ON THE DUAL STABILIZATION OF THE MULTIDIMENSIONAL REGRESSION OBJECT AT A GIVEN LEVEL

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Abstract

The statement of the problem of the dual control of the regression object with multidimensional-matrix input and output variables and dynamic programming functional equations for its solution are given. The problem of the dual stabilization of the regression object at the given level is considered. In order to solve the problem, the regression function of the object is supposed to be affine in input variables, and the inner noise is supposed to be Gaussian. The optimal control action at the last control step is obtained and is proposed to be used at the arbitrary control step. The obtained control algorithm was programmed for numerical calculations and tested for a number of objects.

1 Introduction

The problem of the dual control of the multidimensional regression object is formulated as follows [1, 2, 3]. The control system with controlled object O, controller C, feedback path and driving action g_s is considered (Figure 1). The controlled object O is described at the instant of time s by the probability density function $f_{Y_s}(y_s, \Theta, U_s)$, $s = 0, 1, 2, ..., n$, where $Y_s = (Y_{i_1,i_2,...,i_p,s})$ is the p-dimensional matrix of the output of the object, $U_s = (U_{i_1,i_2,...,i_q,s})$ is the q-dimensional matrix of the input of the object (control action), $\Theta = \{\Theta_1, \Theta_2, ..., \Theta_m\}$ is a set of the parameters of the controlled object consisting of the random multidimensional matrices $\Theta_1, \Theta_2, ..., \Theta_m$ with known priory joint probability density function $f_{\Theta,0}(\theta)$. We will call the set Θ a generalized parameter of the object . It is supposed, that the generalized parameter takes constant value for all of the instants of time $s = 0, 1, 2, ..., n$. The driving action g_s is supposed to be known deterministic multidimensional-matrix sequence. The quality of the functioning of the system at each instant of time s is estimated by a specific loss function $W_s(Y_s, g_s)$. A system, for which the total for $n + 1$ instants of time total average risk

$$R = E(\sum_{s=0}^{n} W_s(Y_s, g_s)) = \sum_{s=0}^{n} R_s, \quad R_s = E\left(W_s(Y_s, g_s)\right), \tag{1}$$

is minimal, is called optimal system. The control action U_s belongs to some permissible area $\bar{U}$. The controller C uses all of the past information in the form of observations $\vec{u}_{s-1} = (u_0, u_1, ..., u_{s-1})$, $\vec{y}_{s-1} = (y_0, y_1, ..., y_{s-1})$ to determine the control action u_s at the instant of time s. The task consists of determining the sequence of the conditional

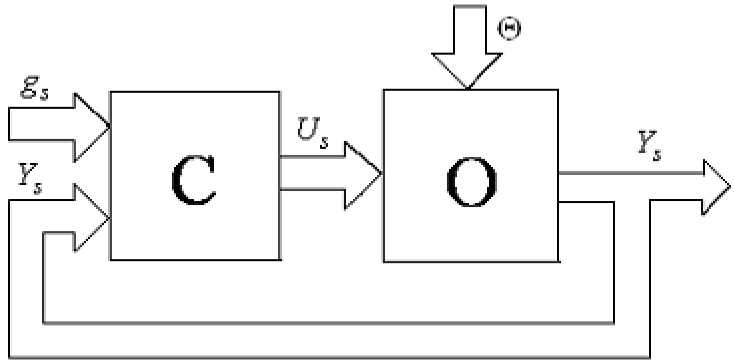

Figure 1: To the statement of the dual control problem

probability density functions $f_{U_s}(u_s/\vec{u}_{s-1}, \vec{y}_{s-1})$, $i = 0, 1, 2, ..., n$, for which the total average risk R (1) is minimal.

As it is known [2, 3], the optimal control actions U_s are not random and will be denoted u_s. In this conditions the controller C will be described by conditional probability density function $f_{Y_s}(y_s, \theta, u_s)$. We will use the following simplified notation: $f_{\Theta,0}(\theta) = f_0(\theta)$, $f_{Y_s}(y_s, \theta, u_s) = f(y_s, \theta, u_s)$.

The optimal control algorithm $u_n, u_{n-1}, ..., u_0$, is determined in pointed inverse order from the following functional equations:

$$f_n^*(\vec{u}_{n-1}, u_n^*, \vec{y}_{n-1}) = \min_{u_n \in \bar{U}} \phi_n(\vec{u}_n, \vec{y}_{n-1}), \tag{2}$$

$$f_{n-m}^*(\vec{u}_{n-m-1}, u_{n-m}^*, \vec{y}_{n-m-1}) = \min_{u_{n-m} \in \bar{U}} [\phi_{n-m}(\vec{u}_{n-m}, \vec{y}_{n-m-1}) +$$

$$+ \int_{\Omega(y_{n-m})} f_{n-m+1}^*(\vec{u}_{n-m}, u_{n-m+1}^*, \vec{y}_{n-m}) f(y_{n-m}/\vec{u}_{n-m}, \vec{y}_{n-m-1}) d\Omega],\ m = 1, 2, ..., n, \tag{3}$$

where ϕ_s is determined by expression

$$\phi_s(\vec{u}_s, \vec{y}_{s-1}) = \int_{\Omega(y_s)} W_s(y_s, g_s) f(y_s/\vec{u}_s, \vec{y}_{s-1}) d\Omega,\ s = 0, 1, 2, ..., n, \tag{4}$$

in which

$$f(y_s/\vec{u}_s, \vec{y}_{s-1}) = \int_{\Omega(\theta)} f(y_s/\theta, u_s) f_s(\theta) d\Omega, \tag{5}$$

$$f_s(\theta) = \frac{f_0(\theta) \prod_{v=0}^{s-1} f(y_v/\theta, u_v)}{\int\limits_{\Omega(\theta)} f_0(\theta) \prod_{v=0}^{s-1} f(y_v/\theta, u_v) d\Omega}, \tag{6}$$

and u_{n-m+1}^* is optimal control action for the instant of time $(n - m + 1)$.

