ABOUT SUBOPTIMAL SOLUTIONS TO THE PROBLEM OF IDENTIFYING OF SPECIAL PROGRAMMABLE DEVICES (SENSORS) AND FLOWS CONTROL

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We are considering the current applied problem of constructing suboptimal solutions to the estimating a homogeneous flow in a bidirectional network. We give an example of constructing a suboptimal solution when installing special programmable devices (sensors) in nodes with variable external flow.

Introduction

We consider a problem of constructing the suboptimal solution for the bidirectional network. We guarantee that the network is fully monitored if the sensor location if special programmable devices (sensors) are installed in the nodes with unknown external flow.

I. Suboptimal solution

For the finite connected bidirectional graph (network) [1] $G = (I,U)$ we represent the traffic flow as follows:

$$\sum_{j\in I_i^+(U)} x_{ij} - \sum_{j\in I_i^-(U)} x_{ji} = \begin{cases} x_i, i \in I^* \\ 0, i \in I\backslash I^* \end{cases} \quad (1)$$

where $I_i^+(U) = \{j \in I{:}(i,j) \in U\}$, $I_i^-(U) = \{j \in I{:}(j,i) \in U\}$, x_{ij} – arc flow, x_i – external flow.

For the external flow the following condition holds: $\sum_{i\in I^*} x_i = 0$. Suppose that for each arc $(i,j) \in U$ one knows the fraction $p_{ij} \in (0, 1]$ of the total flow $\sum_{i\in I^+(U)} x_{ij}$ going out of the node $i \in I$.

As a result of installing sensors in nodes with variable external flow, the system obtained after collecting information about flows has a unique solution. To collect information about the flow function of the form (1), the set of M controlled nodes coincides with the set of I^* nodes with non-zero external flow [1]. As a result, the following information is received from the installed sensors:

$$x_{ij} = f_{ij}, j \in I_i^+(U);\ \ x_{ji} = f_{ji}, j \in I_i^-(U); \\ x_i = f_i, i \in M = I^* \quad (2)$$

We substitute the known values of the variables (2) into the system of equations (1) and remove the arcs of the set H_1 and the monitored nodes $M = I^*$ from the graph G. The graph $G' = (I',U')$ is obtained after removing nodes and arcs with known values (2) from graph G.

Let $CS(M)$ be the cut-set of the cut of the graph (network) G from the source set $M = I^\star$ to the sink set $I\backslash M$. Let $I(CS(M))$ be the set of nodes incident to the edges of $CS(M)$. We form the sets: $M^+ = I(CS(M))\backslash M, M^\star = M \cup M^+$ and $I\backslash M^\star$.

We consider the nodes of the set $i \in M^+ = I(CS(M))\backslash M$. If the conditions $|I_i^+(U)| > 1$ are met and there is an arc (i,v_i) with the known flow $f_{i,v_i} \in U$ going out of the node $i \in M^+$ of the graph G then the unmonitored outgoing flow $x_{ij}, (i,j) \in U'$ for G' can be expressed for the node $i \in M^+$ as follows:

$$x_{ij} = \frac{p_{i,j}}{p_{i,v_i}} f_{i,v_i}, i \in I\backslash M^*, j \in I_i^+(\bar{U}), \\ v_i \in I, |I_i^+(U)| > 1. \quad (3)$$

Let us substitute (3) into the system of equations for the graph G'. Let's continue removing the arcs with known flows (3) from the graph $G' = (I',U')$. We have the resulting graph $\bar{G} = (\bar{I},\bar{U})$ (the unmonitored part of the graph G). The flows $x_{ij}, (i,j) \in \bar{U}$ on the arcs coming from the nodes $I\backslash M^*$ are unknown. So we form the additional equations of type (3) if the conditions $i \in I\backslash M^*, j \in I_i^+(\bar{U}), v_i \in \bar{I}, |I_i^+(\bar{U}) > 1|$ are met. Then the system of equations to get the unknown flows $x_{ij}, (i,j) \in \bar{U}$ has the form

$$\sum_{j\in I_i^+(U)} x_{ij} - \sum_{j\in I_i^-(U)} x_{ji} = a_i, i \in \bar{I}, \quad (4)$$

$$x_{ij} = \frac{p_{i,j}}{p_{i,v_i}} x_{i,v_i}, i \in I\backslash M^*, \\ j \in I_i^+(\bar{U}), v_i \in \bar{I}, |I_i^+(\bar{U})| > 1. \quad (5)$$

where $a_i, i \in \bar{I}$ are constants obtained from the system (1) using the a priori information (2)–(3). In the case of additional equations (5) we apply the constructive theory of decomposition [1] to solve the system (4)–(5). We can not form additional equations (5) if the following conditions are not met: $i \in I\backslash M^*, j \in I_i^+(\bar{U}), v_i \in \bar{I}, |I_i^+(\bar{U})| > 1$. In the absence of additional equations (5) we use the effective algorithms for solving a sparse system (4) with a graph incidence matrix [1].

II. Example

Figure 1 shows the finite connected bidirectional graph $G = (I,U)$, $I = \{1,2,3,4,5,6,7,8\}$, $U = \{(1,2),(1,5),(2,1),(2,6),(3,4),(3,6),(4,3),(4,7),(4,8),(5,1),(6,2),(6,3),(6,7),(7,4),(7,6),(8,4)\}$ and the set of nodes with non-zero external flow $I^* = \{2,3,6,7\}$.

