

# Superspace without torsion and the composite fundamental fermions

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## Abstract

( $N=2$ )-superspace without torsion is described, which is equivalent to an 8-space with a discrete internal subspace. A number and a character of ties determine now an internal symmetry group, while in the supersymmetrical models this one is determined by an extension degree  $N$ . Such a model can be constructed for no less than 4 generations of the two-component fundamental fermions. Analogues of the Higgs fields appear in the model naturally after transition to the Grassmannian extra coordinates. The connection between discrete and continuous internal symmetries of the model is discussed. If one considers gravity as embedding the curved 4-space into a 12-dimensional flat space, a  $U(1)$ -symmetry appears transformations of which should be connected with the ones of  $SU(2)$ -group. If super-strong interacting gravitons are constituents of the composite fermions, all this may open us another way to unify the known interactions. The main feature of this new approach may be the external see of gravitons underlying an internal structure of particles; the lack of any divergencies would be due the Planckian spectrum of external gravitons.

# 1 Introduction

We speak in different languages to describe the unique nature. To avoid divergencies in the theory one introduces supersymmetry. But any super-partner of existing particles is unknown. To rich the accordance with phenomenology, we introduce postulates about the internal symmetry group - but it is reasonable if the fundamental particles have some structure. I would like to tell here about a possibility to tie a few different approaches in particle physics by means of introduction of a superspace without torsion (see [1]). Of course, there is not any symmetry between fermions and bosons in this case. As it was shown by the author, the discrete internal subspace of the two-component fundamental fermions may be replaced with a super-subspace of the Grassmannian extra coordinates in which a set of supermanifolds has been picked out. An extension with  $N = 2$  as minimum is necessary to provide non-zero values of relative coordinates of the composite fermions. The main goal of the supersymmetrical models - the lack of divergencies - perhaps, may be achieved in another way: if we consider super-strong interacting gravitons as constituents of the composite fermions.

## 2 An internal space of the two-component fermions

Internal coordinates  $y_A^\mu$  of the two-component fermions in the model [1] should obey the condition:

$$y_A^\mu = 0 \vee y_A^\mu = \pm l, \quad A = 2, 3, 4; 6, 7, 8; \quad (1)$$

where  $l$  is a constant of the length dimension. It means that the quark sector of the model is characterized by one length  $l$ . This condition does not affect the values of  $y_A^\mu$  for  $A = 1, 5$ , and one has five constants with such dimension (so as  $y_1^\mu = -y_5^\mu$ ) in the model, four of which concern the lepton sector only.

It was shown by the author that the discrete symmetries of the model, which result from the field equations structure in an 8-space, may be interpreted by an observer in the Minkowski space as the continues internal symmetries [1, 2]. They are connected with mixing of different solutions of field equations distinguished with internal coordinate values only. This algebraic structure is such the one that  $SU(3)_c \times SU(2)_l$  is the global symmetry

group of the model with four generations. A permissible multiplet of the group differs from the one taken in the standard model only by an existence of another  $SU(2)_r$ -singlet for states with  $A = 1, 5$  (in the lepton sector) that may be used to provide a non-zero mass of neutrino. After the observation of evidence of non-zero neutrino mass difference by the Super-Kamiokande collaboration [3], my model is more actual than in 1990. It was shown in [1] that transition from the internal coordinates  $y^\mu$  to the Grassmannian ones  $\chi_a, \bar{\chi}_a$ ,  $a = 1, 2, 3, 4$ , leads to the appearance of the effective four-dimensional fields in the model's Lagrangian. The ones are  $SU(2)$ -doublets and analogs of the Higgs fields and may play their part in the mass-splitting mechanism.

The vectorial  $y_\mu$  and spinor  $\chi_a, \bar{\chi}_a$  coordinates are connected between themselves:

$$y^\mu = \bar{\chi} \gamma^\mu \chi, \quad (2)$$

$\gamma^\mu$  are the Dirac matrices. One must consider  $\chi_a, \bar{\chi}_a$ , to be independent of the Minkowskian coordinates  $x^\mu$ , then  $\chi_a, \bar{\chi}_a$  may commute or anticommute between themselves. In last case,  $(x, \bar{\chi}, \chi)$  is the  $(N = 2)$ -superspace without torsion. If  $\chi_a, \bar{\chi}_a$  anticommute, the additional condition must take place:

$$R^2 + 8M = 0, \quad (3)$$

where  $R = \bar{\chi} \gamma^5 \chi$ ,  $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$ ,  $M = \bar{\chi}_1 \chi_1 \bar{\chi}_4 \chi_4 + (\bar{\chi}_2 \chi_2 + \bar{\chi}_1 \chi_1) \bar{\chi}_3 \chi_3 + \bar{\chi}_1 \chi_4 \bar{\chi}_2 \chi_3 + \bar{\chi}_3 \chi_2 \bar{\chi}_4 \chi_1$ . The set of ties (2) picks out 8 supermanifolds; you can name them "branes" if you want. They are "compactified" in the very simple and definite manner. But in another language, we deal here with the discrete internal subspace.

