ABOUT MULTIFLOW DECOMPOSITION METHODS IN FRACTIONAL LINEAR PROGRAMMING PROBLEMS

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We consider a nonlinear flow programming problem with a nested network constraint structure. An example of system decomposition, algorithms and technologies for solving large sparse linear systems with matrices of incomplete rank are given.

Introduction

Currently, there has been a significant increase in interest in the use of streaming programming. Network and streaming models are used in virtually all scientific, social and economic spheres of human activity.

Network models are used in the analysis of a wide variety of systems, for example: inventory management systems, numerous territorial distribution systems (information, transport, energy).

Mathematical models and problems of network flow programming can be formulated in terms of linear and fractional linear programming. Flow models are suitable for analyzing problems that have a network structure that can be conveniently described using certain parameters of arcs and nodes.

I. Mathematical model

For a multinetwork $S = (I,U)$, we consider the following linear-fractional optimization problem with linear constraints

$$f(x)=\frac{p(x)}{q(x)}=\frac{\sum\limits_{(i,j)\in U}\sum\limits_{k\in K(i,j)} p_{ij}^k x_{ij}^k+\beta}{\sum\limits_{(i,j)\in U}\sum\limits_{k\in K(i,j)} q_{ij}^k x_{ij}^k+\gamma}\longrightarrow \max, \quad (1)$$

$$\sum_{j\in I_i^+(U^k)} x_{ij}^k-\sum_{j\in I_i^-(U^k)} x_{ji}^k=a_i^k, i\in I^k, k\in K;$$

$$\sum_{k\in K_0(i,j)} x_{ij}^k\le d_{ij}^0, (i,j)\in U_0; \quad (2)$$

$$\sum_{(i,j)\in U}\sum_{k\in K(i,j)} \lambda_{ij}^{kp} x_{ij}^k=\alpha_p, p=\overline{1,l};$$

$$x_{ij}^k\ge 0, k\in K_0(i,j), (i,j)\in U_0; \quad (3)$$

$$0\le x_{ij}^k\le d_{ij}^k, \in K_1(i,j), (i,j)\in U;\ x_{ij}^k\ge 0,$$

$$k\in K(i,j)\setminus K_1(i,j), (i,j)\in U\setminus U_0;$$

$$I_i^+(U^k)=\{j\in I^k: (i,j)^k\in U^k\};$$

$$I_i^-(U^k)=\{j\in I^k: (j,i)^k\in U^k\}. \quad (4)$$

Here K ($|K|< \infty$) is a set of different products (types of flow) transported through the multinetwork G. Without loss of generality, let's put $K=\{1,\ldots,|K|\}$. Let us denote the connected network corresponding to a certain type k of flow with $S^k=(I^k,U^k)$, where I^k is the set of nodes and U^k is the set of arcs which are available for the flow of type k, $k\in K$. Also, we define for each node $i\in I$ the set of types of flows $K(i)=\{k\in K: i\in I^k\}$ and for each multiarc $(i,j)\in U$ the set $K(i,j)=\{k\in K: (i,j)^k\in U^k\}$. We assume that the denominator $q(x)$ of the objective function (1) does not change sign on a set of multiflows X, $x\in X$.

We use constructive decomposition theory [1] for constructing solutions of the following sparse linear systems: potencials system, system for appropriate direction of multinetwork change and to calculate the increment of the objective function. The work is devoted to methods, algorithms and technologies for constructing optimal and suboptimal solutions in synthesis with modern innovative technologies of sparse matrix analysis [2], algorithmic graph theory, theoretical computer science. The presented algorithms and computing technologies make it possible to construct solutions to large sparse linear systems with matrices of incomplete rank using parallel computing.