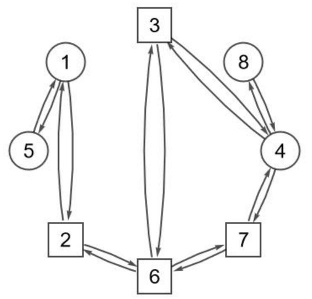

Fig. 1 – Graph G

For the given graph G (Fig. 1) the system of equations of kind (1) has the form

$$\begin{array}{c} x_{1,2}+x_{1,5}-x_{2,1}-x_{5,1}=0 \\ x_{2,1}+x_{2,6}-x_{1,2}-x_{6,2}=x_2 \\ x_{3,4}+x_{3,6}-x_{4,3}-x_{6,3}=x_3 \\ x_{4,3}+x_{4,7}+x_{4,8}-x_{3,4}-x_{7,4}-x_{8,4}=0 \\ x_{5,1}-x_{1,5}=0 \\ x_{6,2}+x_{6,3}+x_{6,7}-x_{2,6}-x_{3,6}-x_{7,6}=x_6 \\ x_{7,4}+x_{7,6}-x_{4,7}-x_{6,7}=x_7 \\ x_{8,4}-x_{4,8}=0 \end{array} \quad (6)$$

We will build the suboptimal solution of the sensor location problem for the graph G. Suppose the set of monitored nodes for the biderectional graph G shown in Figure 1 is $M = I^* = \{2,3,6,7\}$.

Consider the cut of the graph (network) G with the source set $M = I^*$ (and the sink set $I \setminus M = I \setminus I^*$). Let $CS(M)$ be the cut-set (of arcs) and $I(CS(M))$ be the set of nodes incident to the edges of $CS(M)$.

In sensor location problem (SLP) the flows $x_{i,j} = f_{i,j}$ on every incoming and outgoing arcs. For each node $M = I^* = \{2,3,6,7\}$ (is the set of monitored nodes) are known as well as the external flows $x_i = f_i$, $i \in M \bigcap I^* = I^* = \{2,3,6,7\}$: We substitute the known values of the variables into the system of equations (6) and remove the arcs H_1 and the monitored nodes $M = I^* = \{2,3,6,7\}$ from the graph G. The resulting graph G' is shown in Figure 2. Graph $\overline{G}$ – the unobservable part of the graph G is represented in Figure 3.

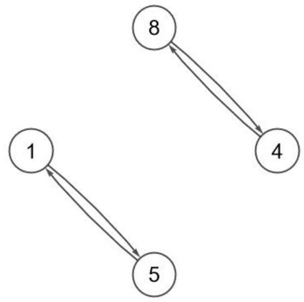

Fig. 2 – Graph G'

If the following conditions are executed: $\left|I_i^+(U)\right| > 1$ and there is an arc (i,v_i) with a known outgoing flow $f_{i,v_i} \in U$ for node $i \in M^+$ of the graph G then unobserved outgoing flow x_{ij},$(i,j) \in U$' for graph G' can be expressed for node $i \in M^+$ through any known flow $f_{i,v_i} \in U$ as follows: $x_{ij} = \dfrac{p_{i,j}}{p_{i,v_i}} f_{i,v_i}$, $j \in I_i^+(U')$.

For the node $i \in M^+ = I(CS(M)) \setminus M$ unobserved outgoing flow x_{ij},$(i,j) \in U$' for graph G' can be expressed by through known observed outgoing flow. If the following conditions are executed: $\left|I_i^+(U)\right| > 1$. $f_{1,2} \in U$ and $f_{4,3} \in U$ as follows:

$$x_{1,5} = \frac{p_{1,5}}{p_{1,2}} f_{1,2},\; x_{4,8} = \frac{p_{4,8}}{p_{4,3}} f_{4,3}. \quad (7)$$

Note that in example 1 the conditions $\left|I_i^+(U)\right| > 1$ for the nodes $i \in I \setminus M^* = \{5,8\}$ are not met: $\left|I_5^+(U)\right| = 1$ and $\left|I_8^+(U)\right| = 1$.

We substitute the known values of the flows (7) along arcs of the set H_2 into the system of equations (6). Let's continue the removal process for the arcs of the set H_2 from the graph G'. The resulting graph $\overline{G} = (\overline{I},\overline{U})$ (the unobservable part of the graph G) is shown in Figure 3.

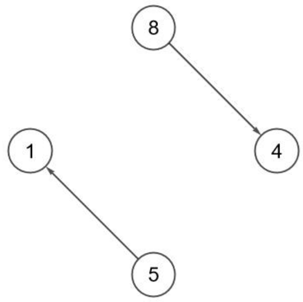

Fig. 3 – Graph $\overline{G}$

We assume that the sensors are ideal, i.e. the data received from the sensors are accurate. As a result of collecting information from sensors, a system with unknown arc flows for the graph $\overline{G}$ will be obtained.

$$\begin{array}{c} x_{5,1} = f_{1,2} - f_{2,1} + \dfrac{p_{1,5}}{p_{1,2}} f_{1,2} \\ x_{8,4} = f_{4,3} + f_{4,7} - f_{3,4} - f_{7,4} + \dfrac{p_{4,8}}{p_{4,3}} f_{4,3} \end{array} \quad (8)$$

For numerical solution of systems with the graph incidence matrix we use effective methods and technologies of sparse analysis.

III. References

1. Pilipchuk L. A., Ramanouski Y. V. Sensor location problem for the bidirectional graph: optimal solutions. Web Programming and Internet Technologies (WebConf2024): materials of the 6th International. scientific-practical conf., Minsk, May 15–16, 2024 / Belarus. State University; Editorial Board: I.M. Galkin (chief editor) [and others]. – Minsk: BSU, 2024. – P. 213–218