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The $(g-2)_\mu$ Anomaly Within the Left-Right Symmetric Model

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Within the model based on the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge group (left-right symmetric model- LRM) the influence of the Higgs sector on the value of the muon magnetic moment (MMM) is investigated. The contributions caused by the doubly charged Higgs boson $\Delta_2^{(--)}$ are found. The obtained value of the MMM is the function of the mass of $\Delta_2^{(--)}$ boson and the triplet Yukawa coupling constants. We demonstrate that at the definite values of these parameters the LRM provides an explanation of the E989 experiment at Fermilab.
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1. Introduction

The predictions of the standard model (SM) has already been corroborated by various contemporary experiments. Nevertheless, some experiments have found several subtle deviations from the SM predictions that remain to be accounted for. Among the experiments that urgently require going beyond the SM are the following.

The first one is the new measurement of the W-boson mass. The CDF collaboration reported their new measurement of the W-boson mass [1]

$$m_W^{exp} = (80.433 \pm 0.0094)\ \text{GeV} \tag{1}$$

which approximately has 7σ deviation from the SM value [2]

$$m_W^{SM} = (80.357 \pm 0.006)\ \text{GeV}. \tag{2}$$

Muon decay can be used to predict m_W in the SM from the more precisely measured inputs, the Z-boson mass (m_Z), the fine structure constant (α), and the Fermi constant (G_μ). Calculations lead to the expression [3]

$$m_W^2 = m_Z^2\left[\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2}(1+\Delta r)}\right], \tag{3}$$

where the quantity Δr summarises the radiative corrections. This expression represents of central importance for precision tests of the electroweak theory.

The second experiment is connected with lepton flavour universality (LFU) violation. Various B meson decays have shown significant deviations from the SM predictions, most of which are related to muon final states. The LFU in B meson decays can be tested by measuring the ratios of the $b \to sll$ transitions

$$R_K = \frac{\text{BR}(B^+ \to K^+\mu^+\mu^-)}{\text{BR}(B^+ \to K^+e^+e^-)}, \qquad R_{K^*} = \frac{\text{BR}(B^0 \to K^{*0}\mu^+\mu^-)}{\text{BR}(B^0 \to K^{*0}e^+e^-)}, \tag{4}$$

and the ratios of the $b \to cl\overline{\nu}_l$ decays

$$R_{D^{(*)}} = \frac{\text{BR}(B \to D^{(*)}\tau\overline{\nu})}{\text{BR}(B \to D^{(*)}l\overline{\nu})}, \tag{5}$$

where $l = e, \mu$. Recently, LHCb has updated the measurement on R_K [4]

$$R_K^{exp} = 0.846^{+0.042+013}_{-0.039-0.012}, \tag{6}$$

which is 3.1σ away from the SM prediction [5]

$$K_R^{SM} = 1.0003 + 0.0001. \tag{7}$$

The third experiment calling for new physics beyond the SM connects with the measuring of the spin dipole magnetic moment (MM) of the muon. It should be reminded that measurements of the MM's of particles have a rich history as harbingers of impressive progress in the quantum theory. For example, registration of the anomalous values of the nucleons MM's was powerful argument for the benefit of the π-meson theory of the nuclear forces formulated by Yukawa while determination of the anomalous MM of the electron has played an important role in development of modern quantum electrodynamics. Let us remind what is measured in the experiment. The magnetic moment of muon is given by

$$\vec{\mu}_\mu = g_\mu \left(\frac{q}{2m}\right)\vec{s}, \tag{8}$$

where g_μ is the gyromagnetic ratio and its value is 2 for a structureless, spin $|\vec{s}| = 1/2$ particle of mass m and charge q. Any radiative correction, which couples the muon spin to the virtual fields, contributes to its magnetic moment and is given by

$$a_\mu = (g_\mu - 2)/2. \tag{9}$$

The recent measurement of the anomalous muon MM, a_μ, by the E989 experiment at Fermilab [6]

$$a_\mu^{FNAL} = 116592040(54) \times 10^{-11} \tag{10}$$

shows a discrepancy with respect to the SM prediction

$$a_\mu^{SM} = 116591810(43)54) \times 10^{-11} \tag{11}$$

which when combined with the previous Brookhaven determination of

$$a_\mu^{BNL} = 116592089(63) \times 10^{-11} \tag{12}$$

leads to a 4.24σ observed excess of

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 251(59) \times 10^{-11}. \qquad (13).$$

