Integration of Fuzzy Systems with Parametric Interpretation for Unified Knowledge Representation

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***Abstract*—The paper considers the problem of stable interpretation of fuzzy logic models. An approach based on parameterized fuzzy logic is proposed, where each logical formula has a set of model parameters in addition to truth values. Parameterized fuzzy logic allows combining different fuzzy logic systems. Model parameters are used to calculate fuzzy truth values as a fuzzy measure. Models and model parameters related to metric spaces, consistent with metric sense spaces and being the basis for interpretation of fuzzy logic formulas on ontological models are considered.**

***Keywords*—Fuzzy logic, Parameterized fuzzy logic, Metric space, Fuzzy measure, Simplicial complex, Residual simplicial complex, Canonical form, Semantic metric, Semantic Space, Integration, Knowledge representation model, Knowledge processing model, Ontology, Unified representation of knowledge, Linear vector space, Parametric t-norm classes, Hilbert cube, Finite structure, Substructural logic**

I. Introduction

Approaches to integration of logical models in a general form are considered in [7].

One of the problems of integration of logical models of knowledge representation and processing [11], [17], [20] is to identify compatible models that provide construction of interpretations of corresponding logical formalisms [6]. If necessary, these models can be considered as part of the corresponding semantic space [7], [8], [19].

One of the broad classes of logical models is fuzzy logics [3]. There is a problem of unreliability of fuzzy logics due to uncertainties [18] existing at different stages of their application [15]. One of the stages is selection of a fuzzy logic model or system with the purpose of application for realization of reasoning and problem solving. It is not always clear how suitable the chosen fuzzy system is. This is due to the fact that interpretations (which are built in the process of fuzzy logical inference) connect logical constructions with abstract algebraic systems that have no definite connection with any subject area or its model. Moreover, for each fuzzy system a different algebraic system is considered, the connection of which with other algebraic systems also remains undefined. This high degree of uncertainty does not allow reliable use of fuzzy logic models which is one of the problems of fuzzy logics [15].

The choice of fuzzy logics is also conditioned by their rich internal and external diversity, which allows fuzzy logics to represent other non-classical logical [3], [9], [10] models by means of fuzzy logics. The diversity of fuzzy logical models leads, among other things, to the diversity of fuzzy logical operations (for example, such as triangular norms and conorms), the emergence of their classes and their parameterization within the corresponding subclasses.

In development of the idea of parameterization of logical operations, the concept of parameterized fuzzy logics [6] is proposed.

The two main parametric families of triangular norms (and corresponding conorms) [14] are: the Frank parametric family [12], [13] and the Schweitzer-Sklar parametric family.

When constructing interpretations of parametrized fuzzy logics we can distinguish their following types: interpretations on algebraic systems, interpretations on "amorphous" models, interpretations on concrete structural-static models.

Interpretations on algebraic systems are largely similar to traditional fuzzy systems, the general scheme of which is given in [6], and therefore we will not consider them in detail in this paper. Further we will consider examples of interpretations of formulas of parameterized fuzzy logics on "amorphous" models and on concrete structural-static models.

II. Interpretation and models of fuzzy logics

As an "amorphous" model, consider a model in which each fuzzy predicate is matched with a vector quantity (vector) A, which can be given by some unit (or zero) vector 1_A, specifying the direction of this quantity, in some linear basis of some vector space and a scalar $||A||$ in the range from 0 to 1, specifying the length of vector A. If and only if the length is 0 or the direction is given by a zero vector, then the vector quantity is equal to a zero vector, its length is 0, but its direction can be a non-zero vector.

The fuzzy negation operation in this model reverses the direction of the vector according to the expression:

$$-1_A$$

and its length according to the expression:

$$1 - ||A||$$

The next operation we will consider is the fuzzy conjunction. It should be noted that the fuzzy conjunction does not fulfill all the properties characteristic, for example, for triangular norms since this conjunction is parameterized, covering more than one triangular norm. A parameterized conjunction can naturally cover several triangular norms in one expression, so the properties of one triangular norm cannot be extended to such a conjunction.

To consider the result of the computation of such a fuzzy conjunction, let us consider 23 cases (variants, see Table I) of the spatial relation of vectors of a pair of its arguments ($A = 1_A * ||A||$ and $B = 1_B * ||B||$), which we will later reduce to a smaller number of cases.

