wt-\cos(w-2\Delta)t-\cos(w+2\Delta)t]=\\ &=4A[1\pm m\cos\Omega t]\sin 2\Delta t\cos wt.\end{aligned} \tag{B11}$$

In deriving (B10) and (B11), we used the well-known trigonometry equations:

$$\begin{aligned}&\sin\alpha\ \sin\beta\ \sin\gamma=\frac{1}{4}[\sin(\alpha+\beta-\gamma)+\sin(\beta+\gamma-\alpha)+\\ &+\sin(\gamma+\alpha-\beta)-\sin(\alpha+\beta+\gamma)],\\ &\sin\alpha\ \sin\beta\ \cos\gamma=\frac{1}{4}[-\cos(\alpha+\beta-\gamma)+\cos(\beta+\gamma-\alpha)+\\ &+\cos(\gamma+\alpha-\beta)-\cos(\alpha+\beta+\gamma)]\end{aligned}$$

with the condition α=β.

The broadening of the spectral lines is due to the interaction of the emitting atom with the surrounding particles: other atoms and molecules, ions and electrons. Therefore, functions of the form (B10) and (B11) continuously fill the frequency band $w-2\Delta \le w \le w+2\Delta$. In addition, it takes some time to establish a signal (transient process), finally, for the components E_x and E_y we have

$$\begin{aligned}&E_x(0,t)=\\ &=A\left[\int_{w-2\Delta}^{w+2\Delta}\left(1-\frac{2}{e^{\lambda\sin^2\varphi t}+e^{-\lambda\sin^2\varphi t}}\right)\sin^2\varphi t d\varphi\right]\times\\ &\times\frac{(1-m\cos\sigma t)\sin wt}{4\Delta},\end{aligned} \tag{B12}$$

$$\begin{aligned}&E_y(0,t)=\\ &=A\left[\int_{w-2\Delta}^{w+2\Delta}\left(1-\frac{2}{e^{\lambda\sin^2\varphi t}+e^{-\lambda\sin^2\varphi t}}\right)\sin^2\varphi t d\varphi\right]\times\\ &\times\frac{(1-m\cos\sigma t)\cos wt}{4\Delta}.\end{aligned} \tag{B13}$$

It is easy to verify that $Ex(0, t)$ and $E_z(0, t)$ satisfy the necessary matching conditions, since $E_x(0,t) = 0$, $E'_x(0, t) = 0$, $Ez(0, t) = 0$ and $E'_z(0, t) = 0$, when the pulse propagates into a medium with zero initial conditions. A group of waves is a kind of oscillatory circuit with distributed parameters, in which "forced" oscillations are not established immediately, but only after a certain time after "including an external emf".

At the initial time moment, when $t\to 0$, the expression in parentheses in the integrand expressions (B12), (B13) exactly corresponds to the "establishment function" in the oscillatory circuit under the action of an external sinusoidal electromotive force.

The problem of propagation of a soliton-like electromagnetic pulse in an inhomogeneous medium is considered in more detail in [7].

APPENDIX C

We denote by $y_{i,j}$ the approximate value of the function P at the grid nodes $\Omega_{h_0,\tau}$. Equation (12) is approximated by a difference scheme taking into account the nonreciprocity relation (10) and the independence of the grid step size from the direction of motion of the medium (averaging of measurements over time)

$$\frac{y_{i,j+1}-2y_{i,j}+y_{i,j-1}}{c_{i,j}^2\tau^2}=\frac{\left(1-\left(u^2/v^2\right)\right)_{i,j}}{2h}\times$$
$$\times\left[\varphi_{i+\frac{1}{2},j+1}\frac{y_{i+1,j+1}-y_{i,j+1}}{h}-\right.$$
$$-\gamma_{i-\frac{1}{2},j+1}\frac{y_{i,j+1}-y_{i-1,j+1}}{h}+ \tag{C1}$$
$$+\gamma_{i+\frac{1}{2},j-1}\frac{y_{i+1,j-1}-y_{i,j-1}}{h}-$$
$$\left.-\varphi_{i-\frac{1}{2},i-1}\frac{y_{i,j-1}-y_{i-1,j-1}}{h}\right],$$
$$i=\overline{2,N_x-1},\ j=\overline{1,N_t-1}$$

where $\varphi=1+u/v$, $\gamma=1-u/v$,

$$\left(1-\frac{u^2}{v^2}\right)_{i,j}=2\left(\left(1+\frac{u}{v}\right)_{i,j}^{-1}+\left(1-\frac{u}{v}\right)_{i,j}^{-1}\right)^{-1}.$$

Boundary conditions:

$$y_{1,j}=P_1\left(t_j\right),\ v_{N_x,j}=P_2\left(t_j\right),\ j=\overline{2,N_t} \tag{C2}$$

Initial conditions:

$$y_{i,0}=P_0\left(x_i\right),\ i=\overline{0,N_x},$$
$$y_{i,1}=P_0\left(x_i\right)+\tau P^*\left(x_i\right)+$$
$$+\frac{v_{i,j}\tau}{2}\left(\frac{y_{i+1,0}-y_{i,0}}{h}-\frac{y_{1,0}-y_{i-1,0}}{h}\right), \tag{C3}$$
$$i=\overline{1,N_x-1}$$

The stability of the sweep algorithm for the difference scheme the Equations (C1)–(C3) are satisfied for any τ and h [4].