Within the SM the muon AMM value is based on up-to-date predictions of QED [7], electroweak [8], hadronic vacuum polarization [9] and hadronic light-by-light [10]. If the deviation of Eq. (13) can be attributed to effects of the physics beyond the SM, then at 95% CL, Δa_μ must lie in the range

$$135.36 \times 10^{-11} \leq \Delta a_\mu \leq 366.63 \times 10^{-11}. \tag{14}$$

The status of the SM calculation of muon MM has been updated recently in Ref. [11]. There are a lot of papers in which the explanations of $(g-2)_\mu$ anomaly are suggested (for a comprehensive review see Ref.[12]). The new Fermilab a_μ measurement provides the best possible starting point for future a_μ determinations. Exciting further progress can be expected from the Run-2-4 results of the FNAL $(g-2)$ experiment, the planned JPARC $(g-2)$ experiment [13] and from further progress on SM theory including the MUonE initiative to provide alternative experimental input to the determination of the hadronic contributions to a_μ [14].

The forth problem of the contemporary physics is connected with existence of a sterile neutrino. Experiments on search for possible neutrino oscillations in sterile state have been carried out for many years. There are experiments at accelerators, reactors, and artificial neutrino sources. Recently, new results in favor of active-sterile neutrino oscillations have appeared after revisions of the reactor data in the Neutrino-4 Experiment [15] ((Dimitrovgrad, Russia). There are several possible schemes of mixing between sterile and active flavors of neutrino. The simplest scenario of mixing is the so called 3+1 scheme, where three active flavors of neutrino and one sterile state are involved. For this case it is reasonable to use the short baseline limit, when leading contribution to oscillations comes only from sterile oscillation parameters (mixing angle and mass square difference). Other oscillation parameters will not impact the oscillation probability. In Ref.[16] all the collected at NEUTRINO-4 data were analyzed. As a result of the analysis, the authors inferred that at currently available statistical accuracy the oscillations have been observed at the 2.9σ level with parameters $\Delta m^2_{14} = (7.3 \pm 1.17)\text{eV}^2$ and $\sin^2 2\theta_{14} = 0.36 \pm 0.12$. The statement was also done that collaboration NEUTRINO-4 plan to improve the currently working experimental setup and create a completely new setup in order to increase the accuracy of the experiment by 3 times. Experimental verification of the results of NEUTRINO-4 is now at the foreground. One of the measurements capable of confirming or refuting the NEUTRINO-4 data will be a series of experiments planned at the Fermi National Laboratory (DANSS and Stereo). The central role in these series of experiments is assigned to the IKARUS neutrino detector, which began its work in 2021. Similar reactor experiments are being carried out, but in terms of sensitivity they are still inferior to NEUTRINO-4. Measurement of the spectrum from the decay of tritium also looks promising (KATRIN, Karlsruhe, Germany).

So, while it is often argued that the SM should be augmented by New Physics at higher energy scales because of some unanswered fundamental questions the new measurement of the W-boson mass, the lepton flavor universality violation, the $(g-2)_\mu$ anomaly and the active-sterile neutrino oscillations may be considered as the New Physics signal already at the weak scale. Special attention must be given to the fact that the first three anomalies are concerned with the muon. It should be note that all these experiments cannot be explained as a mere statistical fluctuation, as several earlier deviations from the SM turned out to be.

In this work we consider the $(g-2)_\mu$-anomaly within the model based on the $SU(2)_R \times SU(2)_L \times U(1)_{B-L}$ gauge group [17–19] (left-right symmetric model - LRM). The aim is to clarify whether this model could explain the results of the E989 experiment at Fermilab. In so doing we shall invoke the results of Ref.[20] in which it was shown that the LRM could give the $(g-2)_\mu$ value coinciding with the value measured in the previous E821 experiment at Brookhaven National Laboratory [21]. In the next Sec. we bring the short description of the neutrino and Higgs sectors of the LRM. In Sec.2 we represent the results of calculations of one-loop electroweak corrections to the muon AMM caused by the doubly charged Higgs bosons $\Delta^{\pm\pm}$. In Sec.3 comparing the theoretical and the experimental values of a_μ we find the bounds on the Higgs sector parameters which provide in its turn information on the heavy neutrino masses and the mixing angles. Sec.4 is devoted to analysis of the derived results.