2 Stabilization of the object at the given level

We will consider the task of reaching the required value of the regression function by sequential control actions in production operation mode and stabilization it at this level. The task is formulated in this case as follows. The controlled object is described at the s-th instant of time by Gaussian probability density function

$$f_{Y_s}(y_s/C, u_s) = N(\psi(C, u_s), d_Y),$$

where $\psi(C, u_s)$ is a regression function, d_Y is a variance-covariance matrix of the inner noise, C is a generalized parameter of the object. Note, that we denote now the generalized parameter C instead of Θ in expressions (5), (6). Let us approximate the regression function by affine function:

$$\psi(C, u_s) = C_0 +^{0,q} (C_1 u_s) = C_{t,0} +^{0,q} (u_s C_{t,1}) = \psi(C_t), \tag{7}$$

where C_k, $k = 0, 1$, are kq-dimensional random matrices, $C_{t,k} = (C_k)^{B_{p+kq,kq}}$, $C_k = (C_{t,k})^{H_{p+kq,kq}}$, and $H_{p+kq,kq}$, $B_{p+kq,kq}$ are the transpose substitutions of the type "back" and "onward" respectively [4]. Let us combine the matrices C_k into a one-dimensional cell $C = \{C_k\}$, $k = 0, 1$. For the task of the object stabilization at the level g we choose the loss function in the form of $W(Y_s) = ||Y_s - g||^2$, where $|| \cdot ||$ is the Euclidean norm of a multidimensional matrix.

Let the random cell $C_t = \{C_{t,k}\}$, $k = 0, 1, ..., m$, $(m = 1)$ has the Gaussian priory probability density function described by the following expression [5]:

$$f(c_t) = M_C exp\left(-\frac{1}{2}\sum_{i=0}^{m}\sum_{j=0}^{m} {}^{0,q_j}\left({}^{0,q_i}\left((c_{t,i} - \nu_{c_t,i})d_{c_t}^{i,j}\right)(c_{t,j} - \nu_{c_t,j})\right)\right).$$

The calculation of the control actions $u_n, u_{n-1}, , ..., u_0$, is connected with the formulae (2)–(6). The posterior probability density function $f_n(c)$ (6) and the probability density function $f(y_n/\vec{u}_n, \vec{y}_{s-1})$ (5) are defined with help of the results of the article [5]. It allowed us to calculate the function $\phi_n(\vec{u}_n, \vec{y}_{n-1})$ (4) provided the loss function $W(Y_n, g) = ||Y_n - g||^2$ and minimize it in accordance with the expression (2). As a result, we received the optimal control action at the last n-th instant of time. The calculation of the optimal control actions for instants of time $n - 1, n - 2, ..., 0$ in accordance with the expression (3) is connected with irresistible both analytical and numerical difficulties. However, the control action obtained at the last instant of time can be used at any instant of time. We will call this control action as the algorithm of the optimal dual control with passive information storage.

3 Computer simulation

The algorithm of the optimal dual control with passive information storage was realized programmatically, utilized at a number of objects and showed results acceptable for practice. For instant, the regression object with vector input and output variables $(p = q = 1)$ and affine regression function (7) was simulated with following parameters:

$$c_0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad c_1 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad d_Y = \begin{pmatrix} 0.001 & 0 \\ 0 & 0.001 \end{pmatrix}. \tag{8}$$

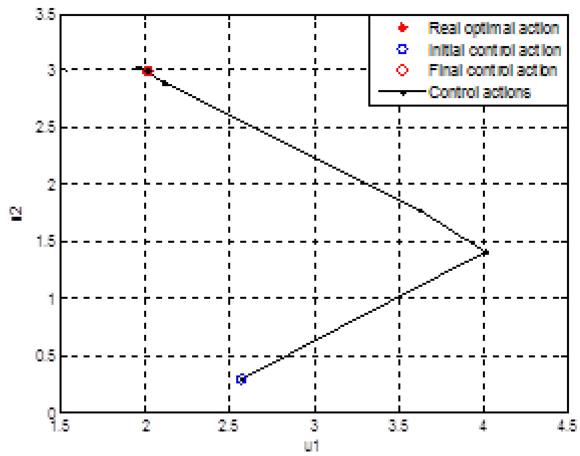

Figure 2: The points of the dual control actions

The prior characteristics of the coefficients of the approximating polynomial (7) and initial control action u_0 are simulated as random.

The sequence of the control actions is showed in the Figure 2 for some variant of the simulation. The figure illustrates the stabilization of the regression function at the level $g = (9 \quad 20)$. As it follows from the object description (8), the regression function has the value $y = g = (9 \quad 20)$ provided control action $u = (2 \quad 3)$. One can sees in the figure that this value of the control action is reached by 5 iterations.

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