Table I
Variants of relations of parameters of "amorphous" parameterized fuzzy logic

№	$\|\|A\|\| * \|\|B\|\|$	$cos(\langle A, B \rangle)$
0	0	$[-1; 1]$
1	0	$[-1; 1]$
2	$(0; 1]$	1
3	$(0; 1]$	$(0; 1)$
4	$(0; 1]$	$(0; 1)$
5	$(0; 1]$	$(0; 1)$
6	$(0; 1]$	0
7	$(0; 1]$	$(-1; 1)$
8	$(0; 1]$	$(-1; 1)$
9	$(0; 1]$	$(-1; 1)$
10	$(0; 1]$	$(-1; 1)$
11	$(0; 1]$	$(-1; 1)$
12	$(0; 1]$	$(-1; 1)$
13	$(0; 1]$	$(-1; 1)$
14	$(0; 1]$	$(-1; 1)$
15	$(0; 1]$	$(-1; 1)$
16	$(0; 1]$	$(-1; 1)$
17	$(0; 1]$	$(-1; 1)$
18	$(0; 1]$	$(-1; 1)$
19	$(0; 1]$	$(-1; 1)$
20	$(0; 1]$	$(-1; 1)$
21	$(0; 1]$	$(-1; 1)$
22	$(0; 1]$	$(-1; 1)$
23	$(0; 1]$	-1

Due to symmetry (commutativity of the fuzzy conjunction operation), the number of these cases (variants) can be reduced to 16 which in turn are reduced to 10 (see Table II) as a result of decomposition. The result of the initial variant (case) is the arithmetic mean of the variants (cases) into which it is decomposed.

Table II
Decomposition of variants of parameter relations of "amorphous" parameterized fuzzy logic

Symmetry of variants		**Decomposition into variants**	
1	1	0	
2	2	5	
3	5	6	
4	4	7	
6	6	8	
7	7	1	1
8	11	1	2
9	15	1	3
10	19	1	4
12	12	2	2
13	16	2	3
14	20	2	4
17	17	3	3
18	21	3	4
22	22	4	4
23	23	9	

Hypothetically, variants 13 and 16 are impossible.

Let us consider these variants sequentially.

As a result of the operation of fuzzy conjunction of two arguments ($A = 1_A * ||A||$ and $B = 1_B * ||B||$), we must obtain a vector quantity given by two parameters: a vector and a scalar.

For the 0th variant, the vector coincides with the vector of a non-zero argument or is calculated by the formula:

$$(1_A + 1_B) / \sqrt{(1_A + 1_B) * (1_A + 1_B)}$$

For all variants except for the 0-th (23rd) we will calculate the vector by the formula:

$$(1_A + 1_B) / \sqrt{(1_A + 1_B) * (1_A + 1_B)}$$

The proposed formula has the advantage of a more convenient model for modeling traditional fuzzy logics but alternative expressions for calculating the vector are possible:

$$(A + B) / \sqrt{(A + B) * (A + B)}$$

$$\frac{(A + B + 1_A + 1_B)}{\sqrt{(A + B + 1_A + 1_B) * (A + B + 1_A + 1_B)}}$$

and others.

For two nonzero noncollinear vectors A and B, the vector $H_{AB} = A * u + B * v$ from their common origin to the intersection of the perpendiculars to their ends can be expressed:

$$H_{AB} = \frac{A * (A * (B - A)) * B^2 + B * ((A - B) * B) * A^2}{(A * B)^2 - A^2 * B^2}$$

from

$$(A * u + B * v - B) * B = (A * u + B * v - A) * A = 0$$

$$\begin{cases} u * (A * B) = (1 - v) * B * B \\ v * (A * B) = (1 - u) * A * A \end{cases}$$

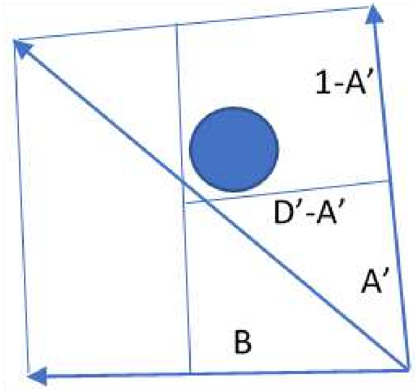

Figure 1. Variant 1 illustration.