2. Brief description of the LRM

There exist two versions of the LRM in relation to the choice of the Higgs sector. In the first case the LRM contains the bi-doublet $\Phi(1/2,1/2,0)$ and two triplets $\Delta_L(1,0,2)$, $\Delta_R(0,1,2)$ [22] (in brackets the values of S_L^W, S_R^W and $B-L$ are given, S_L^W (S_R^W) is the weak left (right) isospin while B and L are the baryon and lepton numbers, respectively), while in the second case the Higgs sector consist of the bi-doublet $\Phi(1/2,1/2,0)$ and two doublets $\chi_L(1/2,0,1)$, $\chi_R(0,1/2,1)$ [23]. Notice that in the LRM the Higgs sector content defines the neutrino nature. Neutrinos are Majorana particles in the first case and they have Dirac nature in the second case. In what follows we shall use the LRM whose Higgs sector consists of the bi-doublet and two triplets. After the spontaneous symmetry breaking we have 14 physical Higgs bosons: four doubly charged scalars $\Delta_{1,2}^{(\pm\pm)}$, four singly charged scalars $h^{(\pm)}$ and $\tilde{\delta}^{(\pm)}$, four neutral scalars S_i ($i=1,2,3,4$) and two neutral pseudoscalars $P_{1,2}$ (S_1 is an analog of the SM Higgs boson). To achieve agreement with experimental data, it is necessary to ensure fulfillment of the condition

$$v_L \ll \max(k_1,k_2) \ll v_R,$$

where v_R (v_L) is the vacuum expectation value (VEV) of the neutral component of the right(left)-handed Higgs triplet, k_1, k_2 are the VEV of the neutral components of the bi-doublet.

We emphasize that in the LRM the Higgs bosons coupling constants determining the interaction of the Higgs bosons both with leptons and with gauge bosons are connected to the neutrino oscillation parameters (NOP's). Therefore the bounds on the Higgs sector parameters could be extended to the bounds on the NOP's.

The choice of the Higgs potential determines both the Higgs boson masses and the form of the Lagrangian describing the Higgs boson interactions with fermions and gauge bosons. The most general Higgs potential $\mathcal{L}_Y^g$ has been proposed in Ref.[24]. Using $\mathcal{L}_Y^g$ one could obtain

$$m^2_{\Delta_2} = \frac{\alpha_3 k_-^2 - (2\rho_1-\rho_3)v_R^2}{2} - \frac{k_-^4(\beta_3 k_+^2 + \beta_1 k_1 k_2)^2}{2k_1^4(4\rho_2+\rho_3-2\rho_1)v_R^2}, \tag{15}$$

where

$$k_0 = \frac{k_-^2}{\sqrt{2}k_+}, \qquad k_\pm = \sqrt{k_1^2 \pm k_2^2}$$

$\rho_{1,3}, \beta_{1,3}$ and α_3 are the constants entering the Yukawa potential. For the mass of this boson to be around the electroweak scale the quantity $(\rho_3/2-\rho_1)$ should have the order of few $\times 10^{-2}$. The $\Delta_2^{(\pm\pm)}$-bosons do not interact with quarks, and as a consequence, the more firm data for obtaining the bounds on their masses come from investigation of the electroweak processes. For example, data of LEP experiments (ALEPH, DELPHI, L3, and OPAL) yield the lower bound equal to 80 GeV [25].

To evaluate v_R one may use the relation [26]

$$v_R = \sqrt{\frac{m_{W_R}^2 - m_{W_L}^2}{g_L^2(1+\tan^2 2\xi)}}, \tag{16}$$

where W_R is an additional (respect to the SM) charged gauge boson and ξ is the mixing angle in the charged gauge boson sector. The current bounds on the LRM gauge boson W_R and the mixing angle ξ are varied within a broad range in relation to what kind of reactions and what assumptions have been used at analysis. For example, the upper bound on m_{W_R} is equal to 1600 GeV, while the lower bound on ξ is few $\times 10^{-3}$.

