Discrete Symmetries Underlying Some Continuous Ones: Two Examples From Gravity And Particle Physics

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Abstract

Two examples, not connected at present, from author's papers (Nuovo Cim., 1992, v.105A, p.77 [hep-th/0207210] and GRG, 1999, v.31, p.1431 [gr-qc/0207017]) are considered here in which a physical model has discrete symmetries and additional non-observable coordinates or parameters. Then it is possible to introduce some apparent continuous symmetries of the model for an observer which cannot know values of these additional quantities.

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1 Introduction

In the standard model of particle physics [1], we see a very complecated primary postulate about the kind of continuous internal symmetry group. In addition, multiplets in different generations should be transformed by the interwoven representations of the group. It is obvious that such the complecated postulates are not good for a fundamental theory.

In this paper, the author describes two own examples from particle physics and gravity in which only some kind of discrete symmetry takes place for the initial model equations but one has a possibility to introduce a continuous symmetry on this base. There is a common feature for the both examples: one must have some additional coordinates or parameters to assume the new simple demands on solutions with a discrete symmetry obeying transition to a continuous one.

As the first example, we consider here a model of composite fermions [2]. The second example is an embedding the general relativity 4−space into a flat 12−space that results in an alternative model of gravity [3].

2 Symmetries of the composite fermions model [2]

By linearization of an equation for an energy E of a two-component system

$$E = (m_1^2 + \bar{p}_1^2)^{1/2} + (m_2^2 + \bar{p}_2^2)^{1/2}$$

(and assuming after it that constituents' masses $m_1 = m_2 = 0$; $\bar{p}_1$ and $\bar{p}_2$ are their momentums), one can get the system of linear quantum equations (13)-(16) from [2] for a wave function $\psi(x_1, x_2)$, where x_1 and x_2 are the coordinates of both constituents. We deal with two 8− spaces in the model: (x_1, x_2) and (x, y), where x belonging to the Minkowski space $R^4_{1,3}$ are coordinates of a centre of inertia, and y are internal coordinates. The author has assumed in [2] that y are transformed independently x.

The equations of motion of such the composite system have the following algebraic property: they permit eight different solutions of the kind $G_A\psi(x, y)$ if $\psi(x, y)$ is some solution. Matrices G_A, $A = 1, ..., 8$, set up a representation of the discrete group D_4. If one assumes that transitions between these solutions are induced by transformations of a space (y), it leads that an algebraic structure of field equations puts hard restrictions on this space: the space (y) should be discrete.

There are two possibilities: to have $y \equiv 0$ for all A or to have two isolated sets of solutions when $y \not\equiv 0$. To get the global symmetry group $SU(3)_c \times SU(2)_l$ for the model in the latter case, one must introduce an additional postulate about conservation of the norm of a set of solutions (see

[2], section 7). The internal coordinates y are not observable; namely it leads to the apparent continuous system's symmetry for any observer in the (x) space. Some other features of the standard model on global level are reflected by this model automatically. A minimal set of generations contains 4 generations. A multiplet of any generation contains two $SU(2)-$singlets that gives a possibility to introduce non-zero neutrino masses that is important after their observation by the Super-Kamiokande collaboration [4].

3 Symmetries of the model of gravity in a flat $12-$space [3]

To embed the general relativity curved $4-$space into a flat $12-$space (x, A, B) with flat $4-$sectors $(x), (A), (B)$, one may map $(x) \to (A) \to (B)$ and get the connection $\Gamma = \tilde{\Gamma} + \bar{\Gamma}$ with $\tilde{\Gamma}$ being its tensor part. This connection has a trivial curvature. After it, we may linear deform the connection: $\Gamma \to \gamma = f\tilde{\Gamma} + \bar{\Gamma}$ where f is a scalar parameter. A non-trivial curvature of the connection γ is proportional to $(f^2 - f)$. The last step is to oblige the according Ricci tensor to Einstein's equations to get an alternative model of gravitation in the flat $12-$space [3].

In this case, f is $U(1)-$symmetry's parameter; the parameter $F \equiv f^2 - f$ gives a curvature scale. Two-valueness of the mapping $F \to f$ gives D_1-symmetry which may be transformed into the $SU(2)-$one as in the previous case. But the parameters of the $U(1)-$ and $SU(2)-$transformations will depend from each other in a general case. We have here a very interesting analogy with the standard model: to provide its independence, one should do a rotation on some angle θ in the parameter plane (f, F) [3]. If we take in the mind a necessity to unify the two described models then the minimum value θ_{min} is determined by the demand that one component of the $SU(2)-$doublet should stay massless or near to it. We will have in such the case: $\sin^2 \theta_{min} = 0,20$ from the pure geometrical reasons. θ_{min} is an analog of the Weinberg angle θ_w for which $\sin^2 \theta_w = 0,215$ from an experiment. It is exiting that these values of angles θ_{min} and θ_w are approximate enough. After the rotation, the $SU(2)-$symmetry will be broken if we take gravitation into account. I would like to note here that as it was shown in my paper [5], quantum gravity would be super-strong on small distances of the order

$\sim 10^{-11}$ m. It would mean that one cannot consider it alone on this scale.

4 Conclusion

In the two cases, which are described here, one can use some constructive (algebraic in both situations) features of the model yielding discrete symmetries to get some continuous ones. Today, these two models are not connected between themselves. I think that to do the first step to unify them, one should linearize Eqs. (9) or (11) from [3] which are uniform relative to the connection $\tilde{\Gamma}$. As the second step, I consider now a possibility to assume that super-strong interacting gravitons [5] are constituents of the composite fermions.

References

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