$$\begin{cases} u = (A*(B-A))*B^2/\left((A*B)^2 - A^2*B^2\right) \\ v = ((A-B)*B)*A^2/\left((A*B)^2 - B^2*A^2\right) \end{cases}$$

Let the following:

$$D = H_{AB}$$

$$D' = H_{A'B}$$

$$1_{A'} = -1_A$$

$$1_D = H_{1_A 1_B}$$

$$1_{D'} = H_{1_{A'} 1_B}$$

Variant 1: The angle between vectors A (vector $A' = A - 1_A$) and B is obtuse ($A*B<0$), the perpendicular to vector A' intersects the perpendicular to B before B $||B|| \leq ||A'||/cos(\langle A', B\rangle)$ (see Fig.1) then the length of the result is equal to the ratio of areas:

$$\frac{(2*||D'-A'|| - ||1_{A'} - A'||)*ctg(\langle A', B\rangle)*||1_{A'} - A'||/2}{(1_{A'}*(1_{D'} - 1_{A'}) + 1_B*(1_{D'} - 1_B))/2}$$

Variant 2. The angle between vectors A (vector $A' = A - 1_A$) and B is obtuse ($A*B<0$), the perpendicular to vector A' intersects B before the perpendicular to it $||B|| > ||A'||/cos(\langle A', B\rangle))$ (see Fig.2) then the length of the result is equal to the ratio of areas:

$$\frac{\frac{\left(||1_{A'}||^2 - ||A'||^2\right)}{ctg(\langle A', B\rangle)} - \left(\frac{||1_{A'}||}{cos(\langle A', B\rangle)} - ||B||\right)^2 * ctg(\langle A', B\rangle)}{2*(1_{A'}*(1_{D'} - 1_{A'}) + 1_B*(1_{D'} - 1_B))/2}$$

Variant 3. The angle between vectors A (vector $A' = A - 1_A$) and B is obtuse ($A*B<0$), the perpendicular to vector A' intersects the perpendicular to B before B $||B|| \leq ||A'||/cos(\langle A', B\rangle)$ (see Fig.3) then the length of the result is equal to the ratio of areas:

$$\frac{||D'-A'||*||D'-A'||*tg(\langle A', B\rangle)/2}{1_{A'}*(1_{D'} - 1_{A'}) + 1_B*(1_{D'} - 1_B)/2}$$

Variant 4. The angle between vectors A (vector $A' = A - 1_A$) and B is obtuse ($A*B<0$), the perpendicular to vector A' intersects B before the perpendicular to it $||B|| > ||A'||/cos(\langle A', B\rangle))$ (see Fig.4) then the length of the result is equal to the ratio of areas:

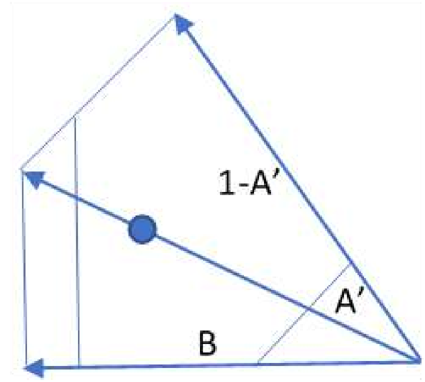

Figure 2. Variant 2 illustration.

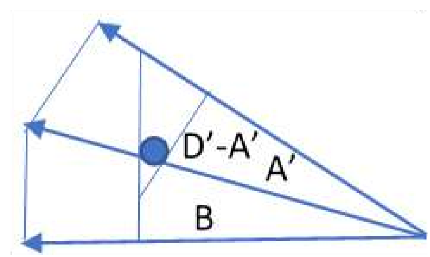

Figure 3. Variant 3 illustration.

$$\frac{||B||*||B|| - ||A'||*||A'||*tg(\langle A', B\rangle)/2}{1_{A'}*(1_{D'} - 1_{A'}) + 1_B*(1_{D'} - 1_B)/2}$$

Variant 5. Vectors A and B are co-oriented then the length of the result is: $min(\{||A||\} \cup \{||B||\})$.