The Lagrangians which are required for our purposes are given by the expressions

$$\mathcal{L}_{\gamma\Delta_2\Delta_2} = 2ie[(\partial_\mu \Delta_2^{(--)*}(x))\Delta_2^{(--)}(x) - \Delta_2^{(--)*}(x)(\partial_\mu \Delta_2^{(--)}(x))]A^\mu + \text{conj.}, \tag{17}$$

$$\mathcal{L}_l^{dc} = -\sum_{a,b} \frac{f_{ab}}{2}[\bar{l}_a^c(x)(1+\gamma_5)l_b(x)s_{\theta_d} + \bar{l}_a^c(x)(1-\gamma_5)l_b(x)c_{\theta_d}]\Delta_2^{(--)*}(x) + \text{conj.}, \tag{18}$$

where the superscript c denotes the charge conjugation operation, $c_{\theta_d} = \cos\theta_d$, $s_{\theta_d} = \sin\theta_d$, θ_d is the mixing angle of the doubly charged Higgs bosons ($\tan\theta_d \sim k_+^2/v_R^2$), f_{ab} is the Yukawa triplet coupling constant, g_R is the gauge coupling of the $SU(2)_R$ subgroup (further we shall speculate that $g_L = g_R$).

In the LRM the Higgs bosons coupling constants are connected with the neutrino sector parameters. For example, in the basis $\Psi^T = \left(\nu_{aL}^T, N_{aR}^T, \nu_{bL}^T, N_{bR}^T\right)$, the neutrino mass matrix

$$\mathcal{M} = \begin{pmatrix} f_{aa}v_L & m_D^a & f_{ab}v_L & M_D \\ m_D^a & f_{aa}v_R & M_D & f_{ab}v_R \\ f_{ab}v_L & M_D & f_{bb}v_L & m_D^b \\ M_D & f_{ab}v_R & m_D^b & f_{bb}v_R \end{pmatrix}, \tag{19}$$

where

$$m_D^a = h_{aa}k_1 + h'_{aa}k_2, \qquad M_D = h_{ab}k_1 + h'_{ab}k_2$$

and h_{ab}, h'_{ab} are the bi-doublet Yukawa coupling constants. Then one could show that the elements of the matrix $\mathcal{M}$ are expressed in the terms the neutrino oscillations parameters by the following way [26]

$$\left.\begin{aligned} m_D^a &= c_{\varphi_a}s_{\varphi_a}(-m_1c_{\theta_\nu}^2 - m_3s_{\theta_\nu}^2 + m_2c_{\theta_N}^2 + m_4s_{\theta_N}^2), \\ m_D^b &= m_D^a(\varphi_a \to \varphi_b, \theta_{\nu,N} \to \theta_{\nu,N} + \tfrac{\pi}{2}), \end{aligned}\right\} \tag{20}$$

$$M_D = c_{\varphi_a}s_{\varphi_b}c_{\theta_\nu}s_{\theta_\nu}(m_1 - m_3) + s_{\varphi_a}c_{\varphi_b}c_{\theta_N}s_{\theta_N}(m_4 - m_2), \tag{21}$$

$$f_{ab}v_R = s_{\varphi_a}s_{\varphi_b}c_{\theta_\nu}s_{\theta_\nu}(m_3 - m_1) + c_{\varphi_a}c_{\varphi_b}c_{\theta_N}s_{\theta_N}(m_4 - m_2), \tag{22}$$

$$\left.\begin{aligned} f_{aa}v_R &= (s_{\varphi_a}c_{\theta_\nu})^2m_1 + (c_{\varphi_a}c_{\theta_N})^2m_2 + (s_{\varphi_a}s_{\theta_\nu})^2m_3 + (c_{\varphi_a}s_{\theta_N})^2m_4, \\ f_{bb}v_R &= f_{aa}v_R(\varphi_a \to \varphi_b + \tfrac{\pi}{2}, \theta_N \to \theta_N + \tfrac{\pi}{2}), \end{aligned}\right\} \tag{23}$$

$$f_{ll'}v_L = f_{ll'}v_R(\varphi_{l,l'} \to \varphi_{l,l'} + \frac{\pi}{2}), \qquad l, l' = a, b, \tag{24}$$

where φ_a is the mixing angle in the a generation between the light and the heavy neutrino entering into the left-handed and the right-handed lepton doublet, $\theta_\nu(\theta_N)$ is the mixing angle between the ν_{aL} and the ν_{bL} neutrino (N_{aR} and N_{bR}), $c_{\varphi_a} = \cos\varphi_a$, $s_{\varphi_a} = \sin\varphi_a$ and so on.