In the model's Lagrangian, which must contain terms of expansion on degrees of  $\bar{\chi}, \chi$ , factors by the first degree of  $\chi_A$  should be sets of scalars with respect to the Lorentz group and  $SU(2)$ -doublets, i.e. they will be analogs of the Higgs fields of the standard model.

### 3 Gravitons as possible constituents of the composite fermions

It was also shown by the author [4] that embedding the general relativity 4-space into a flat 12-space gives an alternative model of gravitation with a global  $U(1)$ -symmetry and the discrete  $D_1$ -one. The last one may lead

to the  $SU(2)$ -symmetry of the unified model. It is an exciting puzzle: may these two  $SU(2)$ -symmetries - from [1] and [4] - be identified or not?

The Ricci tensor  $r_{\mu\nu}$  in the model [4] is equal to:

$$r_{\mu\nu} = 2(f^2 - f)\tilde{\Gamma}_{\epsilon[\alpha}^{\alpha}\tilde{\Gamma}_{\nu]\mu}^{\epsilon}, \quad (4)$$

where  $f$  is a free parameter, and  $\tilde{\Gamma}$  is the connection. The parameter  $f$  can have any value, excluding  $f = 0; 1$ . On the manifold  $\Sigma^4$ , the global variations of  $f$  are not observable, and  $U(1)$  will be the global symmetry group of the model. The discrete  $D_1$ -symmetry will take place on  $\Sigma^4$  due to the quadratic dependence of  $r_{\mu\nu}$  on  $f$ . The last symmetry may be transformed into the global  $SU(2)$ -symmetry. But a variation of the parameter  $f$  can lead to a permutation of a pair of solutions which will be transformed by the group  $SU(2)$ . To avoid this dependence of two transformations, one can perform a rotation in the plane  $(f, F)$  (where  $F = f^2 - f$ ) on some angle  $\theta$ . Under an additional condition that one component of the  $SU(2)$ -doublet should be almost massless after breaking of the  $SU(2)$ -symmetry (i.e.  $f \rightarrow 1$  for this component),  $\theta$  has the minimum value  $\theta_{min}$  with  $\sin^2 \theta_{min} = 0.20$ ; here  $\theta_{min}$  is an analog of the Weinberg angle of the standard model.

A quantum mechanism of classical gravity based on an existence of the external sea of super-strong interacting gravitons was described by the author for the Newtonian limit [5]. This mechanism needs graviton pairing and "an atomic structure" of matter for working it. In this approach, the two fundamental constants - Hubble's and Newton's ones - are connected between themselves (for more details about confrontation of this model with observations, see [6]). If this low-energy quantum gravity is adequate to the nature (an effective temperature of the graviton background must be the same as of CMB) then gravitons might be constituents of the composite fermions. The dimensional constant  $D$  of this model is equal to:  $D = 1.124 \cdot 10^{-27} m^2 / eV^2$ , so that for forehead collisions of any two particles with energies  $E$  and  $\epsilon$  we have for the cross-section  $\sigma(E, \epsilon)$ :  $\sigma(E, \epsilon) = D \cdot E \cdot \epsilon$ . For example, by  $E \sim \epsilon \sim 5 keV$ , such the interaction would have the same intensity as the strong interaction. But any divergence, perhaps, would be not possible because of natural smooth cut-offs of the *external* graviton spectrum from both sides.

## 4 Conclusion

It is obvious that there are much more open problems here than solved ones but some findings may serve as the milestones for future developments. The considered model of the two-component fundamental fermions describes four generations, and an appearance of any particle of the fourth one would be crucial for its progress. The considered idea of underlying discrete symmetries leads us deeper to the machinery of "elementary" particles. The concept of very-low-energy quantum gravity has some support in the discovery of quantum states of ultra-cold neutrons in the Earth's gravitational field by Nesvizhevsky's team [7]: observed energies of levels (it means that their differences, too) have the order of  $10^{-12}$  eV. If the considered quantum mechanism of gravity is realized in the nature, both the general relativity and quantum mechanics should be modified.

## References

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