Variant 6. The angle between vectors A and B is acute ($A*B>0$), the perpendicular to vector A intersects B before the perpendicular to it $||B|| > ||A||/cos(\langle A, B\rangle)$ (see Fig.5) then the length of the result is equal to the

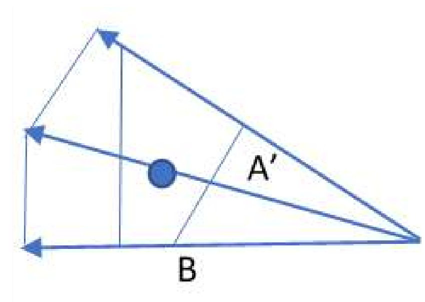

Figure 4. Variant 4 illustration.

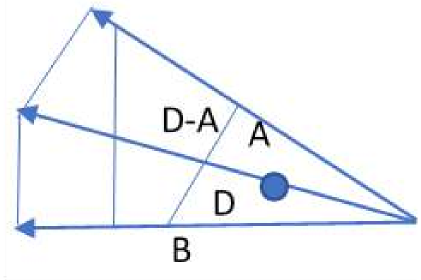

Figure 5. Variant 6 illustration.

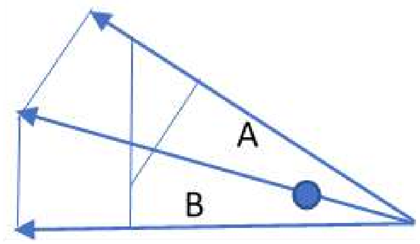

Figure 6. Variant 7 illustration.

ratio of areas:

$$\frac{A*(D-A)/2}{(1_A*(1_D-1_A)+1_B*(1_D-1_B))/2}$$

Variant 7. The angle between vectors A and B is acute ($A*B>0$), the perpendicular to vector A intersects the perpendicular to B before B $||B|| \le ||A||/cos(\langle A,B\rangle)$ (see Fig.6) then the length of the result is equal to the ratio of areas:

$$\frac{((A+B)*D-A*A-B*B)/2}{(1_A*(1_D-1_A)+1_B*(1_D-1_B))/2}$$

Variant 8. Vectors A and B are orthogonal $((A*B=0)\wedge(||A||+||B||>0))$ (see Fig.7) then the length of the result is equal to the ratio of areas: $||A||*||B||/1$.

Variant 9. Vectors A and B are differently directed, then the length of the result is equal to:

$$max(\{0\}\cup\{A+B-1\})$$

Properties of negation:

$$A = \sim(\sim A)$$

$$0 = \sim 1$$

$$1 = \sim 0$$

Properties of conjunction:

- zero element
$$A\widetilde{\wedge}0 = 0$$

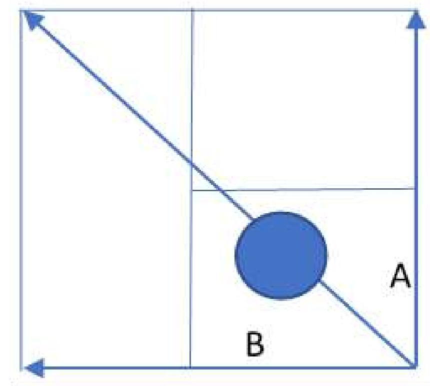

Figure 7. Variant 8 illustration.

- neutral element
$$A\widetilde{\wedge}1 = A$$
- idempotency
$$A\widetilde{\wedge}A = A$$
- commutativity
$$A\widetilde{\wedge}B = B\widetilde{\wedge}A$$
- non-associativity
$$\neg\left(A\widetilde{\wedge}\left(B\widetilde{\wedge}C\right) = \left(A\widetilde{\wedge}B\right)\widetilde{\wedge}C\right)$$
- non-monotonicity
$$\neg\left(A \le B \to A\widetilde{\wedge}C \le B\widetilde{\wedge}C\right)$$
- monotonicity in direction
$$A\overleftarrow{\le}B \to A\widetilde{\wedge}C\ \overleftarrow{\le}B\widetilde{\wedge}C$$

Properties of disjunction:

$$A\widetilde{\vee}B = \sim\left((\sim A)\,\widetilde{\wedge}\,(\sim B)\right)$$

Properties of implication:

$$A \sim> B = \left((\sim A)\,\widetilde{\vee}B\right)$$

Properties of a fuzzy measure [16]:

$$A\widetilde{\wedge}B \le A$$

$$A \le A\widetilde{\vee}B$$

As concrete structural-static models we can consider finite models or simplicial complexes [1], and also we can consider their generalizations, for example, as a residual simplicial complex. Let us consider variants with simplicial complexes. An important question of such consideration is the canonical form of the corresponding simplicial complex.