3. The anomalous muon magnetic moment

Let us consider the contribution to the muon AMM coming from the doubly charged Higgs-boson $\Delta_2^{(--)}$. In the third order of the perturbation theory the corresponding diagrams are represented in Fig.1.

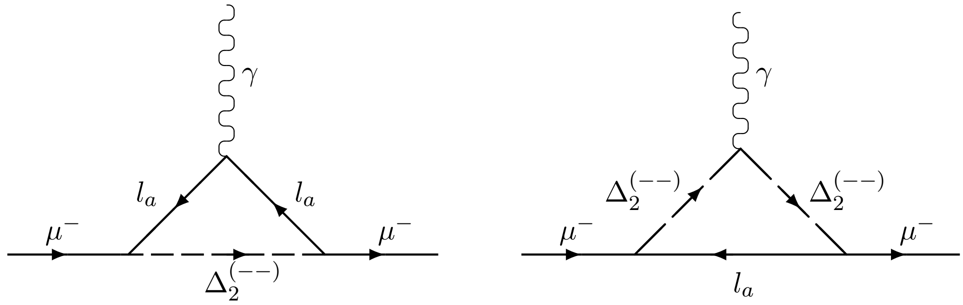

Figure 1: One-loop diagrams contributing to the muon AMM caused by the doubly charged Higgs bosons $\Delta_2^{(--)}$. The wavy line represent the electromagnetic field.

They give the following contribution to the AMM value

$$\Delta a_\mu = \frac{1}{8\pi^2}\left(4f_{\mu e}^2 I_e^{\Delta_2} + f_{\mu\mu}^2 I_\mu^{\Delta_i} + 4f_{\mu\tau}^2 I_\tau^{\Delta_2}\right), \tag{25}$$

where

$$I_{l_a}^{\Delta_2} = \int_0^1 \left(\frac{2m_\mu^2(z^3-z^2)}{m_\mu^2(z^2-z)+m_{\Delta_2}^2 z + m_{l_a}^2(1-z)} + \frac{m_\mu^2(z^2-z^3)}{m_\mu^2(z^2-z)+m_{\Delta_i}^2(1-z)+m_{l_a}^2 z}\right) dz. \tag{26}$$

To perform an exhaustive analysis of the obtained result one should have information about the parameters both the Higgs and the neutrino sectors. Nowadays the information concerning the heavy neutrino sector is very poor. All we have is the upper bound for the heavy electron neutrino mass resulting from the experiments aimed at finding the neutrinoless double β decay

$$m_{N_e} > 63 \text{ GeV}\left(\frac{1.6 \text{ TeV}}{m_{W_2}}\right)^4. \tag{35}$$

Hence, our sole way out in an existing situation is to choose any minimal number of varied parameters and other parameters to express through them. As those we shall take m_{Δ_2} and $f_{\mu\mu}$.

First we express nondiagonal triplet Yukawa coupling constants in terms of $f_{\mu\mu}$. In so doing we shall be constrained by the two flavor approximation. Then from Eqs. (22) and (23) it follows

$$f_{\mu e} \simeq \frac{c_{\varphi_b} s_{2\theta_N}(m_4 - m_2)}{2(c_{\theta_N}^2 m_2 + s_{\theta_N}^2 m_4)} f_{\mu\mu}, \tag{36}$$

where we have neglected the light neutrino masses. Further, for the sake of simplicity, we shall set $f_{\mu e} = f_{\mu\tau}$. Assuming

$$\sin^2\theta_{23} \simeq 0.425, \qquad \sin^2\theta_{12} \simeq 0.297.$$

$$\varphi_a = 33^0, \qquad \theta_N = 45^0, \qquad m_{N_1} = 100 \text{ GeV}, \qquad m_{N_2} = 150 \text{ GeV},$$

we obtain

$$f_{\mu e} = 0.16 f_{\mu\mu}.$$

From Eq.(36) we see that the values of the nondiagonal coupling constants depend predominantly of the heavy neutrino mass difference. For example, setting

$$m_{N_1} = 100 \text{ GeV}, \qquad m_{N_2} = 1000 \text{ GeV},$$

we get $f_{\mu\tau} = 0.69 f_{\mu\mu}$.