II. Example of linear system decomposition

For a multinetwork $S=(I,U)$, $I=\{1,2,3,4,5\}$, $U=\{(1,2),(2,3), (4,1),(4,2), (4,3),(5,1),(5,2), (5,4)\}$ consider a sparse underdetermined system of linear algebraic equations (5) – (6). Multinetwork S presented as a combination of networks S^k (Fig. 1): $S^k=(I^k,U^k)$, $k\in K=\{1,2,3\}$,

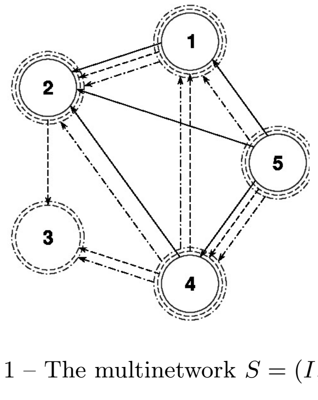

Fig. 1 – The multinetwork $S=(I,U)$

$$I^1=\{1,\ 2,\ 4,\ 5\}, I^2=\{1,\ 2,\ 3,\ 4,\ 5\},$$

$$I^3 = \{1,\ 2,\ 3,\ 4,\ 5\},$$

$$U^1 = \{(1{,}2)^1,(4{,}2)^1,(5{,}1)^1,(5{,}2)^1,(5{,}4)^1\},$$
$$U^2 = \{(1{,}2)^2,\ (2{,}3)^2,\ (4{,}1)^2,\ (4{,}3)^2,\ (5{,}4)^2\},$$
$$U^3 = \{(1{,}2)^3,\ (4{,}1)^3,\ (4{,}2)^3,\ (5{,}1)^3,\ (5{,}4)^3\}.$$

$$x^1_{1,2} - x^1_{5,1} = 5, \quad -x^1_{1,2} - x^1_{4,2} - x^1_{5,2} = -13$$
$$x^1_{4,2} - x^1_{5,4} = -1, \quad x^1_{5,1} + x^1_{5,2} + x^1_{5,4} = 9$$

$$x^2_{1,2} - x^2_{4,1} = -1, \quad x^2_{2,3} - x^2_{1,2} = 7$$
$$-x^2_{2,3} - x^2_{4,3} = -13, \quad x^2_{4,1} + x^2_{4,3} - x^2_{5,4} = -1$$
$$x^2_{5,4} = 8$$

$$x^3_{1,2} - x^3_{4,1} - x^3_{5,1} = -4, \quad -x^3_{1,2} - x^3_{4,2} = -10$$
$$-x^3_{4,3} = -6, \quad -x^3_{4,1} + x^3_{4,2} + x^3_{4,3} - x^3_{5,4} = 12$$
$$x^3_{5,1} + x^3_{5,4} = 8 \qquad (5)$$

$$x^1_{1,2} + 4x^2_{1,2} + 7x^3_{1,2} + 4x^2_{2,3} + 5x^2_{4,1} +$$
$$+4x^3_{4,1} + 3x^1_{4,2} + 6x^3_{4,2} + 6x^2_{4,3} + 2x^3_{4,3} + 9x^1_{5,1} +$$
$$+3x^3_{5,1} + 10x^1_{5,2} + 4x^1_{5,4} + 2x^2_{5,4} + 9x^3_{5,4} = 328$$

$$10x^1_{1,2} + 6x^2_{1,2} + 5x^3_{1,2} + 2x^2_{4,1} + 4x^3_{4,1} +$$
$$+9x^1_{4,2} + 10x^2_{4,3} + 4x^3_{4,3} + 4x^1_{5,1} + 2x^3_{5,1} +$$
$$+4x^1_{5,2} + 7x^1_{5,4} + 7x^2_{5,4} + 10x^3_{5,4} = 412$$

$$5x^1_{1,2} + 8x^2_{1,2} + x^3_{1,2} + 7x^2_{2,3} + 9x^2_{4,1} + 5x^3_{4,1} +$$
$$+2x^3_{4,2} + 6x^2_{4,3} + 5x^3_{4,3} + 4x^1_{5,1} + +x^3_{5,1} +$$
$$+7x^1_{5,2} + 2x^1_{5,4} + 8x^2_{5,4} + 5x^3_{5,4} = 359 \qquad (6)$$

Support $U^k_T \bigcup U^k_C, k \in K = \{1{,}2{,}3\}$ for the multinetwork $S = (I{,}U)$ for the system (5) – (6) [1] is represented on figures 2 – 4, where $U^1_T = \{(1{,}2)^1,\ (4{,}2)^1,\ (5{,}4)^1\}$, $U^2_T = \{(1{,}2)^2,\ (2{,}3)^2,\ (4{,}3)^2,\ (5{,}4)^2\}$, $U^3_T = \{(1{,}2)^3,\ (4{,}2)^3,\ (4{,}3)^3,\ (5{,}4)^3$ – sets of arcs of spanning trees U^1_T, U^2_T, U^3_T of the graphs $S^1 = (I^1{,}U^1)$, $S^2 = (I^2{,}U^2)$, $S^3 = (I^3{,}U^3)$ respectively (marked with bold lines), $U_C = U^1_C \bigcup U^2_C \bigcup U^3_C$ – set of cyclic arcs, $U^1_C = \{(5{,}1)^1{,}(5{,}2)^1\}$, $U^2_C = \{(4{,}1)^2\}$, $U^3_C = \emptyset$.