For simplicial complexes and the corresponding sets defined by them, the (generalized) operations of union $\hat{\cup}$ and intersection $\hat{\cap}$ are naturally defined.

Each argument of the parameterized fuzzy expression can be matched with a residual simplicial complex as one of the parameters. We will also use the notion of residual simplicial complex to represent the results of parameterized fuzzy logics.

The residual simplicial complex can be given by $2*n$-simplicial complexes $\langle C_1, C_2, ..., C_{2*n}\rangle$ through an expression of the form:

$$C_1/(C_2/(\ldots/C_{2*n}))$$

For a residual simplicial complex the following is true:

$$C_{i+1} \subset C_i$$

$$C_{i+2} \subseteq \partial C_i$$

$$(X \in C_i \cap C_{i+1}) \to \exists Y\, (Y \in C_i/C_{i+1}) \wedge (\emptyset \subset Y \cap X)$$

$$\emptyset \subset C_{2*n}$$

$$\partial C = \bigcup_{X \in C} 2^X / \{X\}$$

The height of the residual complex is $2*n$.

We will consider simplicial complexes covering points of subsets of the set of points of the space spanned by the universal simplicial complex U whose residual simplicial complex is $\langle U, \emptyset, ..., \emptyset\rangle$.

The complement of $U\hat{/}C$ of the residual simplicial complex $\langle C_1, C_2, ..., C_{2*n}\rangle$ will be the height complex $2*m(m \le n)$:

$$D_1/(D_2/(\ldots/D_{2*m}))$$

with such smallest T_1, T_k $(2 \le k \le n)$:

$$U/\left(\bigcup_{i=1}^{n} C_{2*i-1}/C_{2*i}\right) \subseteq T_1 \subseteq U$$

$$C_{k-1}/\left(\bigcup_{i=1}^{n-k+1} C_{2*i-1+k}/C_{2*i+k}\right) \subseteq T_k \subseteq C_{k-1}$$

$$D_1/\left(\bigcup_{i=1}^{m} D_{2*i}/D_{2*i+1}\right) = T_1/\left(\bigcup_{i=1}^{n} T_{2*i}/T_{2*i+1}\right)$$

$$((m < i) \wedge (i \le n)) \to (T_i = \emptyset)$$

The intersection $O = I\hat{\cap}E$ of the two residual simplicial complexes I and E $(n \le m)$ is the height complex $2*l(l \le n*(2*m-n+1)$:

$$O_1/(O_2/(\ldots/O_{2*l}))$$

The tiers of the residual simplicial complex are filled in according to the tables Table III ($E_0 = E_{10}$), Table IV) in accordance with the order:

The difference $O = I\hat{/}E$ of the two residual simplicial complexes I and E $(n \le m)$ is the height complex $2*l(l \le n*(2*m-n+1)$:

$$I\hat{/}E = I\hat{\cap}\left(U\hat{/}E\right)$$

Table III
Computable operations for calculating the intersection of two residual simplicial complexes

$I_1 \cap E_1$	$I_2 \cup E_2$	$I_3 \cap E_1$	$I_4 \cup E_4$	$I_5 \cap E_1$	$I_6 \cup E_0$
$I_2 \cup E_2$	$I_2 \cap E_2$	$I_2 \cap E_2$	$I_4 \cup E_4$	$I_5 \cap E_2$	$I_6 \cup E_0$
$I_1 \cap E_3$	$I_2 \cap E_2$	$I_3 \cap E_3$	$I_4 \cup E_4$	$I_5 \cap E_3$	$I_6 \cup E_0$
$I_4 \cup E_4$	$I_4 \cup E_4$	$I_4 \cup E_4$	$I_4 \cap E_4$	$I_4 \cap E_4$	$I_6 \cup E_0$
$I_1 \cap E_5$	$I_2 \cap E_5$	$I_3 \cap E_5$	$I_4 \cap E_4$	$I_5 \cap E_5$	$I_6 \cup E_0$
$I_6 \cup E_6$	$I_6 \cup E_6$	$I_6 \cup E_6$	$I_6 \cup E_6$	$I_6 \cup E_6$	$I_6 \cup E_0$
$I_1 \cap E_7$	$I_2 \cap E_7$	$I_3 \cap E_7$	$I_4 \cap E_7$	$I_5 \cap E_7$	$I_6 \cup E_0$
$I_6 \cup E_8$	$I_6 \cup E_8$	$I_6 \cup E_8$	$I_6 \cup E_8$	$I_6 \cup E_8$	$I_6 \cup E_0$
$I_1 \cap E_9$	$I_2 \cap E_9$	$I_3 \cap E_9$	$I_4 \cap E_9$	$I_5 \cap E_9$	$I_6 \cup E_0$
$I_6 \cup E_0$	$I_6 \cup E_0$	$I_6 \cup E_0$	$I_6 \cup E_0$	$I_6 \cup E_0$	$I_6 \cup E_0$