In what follows we shall be constrained by viewing of two cases: (i) $f_{\mu\tau} = f_{\mu e} = 0.16 f_{\mu\mu}$; (ii) $f_{\mu\tau} = f_{\mu e} = 0.69 f_{\mu\mu}$. For the former case in the m_{Δ_2} *vs.* $f_{\mu\mu}$ parameter space two contour lines marked 135.36 and 366.38 are shown in Fig.2. In the latter case the function $f_{\mu\mu}(m_{\Delta_2})$ is represented in Fig.3. The range of the LRM parameters allowed by the experiments lies between these contours. So we see that the interval of the $\Delta_2^{(--)}$-boson mass at which the satisfaction to the experimental results is possible, also depends on the values of the non diagonal YCC's.

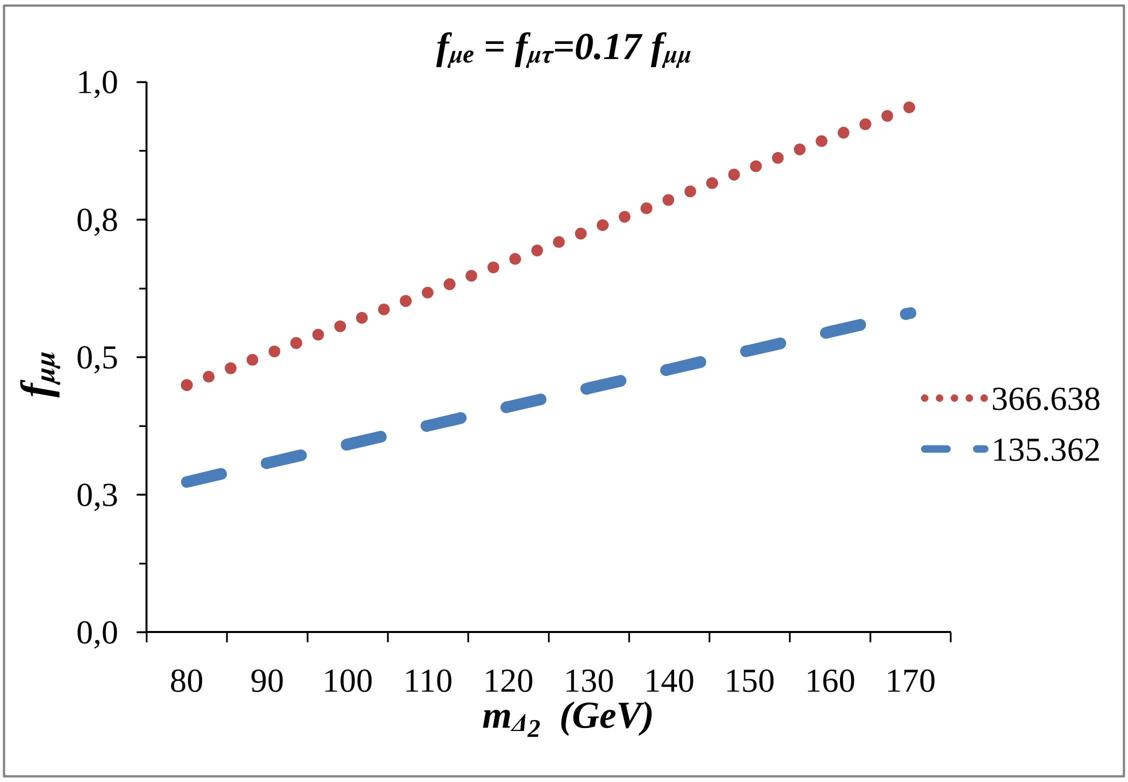

Figure 2: Contours of the one-loop contribution from the $\Delta_2^{(--)}$-boson to the muon AMM at $f_{\mu\tau} = f_{\mu e} = 0.16 f_{\mu\mu}$