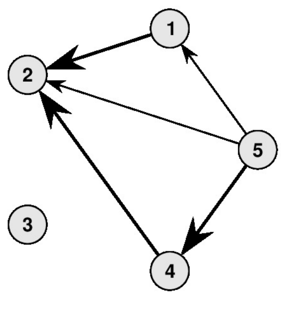

Fig. 2 – Support $U^1_T \bigcup U^1_C$

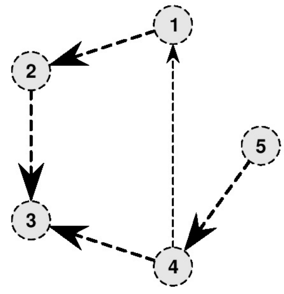

Fig. 3 – Support $U^2_T \bigcup U^2_C$

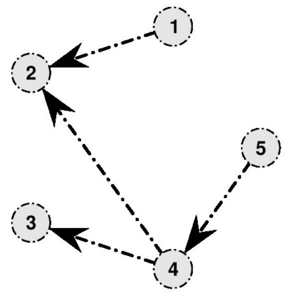

Fig. 4 – Support $U^3_T \bigcup U^3_C$

Construct a general solution to a sparse underdetermined system (5) – (6) relative to the support (Fig. 2 – 4) $U^k_T \bigcup U^k_C, k \in K = \{1{,}2{,}3\}$ of the network $S = (I{,}U)$ for the system (5) – (6).

General solution to sparse underdetermined system (5) – (6) relative to the reference set of arcs $U^k_T \bigcup U^k_C$, $k \in K = \{1{,}2{,}3\}$, which is shown in Fig. 2 – 4, has the form:

$$x^1_{5,2} \to \frac{1}{3{,}4}\left(-449 + 123y^3_{4,1} - 85y^3_{5,1}\right),$$
$$x^1_{5,1} \to \frac{1}{3{,}4}\left(3311 - 889y^3_{4,1} + 697y^3_{5,1}\right),$$
$$x^2_{4,1} \to -\frac{504}{17} + \frac{152y^3_{4,1}}{17} - 7y^3_{5,1},$$
$$x^3_{5,1} \to y^3_{5,1}, \ x^3_{4,1} \to y^3_{4,1},$$
$$x^1_{1,2} \to \frac{1}{3{,}4}\left(3481 - 889y^3_{4,1} + 697y^3_{5,1}\right),$$
$$x^1_{4,2} \to \frac{1}{17}\left(-1295 + 383y^3_{4,1} - 306y^3_{5,1}\right),$$
$$x^1_{5,4} \to \frac{1}{1{,}7}\left(383y^3_{4,1} - 18\left(71 + 17y^3_{5,1}\right)\right)$$
$$x^2_{1,2} \to -\frac{521}{17} + \frac{152y^3_{4,1}}{17} - 7y^3_{5,1},$$
$$x^2_{2,3} \to -\frac{402}{17} + \frac{152y^3_{4,1}}{17} - 7y^3_{5,1},$$
$$x^2_{4,3} \to \frac{623}{17} - \frac{152y^3_{4,1}}{17} + 7y^3_{5,1}, x^2_{5,4} \to 8,$$
$$x^3_{1,2} \to -4 + y^3_{4,1} + y^3_{5,1},$$
$$x^3_{4,2} \to 14 - y^3_{4,1} - y^3_{5,1},$$
$$x^3_{4,3} \to 6, x^3_{5.4} \to 8 - y^3_{5,1}.$$

III. References

1. Pilipchuk L. A. Linear-fractional extremal inhomogeneous problems network flow programming. – Minsk : BSU, 2013. – 235 p. (in Russian).
2. Pilipchuk L.A. Sparse Linear Systems and Their Applications. – Minsk : BSU, 2013. – 235 p.