Table IV
Sequence (transposed) of computable operations to compute the intersection of two residual simplicial complexes

1	2	3	6	7	12	13	18	19	24
2	4	4	6	8	12	14	18	20	24
3	4	5	6	9	12	15	18	21	24
6	6	6	10	10	12	16	18	22	24
7	8	9	10	11	12	17	18	23	24
24	24	24	24	24	24	24	24	24	24

The residuum $O = I\hat{\to}E$ of the two residual simplicial complexes I and E $(n \le m)$ will be the height complex $2*l$ $(l \le n*(2*m-n+1))$:

$$I\hat{\to}E = U\hat{/}\left(I\hat{/}E\right)$$

The union of $O = I\hat{\cup}E$ of the two residual simplicial complexes I and E $(n \le m)$ is the height complex $2*l$ $(l \le n*(2*m-n+1))$:

$$I\hat{\cup}E = U\hat{/}\left(\left(U\hat{/}I\right)\hat{/}E\right)$$

The value of a fuzzy expression (predicate) in parametric fuzzy logic can be calculated as the length $hvol_1\,(C)$, area $hvol_2\,(C)$, volume $hvol_3\,(C)$ or hypervolume $hvol_{dim(C)}\,(C)$ of a simplicial complex. For each simplicial complex with a basis in linear vector space, a minimal covering simplex can be given, and its dimension $dim\,(C)$, equal to the dimension of the maximal simplex in the complex, can also be computed. If the space is n-dimensional, the value of the fuzzy expression can be computed:

$$1 + 2^{-dim(C)} * \left(hvol_{dim(C)}\,(C) - 2\right)$$

For the corresponding fuzzy operations, the properties of the fuzzy measure will also be fulfilled:

$$A\widetilde{\wedge}B \le A$$

$$A \le A\widetilde{\vee}B$$

Another kind of non-classical logics [4], [5] are substructural logics in which (structural) properties of deducibility such as monotonicity, contraction (absorption) and others are violated. These include relevance logics and connexive logics which find out to justify causal implicative properties. Analyzing the properties of these logics involves clarifying the similarities and analogies of the schemes of these logics with other logics and models such as argumentation logics [2]. One of the prospects for further research is to study the connection of non-classical logics of this kind with the fuzzy models considered in this paper in the framework of causal and spatio-temporal relations of the semantic space.

III. Conclusions

Approaches and models to the interpretation of fuzzy logics are proposed. The proposed models can be used in the interpretation of fuzzy logic formulas on the basis of metric meaning space for finite structures in order to analyze or synthesize schemes of fuzzy logic inference systems relevant to the structures of ontologies of subject areas.

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ИНТЕГРАЦИЯ НЕЧЁТКИХ СИСТЕМ И ИХ ПАРАМЕТРИЧЕСКАЯ ИНТЕРПРЕТАЦИЯ ДЛЯ УНИФИЦИРОВАННОГО ПРЕДСТАВЛЕНИЯ ЗНАНИЙ

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В статье рассматривается проблема устойчивой интерпретации нечётких логических моделей. Предлагается подход на основе параметризованной нечёткой логики, где каждая логическая формула кроме значений истинности имеет набор модельных параметров. Параметризованные нечёткая логика позволяет объединить различные нечёткие логические системы. Модельные параметры используются для вычисления значений нечёткой истинности, как нечёткой меры. Рассмотрены модели и модельные параметры, связанные с метрическими пространствами, согласуемыми с метрическим смысловыми пространствами и являющиеся основой для интерпретации нечётких логических формул на онтологических моделях.

Received 03.04.2024