Theoretical and Mathematical Physics, **219**(1): 539–543 (2024)

ON THE PROPERTIES OF SOLUTIONS OF A SYSTEM OF TWO NONLINEAR DIFFERENTIAL EQUATIONS ASSOCIATED WITH THE JOSEPHSON MODEL

V. V. Tsegelnik*

We investigate the analytic properties of solutions of a system of two first-order nonlinear differential equations with an arbitrary parameter l associated with an overdamped Josephson model. We reduce this system to a system of differential equations that is equivalent to the fifth Painlevé equation with the sets of parameters

$$\left(\frac{(1-l)^2}{8}, -\frac{(1-l)^2}{8}, 0, -2\right),\ \left(\frac{l^2}{8}, -\frac{l^2}{8}, 0, -2\right).$$

We show that the solution of the third Painlevé equation with the parameters $(-2l, 2l-2, 1, -1)$ can be represented as the ratio of two linear fractional transformations of the solutions of the fifth Painlevé equation (with the parameters in the above sequence) connected by a Bäcklund transformation.

Keywords: third Painlevé equation, fifth Painlevé equation, Bäcklund transformation, Josephson model

DOI: 10.1134/S0040577924040020

1. Introduction

In Erugin's work [1], where the properties of solutions of six Painlevé equations (currently regarded as nonlinear analogues of special functions) were studied, several new problems were posed. In particular, the irreducibility of the Painlevé equations was questioned in the sense that not all solutions of these equations are new transcendental functions. Namely, it was suggested that among the Painlevé equations, there are those that can be expressed through each other's solutions. This fact was confirmed in [2], where a connection was established between solutions of the third Painlevé equation

$$w'' = \frac{w'^2}{w} - \frac{w'}{z} + \frac{1}{z}(\alpha w^2 + \beta) + \gamma w^3 + \frac{\delta}{w} \tag{1}$$

with $\gamma\delta \neq 0$ and the solutions of the fifth Painlevé equation

$$w'' = \frac{3w-1}{2w(w-1)}w'^2 - \frac{w'}{z} + \frac{(w-1)^2}{z^2}\left(aw + \frac{b}{w}\right) + \frac{cw}{z} + \frac{dw(w+1)}{w-1} \tag{2}$$

with $d = 0$ and $c \neq 0$. We note that in the general case, α, β, γ, δ and a, b, c, d are arbitrary constant parameters in Eqs. (1) and (2).

In [3], the values of the parameters of Eq. (2) are given (without explicit formulas for the correspondences between solutions) in the case $d \neq 0$; for these values, this equations is reducible to Eq. (1).

*Belarusian State University of Informatics and Radioelectronics, Minsk, Belarus, e-mail: tsegvv@bsuir.by.

Translated from *Teoreticheskaya i Matematicheskaya Fizika*, Vol. 219, No. 1, pp. 12–16, April, 2024. Received November 15, 2023. Revised December 29, 2023. Accepted December 31, 2023.