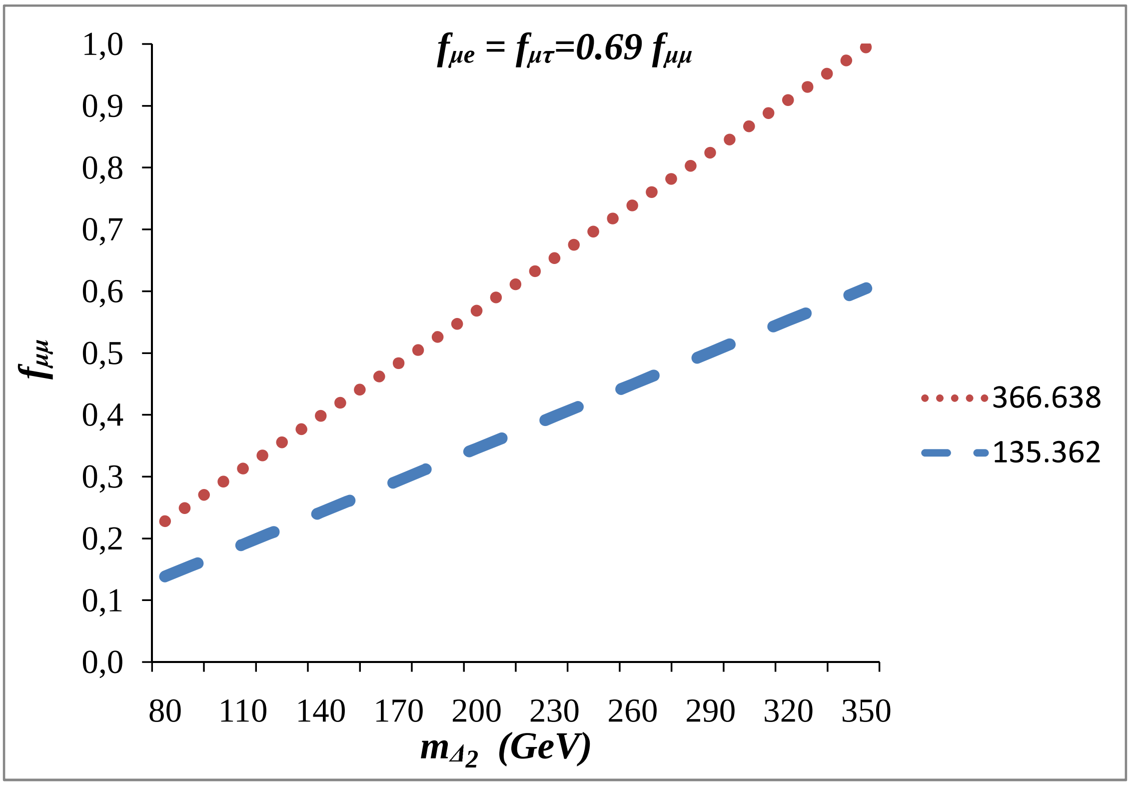

Figure 3: Contours of the one-loop contribution from the $\Delta_2^{(--)}$-boson to the muon AMM at $f_{\mu\tau} = f_{\mu e} = 0.69 f_{\mu\mu}$

4. Conclusions

Within the left-right symmetric model the contribution to the muon anomalous magnetic moment coming from the doubly charged Higgs boson has been obtained. We have shown that this contribution could be represented as the function of the Higgs boson masses and the triplet Yukawa coupling constants (YCC's). Using the connection between parameters of the neutrino and the Higgs sectors the nondiagonal YCC's have been expressed through the neutrino oscillation parameters. By this it turned out that the values of these YCC's are practically not sensitive to the masses and the mixing angles in the light neutrino sector and are mainly defined by the values of the heavy neutrino masses and by the mixing angles between the light and heavy neutrinos. It was demonstrated that at the definite parameter values the model under consideration could explain the results obtained in the E989 experiment at BNL. We emphasize that investigation of the reactions

$$\mu^-\mu^- \to \mu^-\mu^-, \mu^-\tau^-, \mu^-e^-,$$

which may be observed at the muon colliders allows to determine the values both of the triplet YCC's $f_{\mu\mu}$, $f_{\mu\tau}$, $f_{\mu e}$ and of the $\Delta_2^{(--)}$-boson mass. All these reactions go through the s-channels with the exchanges of the $\Delta_2^{(--)}$-bosons. Therefore, the cross section has the resonance peak related to the Higgs boson.

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